

High accuracy simulations of d=4 SU(3) qcd-string

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We present here results from our high accuracy simulations of the $q\bar{q}$ potential in $d = 4$ $SU(3)$ Yang-Mills theory. We measure this quantity by measuring the Polyakov loop correlators using the exponential variance reduction technique (multilevel) of Lüscher and Weisz. Further noise reduction was achieved by replacing temporal links by their multihit averages. In this case we found that the semi-analytic technique proposed by de Forcrand and Roiesnel is much more efficient than the usual monte-carlo multihit. Measurements were carried out on lattices of spatial extents of about 4 and 5.4 fermi. The temporal extent was 5.4 fermi in both cases. Polyakov loop correlators having separations between 0.3 and 1.2 fermi were measured. We analysed the results in terms of the force between $q\bar{q}$ pair as well as in terms of a scaled second derivative of the potential. The data is accurate enough to distinguish between different effective string models and it seems to favour the expression for ground state energy of a Nambu-Goto string.

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1. Introduction

The mechanism of quark confinement continues to be a forefront problem of theoretical physics despite decades of intense research both analytically and through large scale numerical simulations. Lattice simulations strongly point towards the fact that confining flux tubes are indeed formed between $q\bar{q}$ pairs. In recent times there has been intense interest in the possible string-like behaviour of these flux tubes [1]. Progress has been significantly accelerated by the path breaking developments in algorithms like the exponential variance reduction techniques of Lüscher and Weisz [2]. This has allowed one to measure Polyakov loop correlators to unprecedented accuracy thereby increasing sensitivity to the sub-leading behaviour of the $q\bar{q}$ potential. In their work using the multilevel technique Lüscher and Weisz directly measured, with high accuracy, the coefficient of the $1/r$ correction to the linearly rising part of the potential. For $d = 3$ $SU(2)$ lattice gauge theories this had been measured long ago, though not to the same accuracy, by Ambjorn *et. al.* [3]. Asymptotically, at large r this coefficient is expected to have the universal value $c = -\frac{(d-2)\pi}{24}$ (called the Lüscher term [5]) characteristic of a large class of bosonic string theories. They found that at distances of about 1fm the value of c was still about 12% away from this asymptotic value. Lüscher and Weisz initially argued that even this discrepancy could be accommodated through boundary terms in the string action [6] and that the scale of string formation is around 0.5fm. Subsequently they showed that the so called open string - closed string duality forbade such terms. Kuti *et. al.* [7] have undertaken very detailed studies of the spectrum of string excitations for which they have used extended Wilson loops. According to them, the ordering of the spectrum agrees with that of the bosonic string only at distances larger than 2fm. Other recent studies in lower dimensions and simpler gauge groups have been carried out in [8, 9].

In this article we present results of our simulations for $d = 4$ $SU(3)$ Yang-Mills theory and compare with different string models.

2. Our Simulations

2.1 Machine

All simulations were carried out on the teraflop linux cluster KABRU built at IIMSc, Chennai for lattice simulations. The hardware configuration of this machine is: 144 dual Intel Xeons @ 2.4 GHz, 533 MHz FSB and 266 MHz ECC DDRAM (2 GB per node on 120 nodes and 4 GB per node on 24 nodes). The networking is done in 3-D torus topology with SCI technology from Dolphinics, Norway. The sustained node to node bandwidth is 318 MB/s and latency is 3.8 microsecs between different nodes and 0.7 microsecs on the same node. The sustained performance on HPL has been 1.002 teraflops (double precision). Scaling on MILC codes has been very good : `ks_imp_dyn1`(75-80%); on `pure_gauge`(85%). For more details see [13].

2.2 Algorithm

Since we are interested in ground state properties, we measure Polyakov loop correlators as it is well known that the Polyakov loop correlator has the best known projection onto the $q\bar{q}$ ground state. We use the multilevel technique of Lüscher and Weisz for measuring the Polyakov loop correlators very accurately. This algorithm crucially uses locality of the action and the consequent

quasi-factorisation of the partition function. For evaluating Polyakov loop correlators, the spatial-links on certain timeslices are held fixed first while all other links are updated. This is called sub-lattice updating. With each such update the direct-product of the product of temporal links in a sub-lattice at the locations of the Polyakov loop are evaluated and averaged over many sub-lattice updates. Finally these direct-products are multiplied together and traced over to yield a Polyakov loop correlator. This constitutes a single measurement. The final expectation value is an average over usually a few hundred such measurements.

For further details of the algorithm we refer the reader to [2].

This algorithm has several optimization parameters and the most important among them seems to be the number of sub-lattice updates used to compute the intermediate expectation values. We will refer to this number henceforth as "iupd". Another parameter is the thickness of the sub-lattice. For our case a thickness of two was optimal.

It is also well known that improved observables can be constructed by replacing a link by its average as determined by the Boltzmann weight for the link. This is known as multihit and we employ that too on the time-like links for the correlators. The average is computed through the single-link integral defined by

$$\langle U \rangle = Z(J, J^\dagger)^{-1} \frac{\partial Z(J, J^\dagger)}{\partial J^\dagger} \quad \text{where} \quad Z(J, J^\dagger) = \int_{SU(3)} [dU] e^{\text{tr}(UJ^\dagger + U^\dagger J)}. \quad (2.1)$$

For $SU(3)$ this average cannot be efficiently carried out analytically and is most often evaluated using a monte-carlo method. We used a semi-analytic method for this averaging originally due to de Forcrand and Roiesnel [10]. This resulted in a 60% speedup of the code compared to using the monte-carlo method to reach similar levels of accuracy.

2.3 Simulation details

We have carried out simulations at $\beta = 5.7$ on both $24^3 \times 32$ and 32^4 lattices using the Wilson gauge action. The lattice spacing a at this β is 0.17 fm (as estimated through the Sommer scale) so that the temporal extent of the lattice is 5.4fm, while the spatial box is $(4\text{fm})^3$ in one case and $(5.4\text{fm})^3$ in the other.

For Polyakov loop correlators separated by $r = 2 - 6$ each measurement involved simulations on $24^3 \times 32$ lattices with iupd=12000 and 500 measurements were made. Errors were determined by jack-knife analysis. For the larger separations $r = 5 - 7$ simulations were done on 32^4 lattices with iupd=24000 and in all 270 measurements were obtained.

Here it may not be out of place to mention that for the larger separations ($r = 7$ currently), we had to go to larger lattices to continue to gain from the multilevel scheme. We observed that on a $24^3 \times 32$ lattice increasing iupd even by an order of magnitude did not seem to help reduce the error on the $r = 7$ correlator. However going to a larger lattice (32^4) did help significantly. At present we do not understand this very clearly, but we suspect finite volume effects in the sub-lattice expectation values might be the reason.

2.4 Some string actions and corresponding potentials

Before presenting our results we briefly recall some string actions and their ground state energies when their end-points are held fixed.

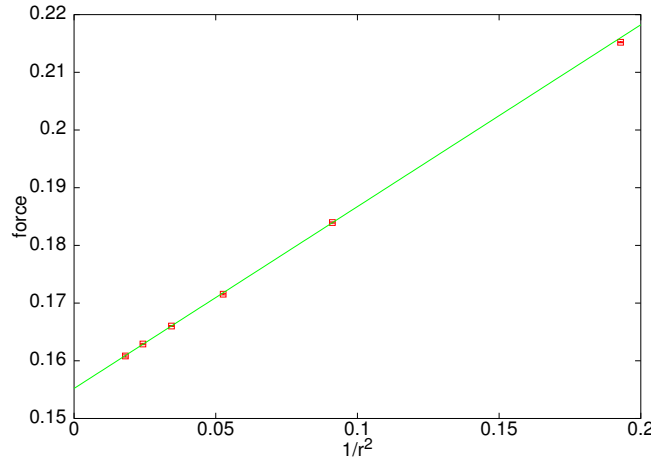


Figure 1: Force in $d=4$ $SU(3)$ case.

Case (a): free bosonic string

$$S = \frac{\sigma}{2} \int_0^T \int_0^R dz_0 dz_1 \partial_a X_\mu \partial_a X^\mu \Rightarrow V(r) = \sigma r - \frac{(d-2)\pi}{24} \frac{\pi}{r} \quad (2.2)$$

Case (b): Nambu-Goto string

$$S = \sigma \int \int \sqrt{\det g} \Rightarrow V_{\text{Arvis}}(r) = \sqrt{\sigma^2 r^2 - \frac{(d-2)\pi}{12} \sigma} \quad (2.3)$$

The potential in the latter case was first given by [11] and it should be noted that it becomes purely imaginary for $r < r_c$ where $r_c = \sqrt{\frac{(d-2)\pi}{12\sigma}}$. This corresponds to the tachyon instability of the Nambu-Goto string. The coefficient of the linear term of the potential (σ) is known as the string tension.

2.5 Results and analysis

In our simulations we measure the Polyakov loop correlator $\langle P^* P \rangle(r)$ where r is the separation between the two Polyakov loops. The $q\bar{q}$ potential is determined as

$$V(r) = -\frac{1}{T} \log \langle P^* P \rangle(r) \quad (2.4)$$

where T is the temporal extent of the lattice. In our analysis we look at the force between the $q\bar{q}$ -pair given by $F(r) = -\frac{\partial V(r)}{\partial r}$ and the scaled second derivative $c(r)$ given by $c(r) = \frac{r^3}{2} \frac{\partial^2 V(r)}{\partial r^2}$. The latter takes the value $-\frac{(d-2)\pi}{24}$ for case (a). The dependence of $c(r)$ on distance distinguishes between different effective string models. On the lattice we define these quantities by

$$F(\bar{r}) = V(r) - V(r-1) \quad (2.5)$$

$$c(\tilde{r}) = \frac{\tilde{r}^3}{2} [V(r+1) + V(r-1) - 2V(r)] \quad (2.6)$$

where \bar{r} and \tilde{r} are defined as in [4] to reduce lattice artifacts.

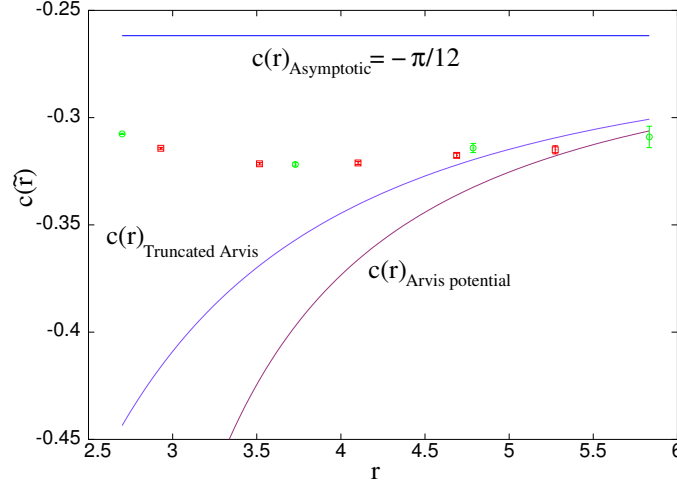


Figure 2: Scaled second derivative for $d=4$ $SU(3)$ case. The red points are from [4]

Since different r values are measured in the same simulation, they are quite strongly correlated. Thus when one evaluates the differences in potentials, it helps to reduce errors if one evaluates the differences for each measurement and then averages over different measurements rather than the other way round. In practice we compute the difference between jackknife bins and then compute the jackknife error for the differences.

In Fig.1 we have plotted the force $F(r)$ as a function of $1/r^2$. Fitting the data to the form $px + q$ with p and q as fit parameters, we obtain the string tension $a^2\sigma = 0.1552(2)$ from q . We compute the Sommer scale r_0 (defined by $r_0^2 F(r_0) = 1.65$) to be $2.93a$ from the force data. In Fig.2 we have plotted the scaled double derivative $c(\tilde{r})$ as a function of r . The horizontal line at $c(r) = -\pi/12$ is the prediction of case (a). The pink curve is $c(r)$ as computed from the Arvis potential (case (b)) while the purple curve is $c(r)$ computed from the so called truncated Arvis potential which is nothing but a series expansion of the Arvis potential truncated at the second order ($O(1/r^2)$). It is clear that in the entire r -range shown, case(a) is clearly excluded by our data. The force data also rules out case (a) quite convincingly as the fit parameter $p = 0.308$ is quite far from the value $\pi/12$. It should be noted that Kuti et al [7] claim that their data fits well the $q\bar{q}$ -potential of case(a) in the this region. Below 0.75fm the data deviates from case (b) also. We interpret this to mean that there is no obvious string behaviour even upto 0.75fm . But as r approaches 1fm we see clear convergence to case (b).

3. Conclusions and future directions

From our data we therefore see clear convergence towards the potential predicted by Nambu-Goto theory for $r > 0.75\text{fm}$. Similar conclusions are reached in [9] for the 3-d $SU(2)$ case as well as by [8] for $r > T/2$ though for $r < T/2$ they report deviations from Nambu-Goto theory. But as noted by many, including Arvis, the quantisations that led to the potentials in eqns(2.2, 2.3) are consistent in only $d = 26$ and can not be used to analyse the qcd-strings in $d = 3, 4$.

The work of Polchinski and Strominger [12] may offer a resolution to this. According to them the free bosonic string action is inconsistent in $d \neq 26$ and it can be made consistent in all di-

mensions by modifying (in a d -dependent manner) both the action and conformal transformations. Miraculously (till we understand this better) the $1/r$ correction to $V(r)$ is the same as in cases (a) and (b)! But it appears highly unlikely that their approach would produce $1/r^2$ and higher order terms that are close to what case (b) demands; yet our present work as well as the works of [8, 9] are suggesting that case (b) works very well even in r -ranges which are not so extremely large as to make $1/r^2$ corrections negligible. It is extremely important to see whether the convergence to the Nambu-Goto case seen in our data holds for larger values. These need to be sorted out through more careful simulations.

Going to larger r -values, larger lattices and larger β -values (closer to continuum) will require even greater computer resources unless we are able to come up with more efficient algorithms. We are currently working on these. We also plan to study the implications of our data for other varieties of strings like those with extrinsic curvature etc. We also plan to investigate the simpler Z_3 -gauge theories to learn about the issues already discussed here as well as to learn about string interactions.

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