

An Abelian two-Higgs model and high temperature superconductivity

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We study a three dimensional Abelian Higgs model containing singly- and doubly-charged scalar fields coupled to a compact Abelian gauge field in the London limit. The model attracts interest because of its relevance to high- T_c superconductors with charge 1 holon and charge 2 spinon-pair fields. It contains two types of vortices carrying magnetic flux and one type of instanton-like monopoles. Using thermodynamic and topological observables we present the phase diagram in the parameter space of the gauge and holon and spinon-pair couplings. The Fermi liquid, the spin gap, the superconductor and the strange metallic phases have been identified in a wide region of parameters. The model may serve as a toy system modelling non-perturbative properties of the Yang-Mills theory.

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1. Introduction

The physics of high- T_c superconductivity [1] is not understood yet. At normal temperatures, all known high- T_c superconductors are ceramic crystals characterized by a poor conductivity. At low temperatures the clean ceramic materials are rather insulators than conductors (Mott insulators). However, as one adds impurities to the clean material (“doping”), a good insulator becomes in certain cases a superconductor at low enough temperatures.

Despite superconducting specimen are three-dimensional structures the physics of high- T_c superconductivity is believed to be essentially two-dimensional [1]. In fact, all known high- T_c superconductors consist of copper oxygen (CuO_2) lattice planes. In the undoped state the crystal does not contain enough free carriers of electric charge. The role of impurities is to provide the carriers – electrons (like in $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ case) or holes (like in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ material) – into the CuO_2 planes, eventually making the CuO_2 planes superconducting 2D systems. Therefore, the physics of high- T_c superconductivity must be understood as an essentially 2D phenomenon.

Physically, there are two basic parameters: the temperature T and the concentration of impurities (doping) x . The phase diagram of a real high- T_c superconductor is the temperature-concentration plane and typically contains the following phases: a metallic, a pseudogap (found in hole doped materials), an anti-ferromagnetic and a superconducting phase.

One popular approach to the physics of superconducting planes is based on a simplified model describing the dynamics of holes and spins. Basically we have a lattice with hopping holes and localized spins acting as dynamical variables. The lattice spacing and possible anisotropy is dictated by the ceramic host material itself. The effective description is provided by the $t - J$ Hamiltonian [1]:

$$H_{tJ} = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger (1 - n_{i,-\sigma})(1 - n_{j,-\sigma}) c_{j\sigma} + J \sum_{\langle ij \rangle} (\vec{S}_i \vec{S}_j - \frac{1}{4} n_i n_j), \quad (1.1)$$

where the first term describes the hopping of holes or electrons without changing spin while the second term describes the anti-ferromagnetic Heisenberg coupling between the spins located at the copper-sites. Here $\vec{S}_i = (1/2) \sum_{\sigma\sigma'} c_{i\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{i\sigma'}$ is the spin operator, $c_{i\sigma}$ and $n_{i,\sigma}$ are the electron creation and occupation number operators for given spin σ , respectively, and $n_i = \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma}$.

Even the simplified $t - J$ model (1.1) is difficult to solve. One successful approach is based on the slave boson formulation [2], which proposes to split the spin and charge variables of the electrons. Let the electron creation operators be written as $c_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i$, where $f_{i\sigma}$ is a spin-particle (“spinon”) operator and b_i is a charge-particle (“holon”) operator. In order to forbid double occupancy of sites one imposes the constraint $f_{i\uparrow}^\dagger f_{i\uparrow} + f_{i\downarrow}^\dagger f_{i\downarrow} + b_i^\dagger b_i = 1$ on the physical states of the system.

The spinon is a fermionic chargeless particle which carries information about the spin of the electron while the holon is a bosonic particle which is responsible for the electron charge. The essential feature of the spin-charge separation approach is that it introduces an additional (internal) compact $U(1)$ degree of freedom which can be formulated as a gauge freedom on the operator level:

$$c_{i\sigma} \rightarrow e^{i\gamma_i} c_{i\sigma} : f_{i\sigma} \rightarrow f_{i\sigma}, \quad b_i \rightarrow e^{i\gamma_i} b_i \quad [\text{usual gauge freedom}], \quad (1.2)$$

$$c_{i\sigma} \rightarrow c_{i\sigma} : f_{i\sigma} \rightarrow e^{i\alpha_i} f_{i\sigma}, \quad b_i \rightarrow e^{i\alpha_i} b_i \quad [\text{internal gauge freedom}]. \quad (1.3)$$

2. The compact $U(1)$ gauge model coupled to matter

The emerging effective theory of superconductivity can further be simplified and reformulated as a lattice gauge model [3]. The basic idea is to neglect – or absorb into the couplings of the effective model – the external gauge degree of freedom (1.2) and to concentrate attention to the internal compact gauge degree of freedom (1.3). In fact, the coupling of the usual electromagnetic field to the charge-carrying holon degrees of freedom (as well as the inter-holon interaction due to the electromagnetic interaction) is relatively weak compared to the strong correlations among the electrons in the lattice environment.

Thus, we go over from the $t - J$ model (1.1) to a compact Abelian gauge model with internal symmetry (1.3), which couples holons and spinons. As in usual BCS superconductivity, at certain parameters of the $t - J$ model the spinons couple and form bosonic quasiparticles. In a mean field theory one can define the fields

$$\chi_{ij} = \sum_{\sigma} \langle f_{i\sigma}^{\dagger} f_{j\sigma} \rangle \rightarrow \chi_{ij} \cdot e^{-i(\alpha_i - \alpha_j)}, \quad \Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle \rightarrow \Delta_{ij} \cdot e^{i(\alpha_i + \alpha_j)}, \quad (2.1)$$

which behave under the internal gauge transformations (1.3) as a neutral (vector-like) particle and a doubly-charged matter field, respectively. The phase of the χ -particle is associated with the compact $U(1)$ gauge field, $\theta_{ij} \equiv \arg \chi_{ij} \rightarrow \theta_{ij} - \alpha_i + \alpha_j$. The radial part of the χ -particle defines the so-called “resonating valence bond” (RVB) coupling $\chi = \langle |\chi_{ij}| \rangle$, and the doubly-charged matter field Δ is called spinon-pair field (an analog of the doubly-charged Cooper pairs).

Usually the RVB coupling is treated in the mean-field approximation and therefore is assumed to be fixed, $\chi(x) = \chi \equiv \text{const}$. Thus, the dynamical content of the effective model is given by a singly-charged boson field (holon) b , a doubly-charged boson field (spinon-pair) Δ and the compact $U(1)$ gauge field θ .

At high temperature the RVB coupling is vanishing, $\chi = 0$, and the system is in the Mott insulator (or, poor metallic) phase, see Fig. 1. As the temperature decreases, the RVB coupling is

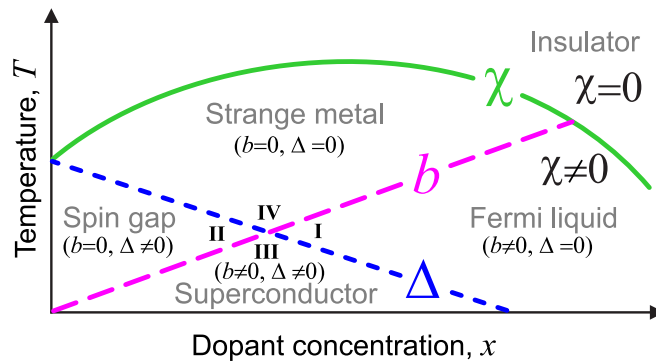


Figure 1: The schematic phase diagram of the $U(1)$ model for a high- T_c superconductor [3].

getting non-zero, $\chi \neq 0$, enabling the formation of the spinon-pair condensate $\Delta = \langle |\Delta_{ij}| \rangle$ and/or of the holon condensate $b = \langle b_i \rangle$. Depending on the presence of the condensates, four phases classified in Ref. [3] may emerge. In Fig. 1 they are sketched and denoted as the Fermi liquid, the spin gap, the superconductor, and the strange metallic phases.

3. The compact Abelian two-Higgs model on the lattice

Following the proposal in Ref. [3] the model described in Section 2 can be studied as compact Abelian two-Higgs model (cA2HM) in *three* dimensions with a $U(1)$ gauge link field θ_l , a single-charged holon field Φ_1 , and a double-charged spinon-pair field Φ_2 . The qualitative characteristics of the model become already visible in the London limit, in which the radial parts of both Higgs fields $\Phi_i = |\Phi_i| e^{i\varphi_i}$ are frozen, $|\Phi_i| = \text{const}$, $i = 1, 2$. The remaining dynamical Higgs variables are the phases of the matter fields $\varphi_{1,2}$. The action of the cA2HM model in the London limit is:

$$S[\theta, \varphi_1, \varphi_2] = -\beta \sum_P \cos \theta_P - \kappa_1 \sum_l \cos(d\varphi_1 + \theta)_l - \kappa_2 \sum_l \cos(d\varphi_2 + 2\theta)_l, \quad (3.1)$$

where θ_P is the standard lattice plaquette field, d denotes the ordinary lattice derivative, $(d\varphi)_{x,\mu} = \varphi_{x+\hat{\mu}} - \varphi_x$. The parameter β is the inverse gauge coupling, $\kappa_1 \propto t \cdot x$ and $\kappa_2 \propto J$ are the hopping parameters of the holon and the spinon-pair field, respectively. They are proportional to the parameters of $t - J$ model. The model (3.1) obeys the $U(1)$ gauge invariance:

$$\theta \rightarrow \theta + d\alpha, \quad \varphi_1 \rightarrow \varphi_1 - \alpha, \quad \varphi_2 \rightarrow \varphi_2 - 2\alpha, \quad (3.2)$$

which is a direct counterpart of the internal gauge symmetry (1.3).

In the limit $\kappa_2 \rightarrow 0$ one gets the compact $U(1)$ gauge model with a charge-1 Higgs field only, the holon. The phase diagram of the $Q = 1$ compact Abelian Higgs model contains [4, 5] two regions: the confinement “phase” (characterized by a low holon condensate b) at small hopping parameter κ_1) and the Higgs “phase” at large κ_1 (where the condensate b becomes large). Both regions are analytically connected for strong enough Higgs selfcoupling, although a Kertész line [6] *physically* separates them also there. A *qualitatively* similar picture (however with a true second order phase transition) is realized [7] in the limit $\kappa_1 \rightarrow 0$ where one gets the $Q = 2$ compact Abelian Higgs model in which the spinon-pair field plays the role of the sole Higgs field. Thus, we expect that the phase diagram of the model (3.1) should contain all four phases depicted in Fig. 1 in the $\chi > 0$ region.

Below we report our Monte Carlo investigation of the cA2HM which is still under way. To scan the phase diagram we have simulated on a 16^3 lattice, choosing three values of the gauge couplings, $\beta = 1.0, 1.5$ and 2.0 , for a huge grid of $\kappa_{1,2}$ hopping parameter pairs covering the range¹ $0 < \kappa_{1,2} \leq 2.5$. We are particularly interested in the strongly coupled case, $\beta \simeq 1$.

The compactness of the gauge field guarantees the presence of instanton-like magnetic monopoles. An elementary monopole is a source of $F^{\text{mon}} = 2\pi$ units of magnetic flux associated with the internal gauge field θ . Due to the matter fields two types of topologically stable vortices exist. The magnetic flux quanta of the vortices corresponding to the holon and the spinon-pair “Higgs” fields are $F_1^{\text{vort}} = 2\pi$ and $F_2^{\text{vort}} = \pi$, respectively. Since the magnetic flux is conserved, one monopole is simultaneously a source of one holon vortex and two spinon-pair vortices.

The densities of the three topological defects are logarithmically plotted in Fig. 2 over the (κ_1, κ_2) plane for a strong gauge coupling $\beta = 1$. With increasing hopping parameters the monopole density gets suppressed. The density of the holon (spinon-pair) vortex becomes suppressed with increasing holon (spinon-pair) hopping parameter κ_1 (κ_2), beyond a line in the κ_1 - κ_2 plane.

¹The actually relevant region of the κ_1 - κ_2 plane depends on β .

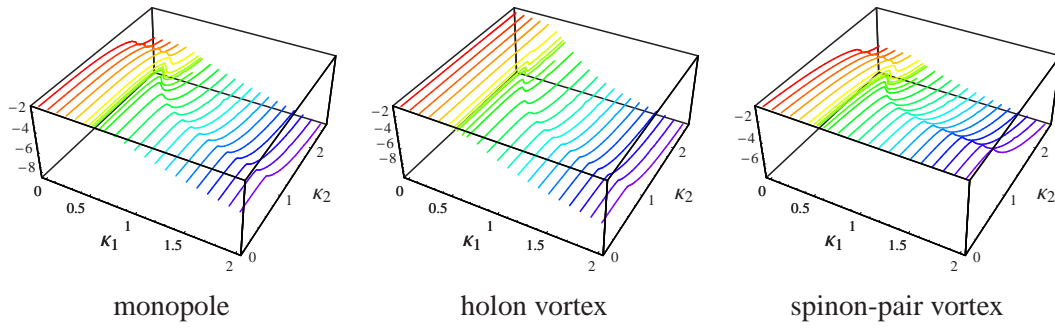


Figure 2: The densities of the topological defects over the (κ_1, κ_2) plane at $\beta = 1$.

The connectivity of the vortex clusters gives a clear view of the phase diagram of the model [5, 7, 6]. A Higgs condensate suppresses the proliferation of the corresponding vortices: they are prevented to percolate over infinitely long distances. We show in Fig. 3 the percolation probabilities

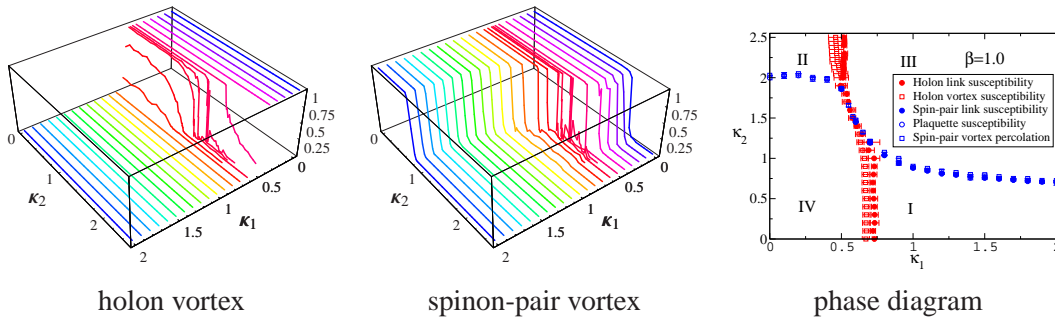


Figure 3: The percolation probability of the infrared vortices and the phase diagram of the system at $\beta = 1$.

dropping to zero (extracted from the cluster correlation functions) and the corresponding phase diagram (classified according to Ref. [3], see Fig. 1). To identify the phases, we measured the average plaquette and both link contributions of the action and their susceptibilities (3.1), respectively. Some examples are shown in Fig 4. From a preliminary finite size analysis from 12^3 to 32^3

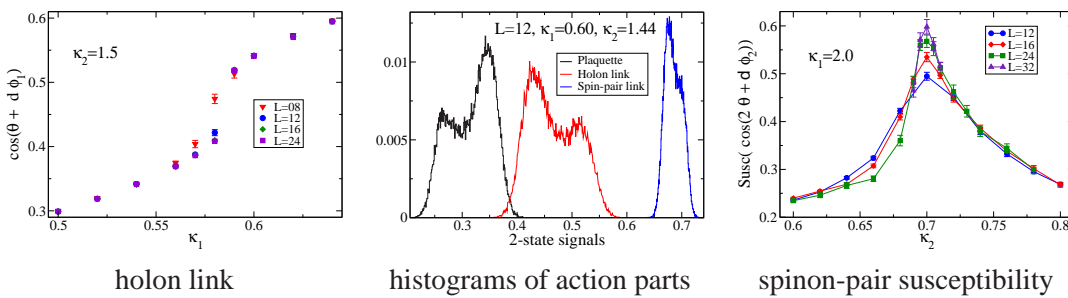


Figure 4: Examples of thermodynamic behaviour at various κ_1, κ_2 and $\beta = 1$.

we found: i) Hint for a first order transition between III and IV in the crossing region of the two percolation lines at strong gauge coupling, ii) No transition along the red (vertical) line for small κ_2 in agreement with the limit $\kappa_2 \rightarrow 0$, iii) Signals for thermodynamic transitions along the remaining transitions lines (second order along the horizontal parts of the blue line).

4. Summary

We have observed two transitions associated with the patterns of vortex percolation in the Abelian two-Higgs model of high- T_c superconductivity. These transition lines are roughly parallel to the corresponding hopping parameter axes κ_i , almost perfectly parallel at weak gauge coupling ($\beta = 2$). The crossing in some region of finite (κ_1, κ_2) gets new features at strong coupling $\beta = 1$. The Fermi liquid, the spin gap, the superconductor and the strange metallic phases have been identified in a wide region of parameters of this model. The percolation transitions are accompanied with ordinary phase transitions except at small κ_1 below the crossing region of the two transition lines. First hints for a changing order along the thermodynamic transition lines are found, and the joint transition in the crossing region seems to be first order. Further studies are underway to fortify our findings.

More complex Higgs sector will be necessary to build more realistic effective models reproducing the confinement mechanism of gluodynamics in the sense of Ref. [7]. The spin-charge separation idea of the high- T_c superconductivity should further be extended to gluodynamics to reformulate it in terms of condensed matter systems [8].

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