PROCEEDINGS OF SCIENCE

The running coupling in lattice Landau gauge with unquenched Wilson fermion and KS fermion

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The running coupling of the Wilson fermon(JLQCD/CP-PACS) and that of Kogut-Susskind(KS) fermion(MILC) are measured in the lattice Landau gauge QCD in $\widetilde{MOM}$ scheme. The quark propagator of the KS fermion is also measured and we find that it is infrared vanishing, which implies that quarks are confined. The renormalization factor of the running coupling and the tadpole renormalization define the scale of the quark wave function. Effects of the A_μ^2 condensates of a few GeV^2 are observed in the running coupling and also in the quark propagator.

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1. Introduction

The mechanism of dynamical chiral symmetry breaking and confinement is one of the most fundamental problem of hadron physics. The propagator of dynamical quarks in the infrared region provides information on dynamical chiral symmetry breaking and confinement. In the previous paper[1], we measured gluon propagators and ghost propagators of unquenched gauge configurations obtained with quark actions of Wilson fermions (JLQCD/CP-PACS) and those of Kogut-Susskind(KS) fermions (MILC) [2, 3] in Landau gauge and observed that the configurations of the KS fermion are closer to the chiral limit than those of Wilson fermions.

In the analysis of running coupling obtained from the gluon propagator and the ghost propagator, with use of the operator product expansion of the Green function, we observed possible contribution of the quark condensates and A^2 condensates in the configurations of the KS fermion[1]. The quark propagator of quenched KS fermion was already measured in [4], and possible contribution of these condensates are reported. Unquenched KS fermion propagator of $20^3 \times 64$ lattice (MILC$_c$) was measured in [5], but to distinguish the gluon condensates and the quark condensates, it is desirable to measure the quark propagator of larger lattice (MILC$_f$) and to compare with data of MILC$_c$. We measure quark propagator of gauge configuration of 1) MILC$_c$ $20^3 \times 64$, $\beta = 6.76$ and 6.83 and 2) MILC$_f$ $28^3 \times 96$, $\beta = 7.09$ and 7.11, using the Staple+Naik action[6].

2. The running coupling in $\widetilde{MOM}$ scheme

The running coupling in $\widetilde{MOM}$ scheme is given with use of the vertex renormalization factor $\tilde{Z}_1$ as

$$\alpha_s(q) = \alpha_R(\mu^2) Z_R(q^2,\mu^2) G_R(q^2,\mu^2)^2 = \frac{\alpha_0(\Lambda_{UV})}{\tilde{Z}_1(\beta,\mu)^2} Z(q^2,\beta) G(q^2,\beta)^2 \tag{2.1}$$

where Z and G are the gluon and the ghost dressing function, respectively, and $\tilde{Z}_1$ is the vertex renormalization factor.

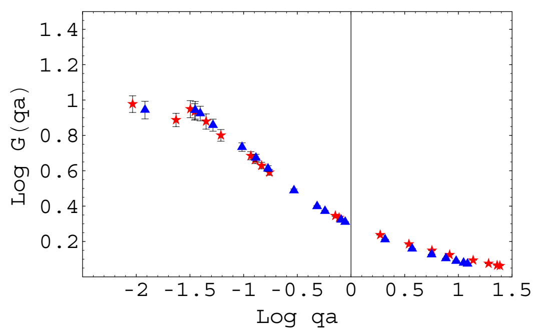

Figure 1: The $\log G(qa)$ as a function of $\log qa$ (a is the lattice spacing of MILC$_f$) of MILC$_f$ $\beta_{imp} = 7.09$(stars) and MILC$_c$ $\beta_{imp} = 6.76$(triangles).

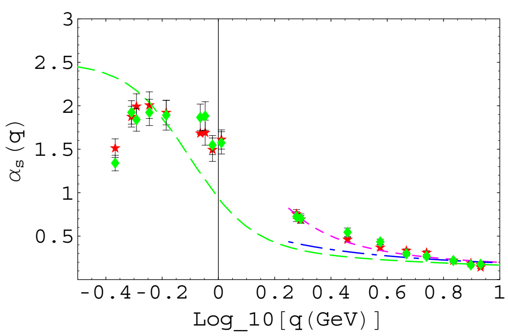

Figure 2: The running coupling $\alpha_s(q)$ as a function of $\log_{10} q$(GeV) of MILC$_f$ $\beta_{imp} = 7.09$(stars) and 7.11(diamonds).

Our data suggests that the gluon propagator is infrared finite as in Dyson-Schwinger equation[9]. The running coupling in the infrared is suppressed as shown in Figure 2,but the main origin is the suppression of the ghost propagator in the infrared.

We parametrize the difference of the lattice data and the pQCD 4-loop result[10, 11] in the 1GeV$< q <$ 6GeV region in the form, with a minor correction term d as

$$\Delta\alpha_s(q) = \alpha_{s,latt}(q) - \alpha_{s,pert}(q) = \frac{c_1}{q^2} + \frac{c_2}{q^4} + d, \tag{2.2}$$

where the A^2 condensates gives c_1 and the gluon condensates and/or quark condensates gives c_2. Although statistics is not large, running coupling of CP-PACS suggests $c_1 \sim 2$GeV. The MILC data suggests $c_1 \sim 4$GeV and $c_2 \sim -2$GeV. There is an analytical calculation that suggests correlation between the A_2 condensates and the the horizon function parameter[12].

3. The quark wave function renormalization

We renormalize the quark field as $\psi_{bare} = \sqrt{Z_2}\psi_R$, and define the colorless vector current vertex[7]

$$\Gamma_\mu(q,p) = S^{-1}(q)G_\mu(q,p)S^{-1}(q+p) \tag{3.1}$$

where

$$G_\mu(p,q) = \int d^4xd^4ye^{ip\cdot y+iq\cdot x}\langle q(y)\bar{q}(x)\gamma_\mu q(x)\bar{q}(0)\rangle \tag{3.2}$$

and $S(q)$ is the quark propagator.

The vertex of the vector current with $p = 0$ is written as

$$\Gamma_\mu(q) = \delta_{a,b}\{g_1(q^2)\gamma_\mu + ig_2(q^2)p_\mu + g_3(q^2)q_\mu \not{q} + ig_4(q^2)[\gamma_\mu, \not{q}]\} \tag{3.3}$$

The Ward identity implies

$$\widetilde{Z_V^{MOM}}\Gamma_\mu(q) = -i\frac{\partial}{\partial q^\mu}S^{-1}(q) \tag{3.4}$$

where $\widetilde{Z_V^{MOM}}g_1(q^2) = Z_\psi(q^2)$ and $\widetilde{Z_V^{MOM}} = 1$ when there is no lattice artefact.

The running coupling g of the ghost, anti-ghost, gluon coupling

$$g(q) = \tilde{Z}_1^{-1}Z_3^{1/2}(\mu^2,q^2)\tilde{Z}_3(\mu^2,q^2) \tag{3.5}$$

and that of quark, gluon coupling

$$g(q) = Z_1^{-1}Z_3^{1/2}(\mu^2,q^2)Z_2(\mu^2,q^2) \tag{3.6}$$

are identical due to the Slavnov-Taylor identity. At the renormalization point $q = \mu$, we fix $Z_2(\mu^2,\mu^2) = 1$ and $\tilde{Z}_3(\mu^2,\mu^2) = 1$, and so $\tilde{Z}_1^{-1}Z_3^{1/2}\tilde{Z}_3 = Z_1^{-1}Z_3^{1/2}Z_2$ implies $\tilde{Z}_1 = Z_1$. On the other hand $Z_\psi(q^2) = \widetilde{Z_V^{MOM}}g_1(q^2)$ where for 16 tastes

$$g_1(q^2) = \frac{1}{48N_c}Tr[\Gamma_\mu(q,p=0)(\gamma_\mu - q_\mu\frac{\not{q}}{q^2})] \tag{3.7}$$

When $\widetilde{Z_V^{MOM}} = 1$, $g_1(\mu^2)$ is identical to $\tilde{Z}_1$ which is defined by the renormalization of the running coupling on the lattice defined at $\mu \sim 6$GeV and summarized in Table 1.

	β_1/K_{sea1}	β_2/K_{sea2}	average	configurations
CP-PACS	1.07(8)	1.21(10)	1.14	$K_{sea1,2} = 0.1357, 0.1382$
MILC$_c$	1.49(11)	1.43(10)	1.46	$\beta_{1,2} = 6.83, 6.76$
MILC$_f$	1.37(9)	1.41(12)	1.40	$\beta_{1,2} = 7.11, 7.09$

Table 1: The $1/\tilde{Z}_1^2$ factor of the unquenched SU(3).

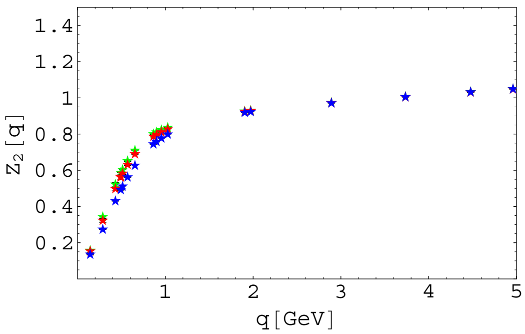

Figure 3: The quark propagator $Z_2(q)$ of MILC$_f$ with bear mass $m_0 = 13.6$MeV(stars), 27.2MeV(diamonds) and 68.0MeV(triangles).

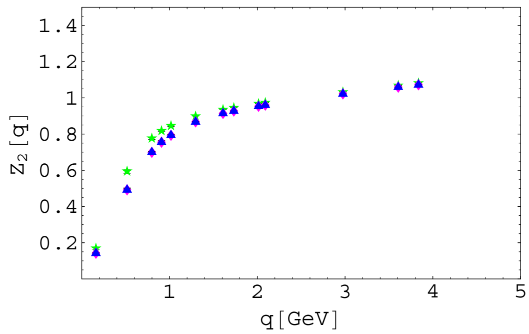

Figure 4: Same as Fig. 3 but quark of MILC$_c$ with bear mass $m_0 = 11.5$MeV(stars), 65.7MeV(diamonds) and 82.2MeV(triangles). The last two almost overlap.

The MILC configurations are produced by the Asqtad action with the bare masses 13.6MeV, 27.2MeV and 68.0MeV in the MILC$_f$, and 11.5MeV, 65.7MeV and 82.2MeV in the MILC$_c$. The gluon wave function renormalization Z_3 of the Asqtad action is renormalized by $1/u_0^2$, where u_0 is the fourth root of the plaquette value. The tadpole renormalization on quark wave function is u_0 and we normalize $S(q)$ by multiplying the average of $u_0\tilde{Z}_1$ i.e. 1/1.36 for MILC$_f$ and 1/1.38 for MILC$_c$. The quark propagators $Z_2(q)$ after this renormalization are infrared vanishing as shown in Figures 3 and 4. The apparent difference in the formulae of [5] and our work are only in the expression and in fact they are equivalent. However, [5] does not find this behavior of $Z_2(q)$.

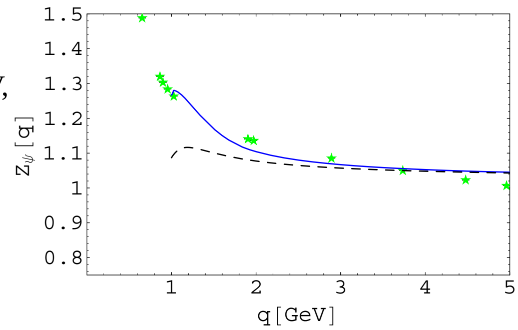

Figure 5: The quark inverse propagator $Z_\psi(q)$ of MILC$_f$ with bear mass $m_0 = 13.6$MeV. Dashed line is the pQCD result and solid line is the result including the A_μ^2 condensates.

Using the pQCD result of the inverse quark propagator[10, 11] and $\langle A^2\rangle$ and $\bar{c}_2$ (the contribution from the mixed condensate $\langle \bar{q}\not{A}q\rangle$[8]) as fitting parameters, we calculate

$$Z_\psi(q) = \frac{1}{Z_2(q)} = Z_\psi^{pert}(q^2) + \frac{\left(\frac{\alpha(\mu)}{\alpha(q)}\right)^{\frac{-\gamma_0+\gamma_{A^2}}{\beta_0}}}{q^2} \frac{\langle A^2(\mu)\rangle}{4(N_c^2-1)} Z_\psi{}^{pert}(\mu^2) + \frac{\bar{c}_2}{q^4} \tag{3.8}$$

where $\alpha(q)$ are data calculated in the $\widetilde{MOM}$ scheme using the same MILC$_f$ gauge configurations[1].

We estimate that $\bar{c}_2$ is small. In Figure 5, we show a fit of the MILC$_f$ $m_0 = 13.6$MeV data, by taking the renormalization point at $\mu \sim 3.8$GeV with use of $\langle A^2(\mu)\rangle \sim 3.2\text{GeV}^2$ and $\bar{c}_2 = 0$. These parameters are consistent with [7].

In $q > 1.5GeV$ region, dynamical mass of a quark in pQCD is expressed as[4]

$$M(q) = -\frac{4\pi^2 d_M \langle\bar{q}q\rangle_\mu [\log(q^2/\Lambda_{QCD}^2)]^{d_M-1}}{3q^2[\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}} + \frac{m(\mu^2)[\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}}{[\log(q^2/\Lambda_{QCD}^2)]^{d_M}}, \quad (3.9)$$

where $d_M = \frac{12}{33-2N_f}$. The second term is the contribution of the massive quark. In the analysis of the lattice data, we observe that the quark condensates $-\langle\bar{q}q\rangle_\mu$ and Λ_{QCD} roughly satisfy $-\langle\bar{q}q\rangle_\mu = (0.70\Lambda_{QCD})^3$[9], with $\Lambda_{QCD} = 0.69$GeV.

For the global fit of $M(q)$, we try the phenomenological monopole type[13]

$$M(q) = \frac{c\Lambda^3}{q^2+\Lambda^2} + m_0 \quad (3.10)$$

where m_0 is the bare quark mass.

Figures 6 and 7 show the mass function $M(0)$ of MILC$_f$ and MILC$_c$, respectively.

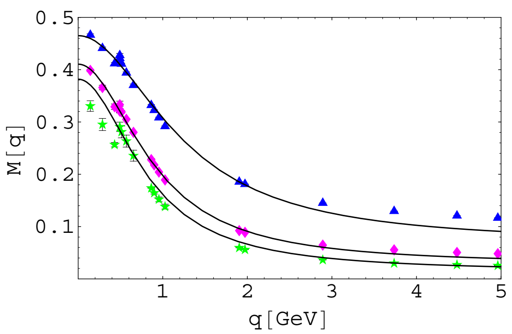

Figure 6: The dynamical mass of the MILC$_f$ quark with bear mass $m_0 = 13.6$MeV(stars), 27.2MeV(diamonds) and 68.0MeV(triangles) and the phenomenological fits.

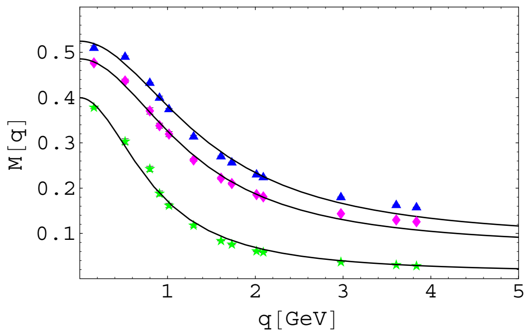

Figure 7: The dynamical mass of the MILC$_c$ quark with bear mass $m_0 = 11.5$MeV(stars), 65.7MeV(diamonds) and 82.2MeV(triangles) and the phenomenological fits.

We observe that the product $c\Lambda$ becomes larger as the bare quark mass becomes heavy and it depends on β in the case of MILC$_f$ but not in the case of MILC$_c$. In the case of MILC$_f$ $m_0 = 13.6$MeV, the lowest three momentum points of $M(q)$ are systematically smaller than the other points. Ignoring these points we find $c\Lambda$ of $\beta = 7.09$ is smaller than that of 7.11 and that of MILC$_c$ as shown in Figure 8. In the chiral limit $m_0 \to 0$, we obtain $M(0) = 0.35 \sim 0.37$GeV, which is larger than [5] and that of the Wilson fermion[13] by about 20%.

4. Discussion and conclusion

We measured running coupling of unquenched Wilson fermion and KS fermion and the quark wave function renormalization factor and mass function of the KS fermion. We observe that the quark propagator vanishes in the infrared, which implies that quark is confined. We fixed the scale of the quark propagator using the renormalization factor of the running coupling and that of the tadpole.

A preliminary study of replacing the Staple+Naik action by the Asqtad action suggests that the $M(q)$ and $Z_2(q)$ of the two agree within 10%.

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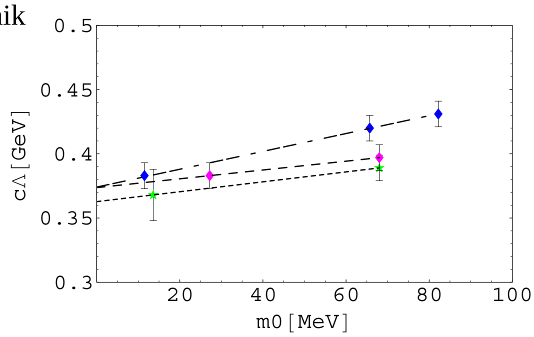

Figure 8: The mass function $c\Lambda$ as a function of bare mass and its chiral limit. Dotted line is the extrapolation of MILC$_f$ $\beta = 7.09$, dashed line is $\beta = 7.11$ and the dash-dotted line is that of MILC$_c$.

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