

Color Confinement in lattice Landau gauge with unquenched Wilson and KS fermions

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The Kugo-Ojima confinement criterion is verified in the unquenched Landau gauge QCD simulation. The valence quark propagator of the Kogut-Susskind fermion with use of the fermion action including the Naik term and the staple contribution is calculated on MILC Asqtad unquenched gauge configurations, and it shows clear evidence of infrared vanishing of the quark propagator.

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1. Introduction

In the lattice Landau gauge QCD simulation, we adopt two types of the gauge field definitions[7]:

$$\text{log } U \text{ type: } U_{x,\mu} = e^{A_{x,\mu}}, \quad A_{x,\mu}^\dagger = -A_{x,\mu},$$

$$U \text{ linear type: } A_{x,\mu} = \frac{1}{2}(U_{x,\mu} - U_{x,\mu}^\dagger)|_{\text{trlp.}},$$

where trlp. implies traceless part. ($A_\mu(x) = i \sum_a A_\mu^a(x) \frac{\Lambda^a}{\sqrt{2}}$, $\text{tr} \Lambda^a \Lambda^b = \delta^{ab}$)

The corresponding optimizing functions are

$$\text{log } U \text{ type: } F_U(g) = \|A^g\|^2 = \sum_{x,\mu} \text{tr} (A_{x,\mu}^g \dagger A_{x,\mu}^g),$$

$$U \text{ linear type: } F_U(g) = \sum_{x,\mu} \text{tr} \left(2 - (U_{x,\mu}^g + U_{x,\mu}^{g\dagger}) \right).$$

Under infinitesimal gauge transformation $g^{-1} \delta g = \varepsilon$, its variation reads for either definition as

$$\Delta F_U(g) = -2 \langle \partial A^g | \varepsilon \rangle + \langle \varepsilon | -\partial D(U^g) | \varepsilon \rangle + \dots,$$

where the covariant derivative $D_\mu(U)$ for two options reads commonly as

$$D_\mu(U_{x,\mu}) \phi = S(U_{x,\mu}) \partial_\mu \phi + [A_{x,\mu}, \bar{\phi}]$$

where $\partial_\mu \phi = \phi(x+\mu) - \phi(x)$, and $\bar{\phi} = \frac{\phi(x+\mu) + \phi(x)}{2}$. Stationarity of the optimizing function implies Landau gauge, the local minimum implies Gribov Region[2] and the global minimum implies Fundamental modular(FM) region[3].

We performed simulations on quenched configurations tabulated in Table 1[9, 10] and unquenched configurations of JLQCD[13], CP-PACS[14], Columbia University[15] and MILC[16] tabulated in Table 2[11].

β	$1/a(\text{GeV})$	L	$aL(\text{fm})$	definition of A
6	1.97	16	1.60	U-linear/ log U
		24	2.40	U-linear/ log U
		32	3.20	U-linear/ log U
6.4	3.66	32	1.72	U-linear/ log U
		48	2.59	U-linear/ log U
		56	3.02	U-linear/ log U
6.45	3.87	56	2.86	log U

Table 1: Configurations used in the quenched QCD simulation

2. The Kugo-Ojima theory

The Kugo-Ojima confinement criterion[1] is given by the fact that the parameter c defined as $u^{ab}(0) = -\delta^{ab}c$ in the eq(2.1) becomes 1

$$\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) u^{ab}(q^2) = \frac{1}{V} \sum_{x,y} e^{-ip(x-y)} \left\langle \text{tr} \left(\Lambda^{a\dagger} D_\mu \frac{1}{-\partial D} [A_\nu, \Lambda^b] \right)_{xy} \right\rangle. \quad (2.1)$$

	β	K_{sea}	am_{ud}^{VWI}/am_s^{VWI}	N_f	$1/a(\text{GeV})$	L_s	L_t	$aL_s(\text{fm})$
JLQCD	5.2	0.1340	0.134	2	2.221	20	48	1.78
	5.2	0.1355	0.093	2	2.221	20	48	1.78
CP-PACS	2.1	0.1357	0.087	2	1.834	24	48	2.58
	2.1	0.1382	0.020	2	1.834	24	48	2.58
CU	5.415		0.025	2	1.140	16	32	2.77
	5.7		0.010	2	2.1	16	32	1.50
MILC _c	6.83(β_{imp})		0.040/0.050	2+1	1.64	20	64	2.41
	6.76(β_{imp})		0.007/0.050	2+1	1.64	20	64	2.41
MILC _f	7.11(β_{imp})		0.0124/0.031	2+1	2.19	28	96	2.52
	7.09(β_{imp})		0.0062/0.031	2+1	2.19	28	96	2.52

Table 2: Configurations used in the unquenched QCD simulation

The Zwanziger horizon condition[3] coincides with Kugo-Ojima criterion provided the covariant derivative approaches the naive continuum limit, i.e., $e/d = 1$ [8]. We observe that in the quenched simulation c saturated at about 0.8, while in the unquenched simulation it is consistent with 1.

	K_{sea} or β	c_x	c_t	c	e/d	h
JLQCD	$K_{sea} = 0.1340$	0.89(9)	0.72(4)	0.85(11)	0.9296(2)	-0.08(11)
	$K_{sea} = 0.1355$	1.01(22)	0.67(5)	0.92(24)	0.9340(1)	-0.01(24)
CP-PACS	$K_{sea} = 0.1357$	0.86(6)	0.76(4)	0.84(7)	0.9388(1)	-0.10(6)
	$K_{sea} = 0.1382$	0.89(9)	0.72(4)	0.85(11)	0.9409(1)	-0.05(9)
CU	$\beta = 5.415$	0.84(7)	0.74(4)	0.81(8)	0.9242(3)	-0.11(8)
	$\beta = 5.7$	0.95(26)	0.58(6)	0.86(28)	0.9414(2)	-0.08(28)
MILC _c	$\beta = 6.76$	1.04(11)	0.74(3)	0.97(16)	0.9325(1)	0.03(16)
	$\beta = 6.83$	0.99(14)	0.75(3)	0.93(16)	0.9339(1)	-0.00(16)
MILC _f	$\beta = 7.09$	1.06(13)	0.76(3)	0.99(17)	0.9409(1)	0.04(17)
	$\beta = 7.11$	1.05(13)	0.76(3)	0.98(17)	0.9412(1)	0.04(17)

Table 3: The Kugo-Ojima parameter for the polarization along the spacial directions c_x and that along the time direction c_t and the average c , trace divided by the dimension e/d , horizon function deviation h of the unquenched Wilson fermion(JLQCD, CP-PACS), and KS fermion (MILC_c, CU, MILC_f). The $\log U$ definition of the gauge field is adopted.

3. Quark propagator

In the unquenched lattice simulation with the improved KS fermion action, the MILC collaboration has replaced the link variables by fattening[5]

$$U_\mu(x) \rightarrow c_1 U_\mu(x) + \sum_{\nu} w_3 S_{\mu\nu}^{(3)}(x) + \dots$$

where $S_{\mu\nu}^{(3)}$ is the staple contribution

$$S_{\mu\nu}^{(3)}(x) = U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})$$

and added the Naik term which is a product of three link variables along one direction.

The gauge configurations of the MILC collaboration are produced by incorporating, larger staple $S_{\mu\nu\rho}^{(5)}$ and $S_{\mu\nu\rho\sigma}^{(7)}$ and the tadpole improvement factor, and the action is called "Asqtad" action.

In the present calculation of valence quark propagator, we do not include all improvements of the "Asqtad" action but incorporate $S^{(3)}$ staple term and the Naik term, i.e. the propagator we measure is the same as that of [5], and we put for Dirac operator $\mathcal{D} + m$ as

$$\begin{aligned} \mathcal{D}(U)_{x,y} = & \frac{1}{2} \sum_{\mu=-4}^4 \eta_\mu(x) \text{sign}(\mu) [(c_1 U_\mu(x) + w_3 \sum_{\nu \neq \mu} S_{\mu\nu}^{(3)}(x)) \delta_{y,x+\hat{\mu}} \\ & + c_3 U_\mu(x) U_\mu(x+\hat{\mu}) U_\mu(x+2\hat{\mu}) \delta_{y,x+3\hat{\mu}}] \end{aligned}$$

where $w_3 = 9/64$, $c_1 = 9/32$ and $c_3 = -1/24$, and $\eta_\mu(x)$ is given as

$$\eta_\mu(x) = (-1)^{\zeta^{(\mu)}x}, \quad \zeta_\nu^{(\mu)} = \begin{cases} 1 & \nu < \mu \\ 0 & \text{otherwise} \end{cases}$$

For KS fermion, one defines translation invariant states as

$$\chi_{p,\alpha}(x) = \frac{1}{\sqrt{V}} e^{ikx}, \quad k_\mu = p_\mu + \pi\alpha_\mu$$

where $p_\mu = 2\pi m_\mu / L_\mu$ ($m_\mu = 0, \dots, (L_\mu/2) - 1$), $\alpha_\mu = 0, 1$. Then the Dirac operator in the tree level, $\mathcal{D}(I) + m$, has a proper form in the above basis as

$$\langle \chi_{p',\beta} | \mathcal{D}(I) + m | \chi_{p,\alpha} \rangle = [i \sum_{\mu} (\bar{\gamma}_\mu)_{\alpha\beta} (\frac{9}{8} \sin p_\mu - \frac{1}{24} \sin 3p_\mu) + m \bar{\delta}_{\alpha\beta}] \delta_{p'p}$$

where the Dirac gamma matrices of KS fermions appear as

$$(\bar{\gamma}_\mu)_{\alpha\beta} = (-1)^{\alpha_\mu} \bar{\delta}_{\alpha+\zeta^{(\mu)},\beta} \quad \text{and} \quad \bar{\delta}_{\alpha\beta} = \prod_{\mu} \delta_{\alpha_\mu\beta_\mu}.$$

4. Calculation of the propagator

The quark propagator is calculated by statistical average over Landau-gauge-fixed samples as

$$S_{\alpha\beta}(p) = \left\langle \langle \chi_{p,\alpha} | \frac{1}{\mathcal{D}(U) + m} | \chi_{p,\beta} \rangle \right\rangle$$

The inversion, $\frac{1}{\mathcal{D}(U)+m}$, is performed via conjugate gradient method after preconditioning as follows.

We define the operator $\mathcal{M} = (I + \frac{1}{m}\mathcal{D})$ with use of even- odd- sites decomposition

$$\mathcal{M} = \begin{pmatrix} I & \frac{1}{m}\mathcal{D}_{oe} \\ \frac{1}{m}\mathcal{D}_{eo} & I \end{pmatrix} = I - L - U$$

where lower triangle and upper triangle matrices, L and U , should be properly understood.

Using the Eisenstat trick[6], we define

$$\begin{aligned}\tilde{\mathcal{M}} &= (I-L)^{-1}\mathcal{M}(I-U)^{-1} = (I+L)(I-U-L)(I+U) \\ &= I-LU = \begin{pmatrix} I & 0 \\ 0 & I - (\frac{1}{m})^2 \mathcal{D}_{eo}\mathcal{D}_{oe} \end{pmatrix}\end{aligned}$$

where $L^2 = U^2 = 0$ understood.

We note that $\tilde{\mathcal{M}}$ is hermitian and the conjugate gradient method and/or BiCGstab method are applicable for its inversion. Thus for solution of the equation $m\mathcal{M}\phi = \rho$, we obtain that

$$\phi = (I+U)\tilde{\mathcal{M}}^{-1}(I+L)\frac{1}{m}\rho$$

For the right hand side, ρ , of the equation $\mathcal{M}\phi = \rho$, we put matrix site field $\rho_x = \chi_{p,\beta}(x)I_{3\times 3}$ where $I_{3\times 3}$ is the unit color source matrix. Then calculation with use of matrix site field, ϕ , $\langle \chi_{p,\alpha} | \phi \rangle$ yields the sample contribution to $S_{\alpha\beta}(p)$ which is color 3×3 matrix.

Our error estimate of the inversion calculation is

$$\frac{\|\mathcal{M}\phi - \rho\|}{\|\rho\|} < \text{a few per cent at most}$$

where the used norm is maximum norm in the space of site, color and flavor, and the accuracy gets 10^{-1} higher if L^2 norm is used.

With use of momentum definition

$$q_\mu = \frac{9}{8}\sin(p_\mu) - \frac{1}{24}\sin(3p_\mu) = \sin p_\mu \left(1 + \frac{1}{6}\sin^2(p_\mu) \right),$$

one obtains that

$$S_{\alpha\beta}(q) = Z_2(q) \frac{-i(\bar{\gamma}q)_{\alpha\beta} + M(q)\bar{\delta}_{\alpha\beta}}{q^2 + M(q)^2}$$

Since $\text{tr}\bar{\gamma}_\mu = 0$, trace over color and flavor yields

$$\text{tr}S(q) = 16N_c \frac{Z_2(q)M(q)}{q^2 + M(q)^2} \equiv 16N_c \mathcal{B}(q) \quad (4.1)$$

On the other hand

$$\text{tr}(i\bar{\gamma}qS(q)) = 16N_c q^2 \frac{Z_2(q)}{q^2 + M(q)^2} \equiv 16N_c q^2 \mathcal{A}(q) \quad (4.2)$$

The dynamical mass function of the quark is $M(q) = \frac{\mathcal{B}(q)}{\mathcal{A}(q)}$, and the quark wave function renormalization is $Z_2(q) = \frac{\mathcal{A}(q)^2 q^2 + \mathcal{B}(q)^2}{\mathcal{A}(q)}$.

5. Discussion and conclusion

We found that the Kugo-Ojima parameter is consistent with 1 in the MILC configuration.

The quark field renormalization Z_2 is found infrared vanishing[12], in contrast to [4]. It implies confinement rather than dynamical chiral symmetry breaking. We observed further, the renormalization of Z_2 and that of the running coupling are correlated[12]. On the extraction of the continuum limit of the running coupling from compact lattice simulations, there is a warning from the Dyson-Schwinger approach[17].

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