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Geometry of three dimensional vacuum domains in four dimensional SU(2) gluodynamics

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We review briefly recent results of lattice simulations on 3d domains in the vacuum state of $SU(2)$ gluodynamics. The defects are defined as unification of all the negative links in central projection under condition that the total number of negative links is minimized. In the continuum limit, negative links correspond, generally speaking to singular fields. The data indicate that total volume of the defects scales in physical units. We consider also correlator of negative links. The correlator scales in physical units as well, within the error bars. A new observation reported here is a strong anisotropy of the correlator.

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1. Introduction

Understanding of non-perturbative fluctuations in the QCD vacuum has been fast changing recently. The reason is the results of the lattice simulations. In continuum theory one thinks mostly in terms of soft, spherically symmetric excitations like instantons. Lattice simulations, on the other hand, indicate strongly that lower-dimensional defects are in fact crucial for confinement, for a recent review see, e.g., [1]. Indeed, monopoles are trajectories, or 1d defects, while the central vortices are surfaces, or 2d defects. Moreover, the 1d and 2d defects appear to be fine tuned, for review see [2]. Namely, in case of the vortices the total area scales in physical units [1] while the the total *non-Abelian* action associated with the vortices is singular in the limit of vanishing lattice spacing a [3]:

$$A_{vort} \;\approx\; 24(fm)^{-2}V_4 \;\;,\;\; S_{vort} \;\approx\; 0.54\frac{A_{vort}}{a^2} \;\; . \tag{1.1}$$

where A_{vort} is the total area of the surfaces while V_4 is the total volume of the lattice. Theoretically, the only way to explain the observations (1.1) is to assume that the surfaces possess also ultraviolet divergent entropy which almost cancels the effect of suppression due to the action (1.1). Moreover, in case of trajectories similar fine tuning is an indispensable part of the so called polymer representation of field theory, see e.g. [4].

In case of surfaces, however, even imposing the fine tuning between entropy and action 'by hand' does not help much. In particular, if one starts with the Goto-Nambu action and tunes it to the entropy strings decay into what is called branched polymers [4]. The branched polymers are effectively 1d structures. The branched polymers are known to be relevant also to $Z(2)$ gauge theory in 3d [5] and in 4d [6].

The vortices observed in the vacuum state of gluodynamics are defined and studied phenomenologically, through use of projected fields. How then can one distinguish between 'true' vortices (that is, Euclidean strings) and branched polymers? The answer appears simple. Consider minimal *three-dimensional* volume bound by the central vortices. If the central vortices in the non-Abelian case are similar to the central vortices of $Z(2)$ gauge theory then:

$$(V_3)_{branched\;\;polymers} \sim a \cdot A_{vort} \;\; . \tag{1.2}$$

If, on the other hand, the central vortices are true 2d dimensional defects then, generally speaking,

$$(V_3)_{strings} \;\sim\; \Lambda^{-1}_{QCD} \cdot A_{vort} \;\; . \tag{1.3}$$

In other words, the minimal 3d volume bound by the vortices is to scale in the physical units. Results of measurements of the 3d volume were reported first in [7] and favor the possibility (1.3).

Further results on geometrical properties of the 3d volume were obtained in [8]. In the subsequent sections we review these results and their implications. Moreover, we add some further preliminary results demonstrating anisotropy of the 3d volumes.

2. Three-dimensional volumes

2.1 Scaling of 3d volumes

Central vortices are defined as unification of negative plaquettes in $Z(2)$ projection of the original non-Abelian fields [1]. Plaquettes evaluated in projected fields are invariant under remaining

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(after the projection) $Z(2)$ gauge transformations. And in this sense the vortices themselves are uniquely defined. The volume bound by the vortices is not uniquely determined, on the other hand. The *minimal* volume bound by the central vortices can be found by minimizing the number of negative links. In other words, one fixes the remaining $Z(2)$ gauge invariance by minimizing the total number of negative links. In analogy with the $U(1)$ case, the $Z(2)$ gauge considered can be called $Z(2)$ Landau gauge fixing. It turns out that the volume occupied by the minimal number of negative links scales in the physical units [7]:

$$V_3 \approx 2(fm)^{-1}V_4 \quad , \tag{2.1}$$

where V_4 is the total volume of the lattice. As is explained in the Introduction, result (2.1) implies that the central vortices of gluodynamics are not branched polymers but rather look as true 2d surfaces.

Let us also mention that there is no extra non-Abelian action associated with the 3d volume under discussion [7]. The action is the same as on average over the whole lattice. This is in contrast with the case of monopoles and central vortices, which are distinguished by an ultraviolet divergent non-Abelian action, see [9] and [3], respectively. However, this difference can be readily understood theoretically along the lines of argumentation presented in [2].

2.2 Removal of P-vortices

Measurements of the 3d volumes are also relevant to appreciate the meaning of the so called removal of P-vortices introduced in [10]. One determines first central projected values of the link variables, $Z_\mu(x)$. And then modifies the original link matrices $U_\mu(x)$ in the following way:

$$U_\mu(x) \rightarrow \tilde{U}_\mu(x), \quad \tilde{U}_\mu(x) \equiv Z_\mu(x)U_\mu(x) \quad . \tag{2.2}$$

The effect of (2.2) is disappearance of confinement. Although this is very impressive, there remains a question to be answered, how serious is the damage to the original fields produced by an *ad hoc* procedure (2.2). If one judges by the number of plaquettes affected by (2.2) then the change affects a small fraction of the whole lattice. Indeed, only plaquettes belonging to the P-vortices are changing their sign and the probability of a given plaquette to belong to P-vortices is small:

$$\theta_{plaquette} \sim (a \cdot \Lambda_{QCD})^2 \quad , \tag{2.3}$$

where a is the lattice spacing and the probability (2.3) tends to zero with $a \rightarrow 0$.

However, this cannot be a final answer to our question. Indeed, the 'vanishing of confinement' means that the value of the Wilson line for a typical field configuration is changing its sign under (2.2) with a probability of order unit. Moreover, the Wilson line is 1d object and P-vortices are 2d objects. Thus, generally speaking, they do not intersect in d=4. Therefore, change of the sign of plaquettes belonging to the P-vortices cannot be the reason for disappearance of confinement under (2.2). The way out of the paradox is apparently that we should follow change not only in plaquettes but in the potentials (or links) as well and we come again to the 3d volumes considered above.

Namely, the minimal number of links which are affected by (2.2) is vanishing in the limit of $a \rightarrow 0$ as a 3d volume. It vanishes, however, not so strongly as the number of plaquettes belonging to P-vortices, see (2.3). Note that the original version of the removal of the P-vortices did not use the $Z(2)$ Landau gauge and approximately half of the links were modified by (2.2).

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3. Correlator of negative links

3.1 Definitions

As the next step, one can introduce correlator of negative links in the $Z(2)$ Landau gauge. Since the total volume V_3 scales in the physical units one may hope that the correlator scales as well [1]. In the continuum theory, if one imposes Landau gauge correlator of vector potentials is described by a single form factor. We are using $Z(2)$ Landau gauge and, at first sight, there is a single independent form factor as well. However, the notion of a negative link does not have meaning in the continuum. More precisely, negative links correspond to singular fields, $A_\mu \sim 1/a$. Therefore, we cannot rule out a priori more complicated dependencies on the mutual orientation of the links and of the displacement x. We will consider, therefore, the correlator of the negative links in its generality and begin with corresponding definitions. Consider first correlator of parallel links:

$$G^{\|}_{\mu\nu}(x) = \langle Z_{0,\mu} Z_{x,\nu} \rangle,\ \mu = \nu \tag{3.1}$$

Moreover, orientation of links and separation x can be either mutually transversal so that the scalar product of x_ρ and of the unit vector in the μ-direction vanishes, or longitudinal so that the vector x_ρ is directed along the μ direction as well. One can also consider correlation of perpendicular links,

$$G^{\perp}_{\mu\nu}(x) = \langle Z_{0,\mu} Z_{x,\nu} \rangle, \qquad \mu \neq \nu\ . \tag{3.2}$$

Again, there are further sub-cases depending on the mutual orientation of x_ρ and links looking in the $\mu-, \nu-$ directions.

3.2 Isotropic case

Let us change variables, $\hat{Z}_\mu(x) = \{1,\ if\ Z_\mu(x) = -1;\ \ 0\ if\ Z_\mu(x) = 1\}$. Moreover, consider first the isotropic correlator:

$$G \equiv \frac{1}{N_r} \Sigma_{r<|x|<r+a/2} \langle \hat{Z}_\mu(0), \hat{Z}_\nu(x) \rangle\ , \tag{3.3}$$

where the summation runs over all links $Z_\mu(x)$ for x lying in the spherical layer $r < |x| < r + a/2$ and N_r is the total number of links in the layer. At large distances the function $G(r)$ can be fitted by a constant plus an exponent. The corresponding mass turns to be close to the lowest glueball mass

$$m \approx (1.4 - 1.6)\ GeV\ . \tag{3.4}$$

Note also that measurements at finite temperature were performed very recently as well [12].

3.3 Anisotropy in the correlator of negative links

A new point which we are reporting here is observation of a strong anisotropy of the correlator of the negative links in the $Z(2)$ Landau gauge. We will concentrate on the case $\mu = \nu$, see Eq. (3.1). Defining

$$F^{\|,\perp} = \frac{G^{\|,\perp}_{\mu\nu}(x)}{G^{\|,\perp}_{\mu\nu}(\infty)} - 1, \tag{3.5}$$

[1] Minimization of the number of negative links can be considered as a discrete analog of minimization of $<(A^a_\mu)^2>$. Although the latter vacuum expectation value is gauge dependent, its minimal value may have a physical meaning [11].

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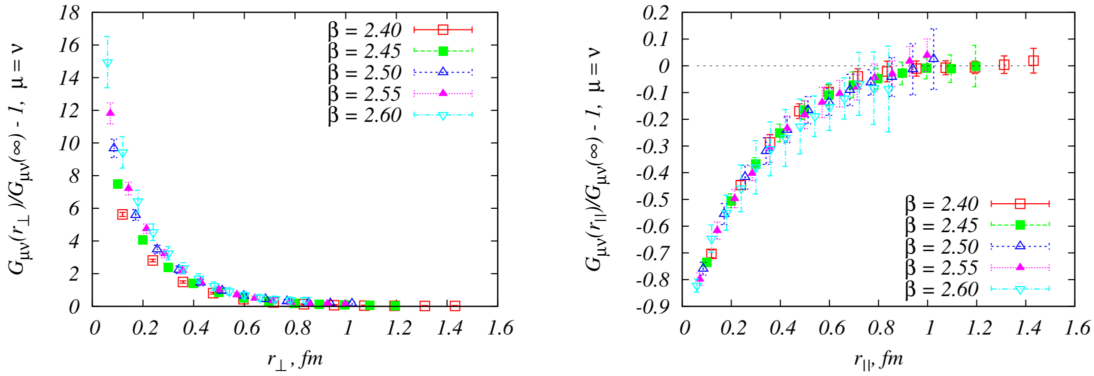
we observe that the correlators have *opposite signs*. Namely, the correlation of the parallel negative links which we are considering is positive if the displacement x is perpendicular to the links and is negative if the displacement vector is parallel to the links. Our preliminary data are presented in Fig 1. The both correlators exhibit scaling. The mass values associated with the transversal correlator



(a) (b)

Figure 1: Correlation of parallel links: (a) transversal correlation, $x = r_\perp$ is transversal to links, and (b) longitudinal, $x = r_{||}$ is along the links.

are presented in Table 1.

β	a, fm	L	m, fm^{-1}
2.40	0.1183	24	5.30 ± 0.10
2.45	0.0996	24	5.35 ± 0.12
2.50	0.0854	24	5.20 ± 0.15
2.55	0.0713	28	5.40 ± 0.20
2.60	0.0601	28	5.50 ± 0.10

Table 1: Mass parameter in the transversal correlator of parallel links.

3.4 Mass scales

Since the longitudinal correlator is negative, the correlator of negative links cannot be interpreted as propagator of a physical degree of freedom. Rather, the properties of the correlator reflect geometry of the lower-dimensional defects. And at scale of GeV^{-1} the geometry is not isotropic (for a given field congfiguration). The correlators scale in physical units and the corresponding mass scales are having physical meaning. In particular, the mass fitted above has the meaning of inverse typical size of the 3d volume in the transverse direction. In other words, existence of lower-dimensional defects brings in mass scales which are not glueball masses.

One can speculate, though, that at very large distances $x \gg (GeV)^{-1}$ these masses are unobservable. Indeed, at such distances the negative links will be separated by a few boundaries of the percolating 3d volumes and the anisotropy should vanish. This guess is partly confirmed by the observation that if, instead of going to limit of very large x, one averages over the directions the correlation length is indeed close to the inverse glueball mass, see Eq. (3.4).

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References

[1] J. Greensite, *The confinement problem in lattice gauge theory*, *Prog. Part. Nucl. Phys.* **51** (2003) 1 [hep-lat/0301023].

[2] V.I. Zakharov, *Lattice SU(2) theory projected on scalar particles*, *Phys. Usp.* **47** (2004) 37, *Usp. Fiz. Nauk* **47** (2004) 39; V.I. Zakharov, *Nonperturbative match of ultraviolet renormalon*, [hep-ph/0309178].

[3] F.V. Gubarev, A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, V.I. Zakharov, *Fine tuned vortices in lattice SU(2) gluodynamics,Phys. Lett. B* **574** (2003) 136 [hep- lat/0212003]; A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn,V.I. Zakharov, *Properties of P-vortex and monopole clusters in lattice SU(2) gauge theory*, *Phys. Rev. D* **71** (2005) 054511 [hep-lat/0402017].

[4] J. Ambjorn, *Quantization of Geometry*, [hep-th/9411179].

[5] V.S. Dotsenko *et al.*, *Critical and topological properties of cluster boundaries in the 3-d Ising model*, *Phys. Rev. Lett.* **71** (1993) 811 [hep-th/9304088].

[6] A.V. Kovalenko, unpublished.

[7] A.V. Kovalenko *et al.*, *Three dimensional vacuum domains in four dimensional SU(2) gluodynamics*, *Phys. Lett. B* **613** (2005) 52 [hep-lat/0408014].

[8] M.I. Polikarpov, S.N. Syritsyn, V.I. Zakharov, *A novel probe of the vacuum of the lattice gluodynamics*, *JETP Lett.* **81** (2005) 143 [hep-lat/0402018].

[9] V.G. Bornyakov, *et al.* , *Anatomy of the lattice magnetic monopoles*, *Phys. Lett. B* **537** (2002) 291 [hep-lat/ 0103032].

[10] P. de Forcrand, M. D'Elia, *On the relevance of center vortices to QCD*, *Phys. Rev. Lett.* **82** (1999) 4582 [hep-lat/9901020].

[11] F.V. Gubarev, L. Stodolsky, V.I. Zakharov, *On the significance of the vector potential squared*, *Phys. Rev. Lett.* **86** (2001) 2220 [hep-ph/0010057]; F.V. Gubarev, V.I. Zakharov, *On the emerging phenomenology of* $<(A_\mu)^2>$, *Phys. Lett*, **B501** (2001) 28 [hep-ph/0010096]; M.N. Chernodub *et al.* ,*Vacuum type of SU(2) gluodynamics in maximally abelian and landau gauges*, [hep-lat/0508004].

[12] K. Langfeld, G. Schulze, H. Reinhardt, *Center flux correlation in SU(2) Yang-Mills theory*, [hep-lat/0508007].