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Liquid crystal defects of Yang-Mills theory in Landau gauge

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We point out that the global symmetries of the Yang-Mills theory in the Landau gauge support existence of variables similar to director fields in liquid crystal systems. We discuss two different types of the liquid crystal variables associated with the group of global Euclidean rotations and with a residual global group of color transformations. These global symmetries allow to identify various topological defects which resemble, in particular, vortex-like disclination defects and monopole-like objects in nematic liquid crystals. We suggest that the deconfinement phase transition in the Yang-Mills theory may be associated with a phase transition in a liquid crystal.

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1. Introduction

Confinement of color is one of the most important problems in the modern quantum field theory. The confinement is certainly caused by a complicated non-perturbative dynamics of the gluon fields. There are various proposals to identity gluon field configurations which are relevant for confinement. The relevant gluon fields can be associated either with particular topological defects [1, 2] or with specific non-topological, stochastic or classical configurations [3]. The popular approaches which are based on the topological defects include the dual superconductor [1] and center vortex [2] mechanisms. The first approach suggests that the relevant gluon configurations resemble the Abelian monopoles in particular Abelian gauge(s) of the Yang-Mills (YM) theory while the second approach utilizes the vortex-like defects associated with the center of the color group in a Center gauge. The gauge fixing is required in both cases since the pure YM theory does not contain the topologically stable monopole– and vortex–like configurations.

Among other gauges, the Landau gauge is one of the simplest, very well-studied and smooth gauges. Recently, attempts to approach the confinement problem in terms of particular gluon configurations in this gauge were made in Ref. [4, 5, 6, 7]. It was shown numerically [4] that in the Landau gauge the formation of the confining string is caused by magnetic displacement currents. Moreover, a particular structure of the YM string in the Landau gauge allowed authors of Ref. [5] to make a relation between the string and an Abrikosov vortex in a particular type of an ordinary superconductor.

In Ref. [6] the YM theory in the Landau gauge is reformulated as a nematic liquid crystal [8] (LC) in an internal space-time. The approach of Ref. [6] is based on the spin–charge separation idea [9] which is well known in theories of high-T_c cuprate superconductors [10]. Another way [7] to make a link between the YM theory and the nematic LC is to use (residual) global gauge symmetry, which remains unfixed after the Landau gauge is imposed. Below we describe examples of the LC variables in the Euclidean SU(2) Yang-Mills theory, and discuss a possible role of the LC variables in the dynamics of the system.

2. Two types of nematic liquid crystals in SU(2) Yang-Mills theory

2.1 An example of realization of the liquid crystal variables in color

A color realization of a liquid crystal in the Yang-Mills theory should be done in a gauge which fixes the gauge (local) color freedom while leaving a particular global (sub)group of the gauge group intact. The unfixed global group should be isomorphic to the group of rotations of a Euclidean space. This group is to be associated with the group of rotations of the director field in the color realization of the liquid crystal.

The simplest example of the gauge with the described properties is the Landau gauge which is formulated via the minimization of the gauge fixing functional,

$$\min_{\Omega} F[A^{\Omega}]\,, \qquad F[A] = \int \mathrm{d}^4 x \, \big[A^a_\mu(x)\big]^2\,, \qquad \Omega \in SO(3)_{\mathrm{gauge}}\,, \tag{2.1}$$

over the gauge transformations Ω. Since the gluon field is not sensitive to the center $\mathbb{Z}_2$ of the gauge group $SU(2)$, the gauge symmetry is $SO(3) \sim SU(2)/\mathbb{Z}_2$. The gauge condition (2.1) fixes the gauge color freedom up to the global color group, $SO(3)_{\rm gauge} \to SO(3)_{\rm global}$, because the gauge-fixing functional $F[A]$ and its local counterpart, $\partial_\mu A^a_\mu = 0$, are both invariant under the global (coordinate-independent) transformations, $A^a_\mu(x) \to \Omega^{ab}_{\rm gl} A^b_\mu(x)$, with $\Omega_{\rm gl} \in SO(3)_{\rm global}$.

Following Ref. [7] we construct the composite color-spin field

$$C^{ab}(x) = A^c_\mu(x) A^c_\mu(x) \cdot \delta^{ab} - A^a_\mu(x) A^b_\mu(x) \qquad \text{[color realization]}\,, \tag{2.2}$$

which is a scalar and a rank-2 symmetric tensor with respect to, correspondingly, space-time rotations and the global color transformations. The definition (2.2) is similar to the moment of inertia tensor of a body composed of mass-points (labelled by the integer n) with the masses m^n located at the positions r_n:

$$I^{ij} = \sum_n m_n (r^2_n\, \delta^{ij} - r^i_n r^j_n)\,. \tag{2.3}$$

At each point x of the space-time the matrix C^{ab}, Eq. (2.2), corresponds to the “moment of inertia tensor” of a “solid body” consisting of four mass-centers with equal “masses” $m_1 = 1$, $\mu = 1,\dots,4$. The gauge field A^a_μ plays a role of a “coordinate” of the $\mu^{\rm th}$ mass-center, and the color indices a and b play roles of the coordinate indices (analogues of i and j in Eq. (2.3)). Under the global color transformations $\Omega_{\rm gl}$ the matrix C transforms in the adjoint representation, $C(x) \to \Omega C(x) \Omega^T$, similarly to transformations of the moment of inertia tensor (2.3) under usual spatial rotations. We call Eq. (2.2) as a *color realization* of the discussed solid body analogy.

The matrix $C^{ab}(x)$ can be represented in the form $C^{ab}(x) = \Theta(x)\,{\rm diag}(c_1,c_2,c_3)\,\Theta^T(x)$, where Θ is a local $SO(3)$ transformation. The eigenvalues $c_1 \geqslant c_2 \geqslant c_3$ are interpreted as “moments of inertia” defined with respect to the orthonormal “principal axes of inertia” $\mathbf{e}_k$, $k = 1,2,3$, respectively. The axes $\mathbf{e}_k$ are normalized eigenvectors of the “moment of inertia tensor” C. The eigenvector $\mathbf{n}(x) \equiv \mathbf{e}_3(x)$ corresponding to the lowest highest eigenvalue of the composite symmetric field (2.2) defines a local direction of the “ellipsoid of inertia” associated with the moment of inertia tensor (2.2).

The LC analogy appears naturally after one realizes that the field $\mathbf{n}(x)$ can be associated with the direction (the "director field") of the longest principal axes of an axially symmetric molecule in nematic LC’s. The ordinary nematic crystals [8] are liquids composed of rod-like direction-less molecules. The molecules are invariant under (i) the $Z\!\!\!Z_2$ group consisting of the π-rotations about any axis perpendicular to $\mathbf{n}$ and (ii) the $SO(2)$ group of rotations about the vector $\mathbf{n}$. Thus, the physical space of the axial molecule – corresponding to the ellipsoid of inertia defined by the color tensor (2.2) in the YM theory – is the coset

$$G/H = SO(3)/(Z\!\!\!Z_2 \times SO(2)) \equiv \mathbb{R}P(2) \qquad \text{[color realization]} \tag{2.4}$$

The non-unit elements of this group make physically distinct changes to the director field $\mathbf{n}$.

Note that in principle any eigenvector a color tensor of the type (2.2) can be chosen as the director field $\mathbf{n}$. However, the relevance of the particular definition of the director field to the dynamics of the system should tested by numerical simulations.

2.2 An example of realization of the liquid crystal variables in space

A space realization of the moment of inertia tensor (2.3) can be done analogously to the color realization (2.2). The "tensor of inertia" is now taking values in the Euclidean coordinate space, and the simplest analogue of (2.3) is

$$S_{\mu\nu}(x) = A^a_\alpha(x) A^a_\alpha(x) \cdot \delta_{\mu\nu} - A^a_\mu(x) A^a_\nu(x) \qquad \text{[space realization]}. \tag{2.5}$$

This is a rank-2 symmetric tensor in the coordinate space and is a singlet in the color space. At each point x of the coordinate space the tensor (2.5) describes a solid body made of three (for the SU(2) gauge group) point-like objects of equal masses. The objects are located at points $r^a_\mu \equiv A^a_\mu$ where the superscript a labels the "masses".

We identify the nematic variable **n** (*i.e.*, the director field) with an eigenvector of the "moment of inertia" tensor (2.5) corresponding to a lowest eigenvalue of. The physical space of the director field **n** is now

$$G/H = SO(4)/(\mathbb{Z}_2 \times SO(3)) \equiv \mathbb{R}P(3) \qquad \text{[space realization]}. \tag{2.6}$$

3. Defects in liquid crystals

Non-perturbative dynamics of the gauge theories is often associated with presence of topological defects. The existence of a topologically stable defect of a particular dimensionality depends [11] on non-triviality of a corresponding homotopy group of the physical space G/H. In Table we show the first four homotopy groups of the color and space realization of the nematic LC in the four-dimensional SU(2) YM theory.

color realization					space realization				
G/H	π_0	π_1	π_2	π_3	G/H	π_0	π_1	π_2	π_3
$\mathbb{R}P(2)$	1	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{R}P(3)$	1	$\mathbb{Z}_2$	1	$\mathbb{Z}$

Table 1: The physically interesting homotopy groups of the physical spaces G/H for the color and space realizations of the liquid crystals in the SU(2) YM theory.

- Both the color and the space LC realizations of the four-dimensional SU(2) YM theory do not possess domain walls since their physical spaces are the connected spaces, $\pi_0 \equiv 1$.
- The fact that the both spaces are not simply-connected, $\pi_1 = \mathbb{Z}_2$ tells that both realizations have topologically stable $\mathbb{Z}_2$ vortices called disclinations in the nematic LC's and, simultaneously, the Alice vortices in the superfluid Helium-3 in the A-phase.
- The existence of the monopoles is characterized by the π_2 homotopy group, which is non-trivial in the color realization (where the monopoles exist) and is trivial for the space realization (no monopoles present).
- Finally, the non-triviality of the π_3 homotopy groups indicate that both realizations contain instantons labelled by an integer number.

4. Discussion

There is a crucial difference between the color and the space realizations of the LC variables in the YM theory. In the real liquid crystals [8] the global rotations $\mathbf{n} \to \Omega_{\mathbf{n}} \mathbf{n}$ of the director field $\mathbf{n}$, are tightly linked with the rotations of the coordinate space, $\mathbf{x} \to \Omega_{\mathbf{x}} \mathbf{x}$. In the color realization of the LC in the YM theory (2.2) the rotation of the director field $\mathbf{n}$ and the Euclidean space rotations are completely independent, $\Omega_{\mathbf{n}} \neq \Omega_{\mathbf{x}}$. On the other hand, in the space realization (2.5) these transformations are linked together, $\Omega_{\mathbf{n}} \equiv \Omega_{\mathbf{x}}$. Therefore the space realization of the LC in the YM theory has a closer (compared to the space realization) analogy to the *real* liquid crystals.

The physical space, $G/H = \mathbb{R}P(2)$, of the color realization of the LC in the SU(2) YM theory depends on the number of colors $N_c = 2$ and is independent on the dimensionality of the coordinate space. Contrary to the color realization, the physical space of the director field in the space realization does not depend on the number of colors N_c and is always $G/H = \mathbb{R}P(D-1)$, where $D = 4$ is the dimensionality of the Euclidean coordinate space. The Landau gauge fixing plays an auxiliary role in the definition of the inertial tensor of the sort (2.5), and therefore any other gauge (or, maybe, no gauge fixing at all!) can be used to define the tensor similar to Eq. (2.5).

Let us discuss a possible role of the LC variables in the phase structure of the YM theory. Consider the color, $\mathbb{R}P(2)$, realization of the LC in the SU(2) YM theory. Since the color is unbroken both in the confinement and in the deconfinement phases, the nematic LC phase (corresponding to broken color symmetry) is not allowed. Thus, the YM finite-temperature phase transition may only be associated with a phase transition between one isotropic phase of the liquid crystal and another isotropic phase. The two distinct isotropic phases were indeed observed in the lattice numerical simulations of nematic liquid crystals [12]. One of such isotropic phases is characterized the condensation of topological defects. In the language of the YM theory this phase should correspond to the confinement phase. Another isotropic phase of the LC – called the topologically ordered phase [12] – is characterized by absence of the condensate of the topological defects. This second LC phase can be considered as an analog of the deconfinement phase in the YM theory. Thus, in a suitable color LC variables the deconfinement phase transition in the YM theory may be viewed as a transition between the "topologically disordered" ($T < T_c$) phase and the "topologically ordered" ($T > T_c$) phase of the $\mathbb{R}P(2)$ liquid crystal.

Next, consider the space, $\mathbb{R}P(3)$, realization of the LC in the YM theory. Since at finite temperature the SO(4) group of the Euclidean rotations is anyway broken by the compactified ("temperature") direction of the space-time, then the nematic LC phase is not forbidden. In the nematic phase the director field points into a particular direction of the space-time while in the isotropic phase the director field is disordered. It is natural to associate the nematic LC phase with the deconfinement YM phase, since in this phase the spatial (magnetic) and temporal (electric) variables are clearly different. In the deconfinement phase the director field should naturally be pointing along the temporal direction. Moreover, the topological defects should therefore be condensed in the confinement ("isotropic") phase and should be dilute in the deconfinement ("nematic") phase.

In the language of the LC variables, the disorder caused by the LC topological defects in the low-temperature phase of the YM theory may lead to the confinement of color. Investigation of suitable LC realizations of the lattice SU(2) gauge theory is underway [13].

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