PROCEEDINGS OF SCIENCE

Non-degenerate quark mass effect on B_K with a mixed action

Taegil Bae[*]
Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: esrevinu@phya.snu.ac.kr

Jongjeong Kim
Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: rvanguard@phya.snu.ac.kr

Weonjong Lee [†]
Center for Theoretical Physics, Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: wlee@phya.snu.ac.kr

We investigate the effect of non-degenerate quarks on B_K. This effect is noticeably large for B_K (significantly larger than statistical uncertainty). Hence, it is important to include this effect in order to determine B_K with higher precision. We also observe that the quality of fitting for B_K gets better when we include non-degenerate combinations to fit to the prediction by Van de Water and Sharpe. However, this effect on $B_7^{(3/2)}$ and $B_8^{(3/2)}$ turns out to be relatively small.

XXIIIrd International Symposium on Lattice Field Theory
25-30 July 2005
Trinity College, Dublin, Ireland

[*]Speaker.

[†]This research is supported in part by the KOSEF grant (R01-2003-000-10229-0), by the MOST/KISTEP grant of international collaboration, and by the BK21 program.

PoS(LAT2005)335

1. Effect of non-degenerate quarks on B_K

This paper is a follow-up from the previous paper [1] on B_K. Hence, in order to avoid repeating the same explanation, we will quite often refer to Ref. [1] for details.

In this paper, we focus on B_K, the kaon bag parameter for indirect CP violation. In particular, we are interested in the effect of non-degenerate quarks (*i.e.* $m_x \neq m_y$). Before we go into the details, we want to address couple of issues which covers the historical evolution of the staggered ε'/ε project. When we used unimproved staggered fermions to calculate weak matrix elements, we have observed three major problems: (1) large scaling violation, (2) large perturbative corrections at the one loop level, and (3) large uncertainty due to the quenched approximation. We use improved staggered fermions with HYP fat links which reduce the scaling violations and decrease the one-loop correction down to $\approx 10\%$ level [2, 3, 4]. We measure weak matrix elements over a subset of the MILC gauge configurations in order to remove the uncertainty originating from quenched approximation [5]. In other words, we numerically simulate the unquenched QCD where the valence quarks are HYP staggered fermions and the sea quarks are AsqTad staggered fermions (we call this a "mixed action"). Hence, in this paper, we report recent progress in calculating B_K in unquenched QCD using improved staggered fermions.

Details of input parameters for this numerical study are provided in Ref. [1] and so we do not repeat them here. We use 5 degenerate quark combinations and 4 non-degenerate combinations to probe the effect of non-degenerate quarks. These 9 combinations are listed in Table 1. Note that there are two pairs of which the degenerate and non-degenerate combinations share the same kaon mass at the leading order (for example, $m_x = m_y = 0.03$ and $m_x = 0.01$ & $m_y = 0.05$).

m_x	m_y	B_K
0.01	0.01	0.6108 ± 0.0164
0.02	0.02	0.6421 ± 0.0065
0.03	0.03	0.6722 ± 0.0039
0.04	0.04	0.6955 ± 0.0029
0.05	0.05	0.7144 ± 0.0024
0.01	0.05	0.6869 ± 0.0068
0.02	0.05	0.6907 ± 0.0039
0.03	0.05	0.6976 ± 0.0030
0.04	0.05	0.7058 ± 0.0027

Table 1: Quark mass combinations and corresponding B_K values

In Fig. 1, we present B_K as a function of Euclidean time with a degenerate quark combination ($m_x = m_y = 0.03$) in the lefthand side and with a non-degenerate quark combination ($m_x = 0.01$ and $m_y = 0.05$) in the righthand side. The best fitting range is $10 \leq t \leq 15$ as explained in Ref. [1]. Here, note that the quark masses are chosen such that the kaon masses are the same at the leading order for both the degenerate and non-degenerate combinations. In Fig. 1, we observe two things: (1) the signal for the non-degenerate combination is noisier than that for the degenerate one and (2) the B_K value of the non-degenerate combination is noticeably higher than that of the degenerate one.

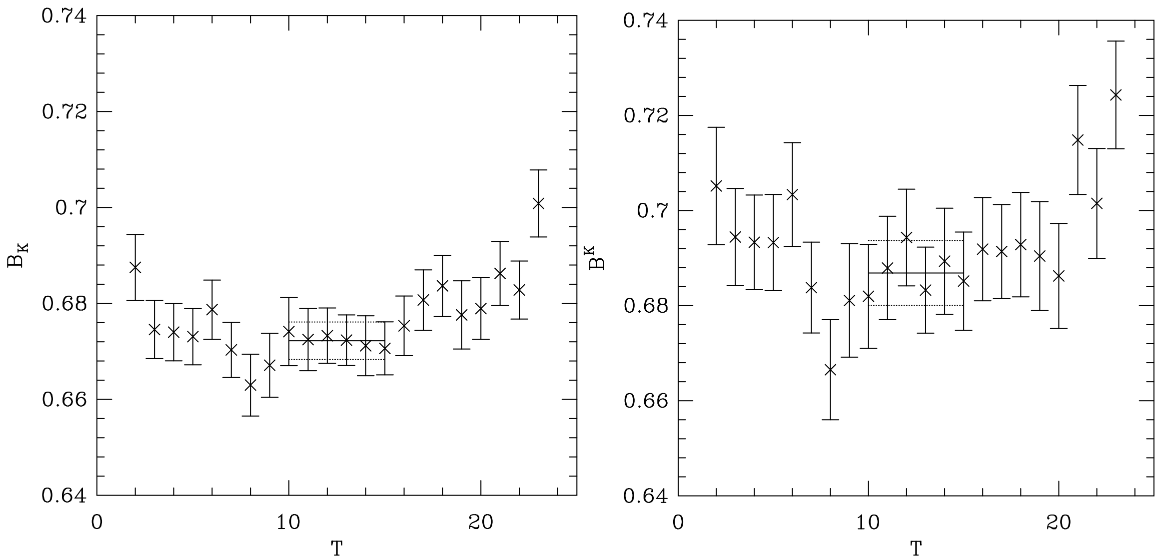

Figure 1: B_K vs. T for degenerate valence quarks with $m_x = m_y = 0.03$ (left) and for non-degenerate valence quarks with $m_x = 0.01$ and $m_y = 0.05$ (right)

In Ref. [6], Van de Water and Sharpe calculated the chiral behavior of B_K using staggered chiral perturbation. They provide results for the case of $N_f = 2+1$ partially quenched QCD ($m_u = m_d \neq m_s$) in the continuum as well as at a finite lattice spacing with staggered fermions. For the degenerate combination ($m_x = m_y$), the chiral behavior of B_K is given in Ref. [1] and we do not repeat it here. For the non-degenerate combination ($m_x \neq m_y$), they predict the chiral behavior of B_K in the continuum limit as follows:

$$B_K = \tilde{c}_1 \Bigg(1 + \frac{1}{48\pi^2 f^2} \Big[I_{\rm conn} + I_{\rm disc} + \tilde{c}_2 m_{xy}^2 + \tilde{c}_3 \frac{(m_X^2 - m_Y^2)^2}{m_{xy}^2} + \tilde{c}_4 (2m_U^2 + m_S^2) \Big] + \cdots \Bigg) \tag{1.1}$$

where $f = 132$ MeV and $\tilde{c}_i$ are unknown dimensionless low-energy constants. The chiral logarithms are given by $I_{\rm conn}$ and $I_{\rm disc}$. The contribution from the quark connected diagrams is

$$I_{\rm conn} = 6m_{xy}^2 \tilde{l}(m_{xy}^2) - 3l(m_X^2)\left(1 + \frac{m_X^2}{m_{xy}^2}\right) - 3l(m_Y^2)\left(1 + \frac{m_Y^2}{m_{xy}^2}\right) \tag{1.2}$$

The contribution from the diagrams involving a hairpin vertex is

$$\begin{aligned} I_{\rm disc} &= I_X + I_Y + I_\eta \qquad (1.3) \\ I_X &= \tilde{l}(m_X^2) \frac{(m_{xy}^2 + m_X^2)(m_U^2 - m_X^2)(m_S^2 - m_X^2)}{m_\eta^2 - m_X^2} \\ &\quad - l(m_X^2) \frac{(m_{xy}^2 + m_X^2)(m_U^2 - m_X^2)(m_S^2 - m_X^2)}{(m_\eta^2 - m_X^2)^2} \\ &\quad - l(m_X^2) \frac{2(m_{xy}^2 + m_X^2)(m_U^2 - m_X^2)(m_S^2 - m_X^2)}{(m_Y^2 - m_X^2)(m_\eta^2 - m_X^2)} \end{aligned}$$

$$-l(m_X^2)\frac{(m_U^2-m_X^2)(m_S^2-m_X^2)-(m_{xy}^2+m_X^2)(m_S^2-m_X^2)-(m_{xy}^2+m_X^2)(m_U^2-m_X^2)}{m_\eta^2-m_X^2} \quad (1.4)$$

$$I_Y = I_X(m_X^2 \leftrightarrow m_Y^2) \quad (1.5)$$

$$I_\eta = l(m_\eta^2)\frac{(m_X^2-m_Y^2)^2(m_{xy}^2+m_\eta^2)(m_U^2-m_\eta^2)(m_S^2-m_\eta^2)}{(m_X^2-m_\eta^2)^2(m_Y^2-m_\eta^2)^2} \quad (1.6)$$

Here, note that the chiral logarithms appear through $l(X)$ and $\tilde{l}(X)$, which are defined as

$$l(X) = X\log(X/\Lambda^2)+\text{F.V.} \quad (1.7)$$

$$\tilde{l}(X) = -[\log(X/\Lambda^2)+1]+\text{F.V.} \quad (1.8)$$

where F.V. represents a finite volume correction. The notations for the various meson masses are as follows for those composed of sea quarks:

$$m_U^2 = 2\mu m_d, \qquad m_S^2 = 2\mu m_s, \qquad m_\eta^2 = (m_U^2+2m_S^2)/3 \quad (1.9)$$

and as follows for those composed of valence quarks

$$m_X^2 = 2\mu m_x, \qquad m_Y^2 = 2\mu m_y, \qquad m_{xy}^2 = \mu(m_x+m_y) \quad (1.10)$$

Here, we set sea quark masses to $m_u = m_d \neq m_s$ and the two valence quark masses are m_x and m_y.

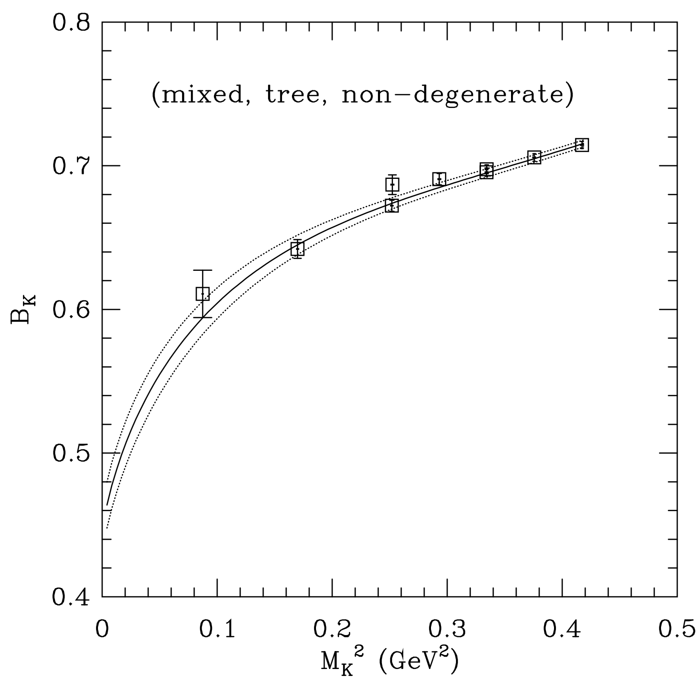

Figure 2: B_K vs. M_K^2

In Fig. 2, we plot B_K data as a function of M_K^2. Here, it includes both degenerate combinations ($m_x = m_y$) and non-degenerate combinations ($m_x \neq m_y$). We fit the data to the form of Eq. (1.1) suggested by chiral perturbation theory:

$$B_K = c_1\left(1+\frac{1}{48\pi^2 f^2}\Big[I_{\rm conn}+I_{\rm disc}\Big]\right)+c_2 m_{xy}^2+c_3\frac{(m_X^2-m_Y^2)^2}{m_{xy}^2}+c_4 m_{xy}^4 \tag{1.11}$$

where the cut-off scale Λ is set to $\Lambda = 4\pi f$. The fitting results are summarized into Table 2.

parameters	average	error
c_1	0.4449	0.0158
c_2	-1.4358	0.1779
c_3	0.0079	0.0073
c_4	1.3825	0.2120
χ^2/d.o.f.	0.3430	0.2789

Table 2: Fitting results for B_K

When we compare the fitting results of Table 2 with those in Ref. [1] (only for the degenerate combinations), we notice three things: (1) χ^2 is much smaller for the whole fitting (including both degenerate and non-degenerate combinations) than that only for the degenerate combinations, (2) the low-energy constants c_1, c_2 and c_4 are consistent with those of Ref. [1] within statistical uncertainty, and (3) for a given choice of Λ, the c_3 term is essentially zero within statistical uncertainty. Judging from these observations, we understand that the effect of non-degenerate quarks are dominated by the chiral logarithmic terms such as $I_{\rm conn}$ and $I_{\rm disc}$ whereas the c_3 term does not play much role at all. Judging from the fact that the c_3 term is zero, we know that we can determine all the low-energy constants (c_1, c_2, c_4) using only degenerate combinations. In addition, using these low-energy constants, we can determine the B_K value at the physical kaon mass with physical valence quarks ($m_x = m_d$ and $m_y = m_s$). However, we know that χ^2 is substantially smaller when we include non-degenerate combinations in fitting, which leads us to the conclusion that it would be significantly better for data analysis to measure enough non-degenerate combinations as well as degenerate combinations.

In Table 2, the χ^2 is rather high when we take into account the fact that all the data are correlated. In fact, one can observe that the fitting curve does not fit the lightest two data points very well in Fig. 2. In other words, the fitting curve miss the lightest two data points in the opposite direction. In Ref. [6], it is pointed out that the contribution from the non-Goldstone pions are so significant that the curvature of the fitting curve becomes smoother, which is consistent with what we observe in Fig. 2. However, the full prediction from staggered chiral perturbation contains 21 unknown low-energy constants for a single lattice spacing and 37 unknown low-energy constants for the full analysis [6]. In order to determine all of them, it is necessary to carry out a significantly more extensive numerical work.

In the current analysis of B_K data, we match the lattice results to the continuum values at the tree level. In this respect, the results are preliminary.

2. Effect of non-degenerate quarks on $B_7^{(3/2)}$ and $B_8^{(3/2)}$

In Fig. 3, we plot $B_7^{(3/2)}$ in the lefthand side and $B_8^{(3/2)}$ in the righthand side as a function of

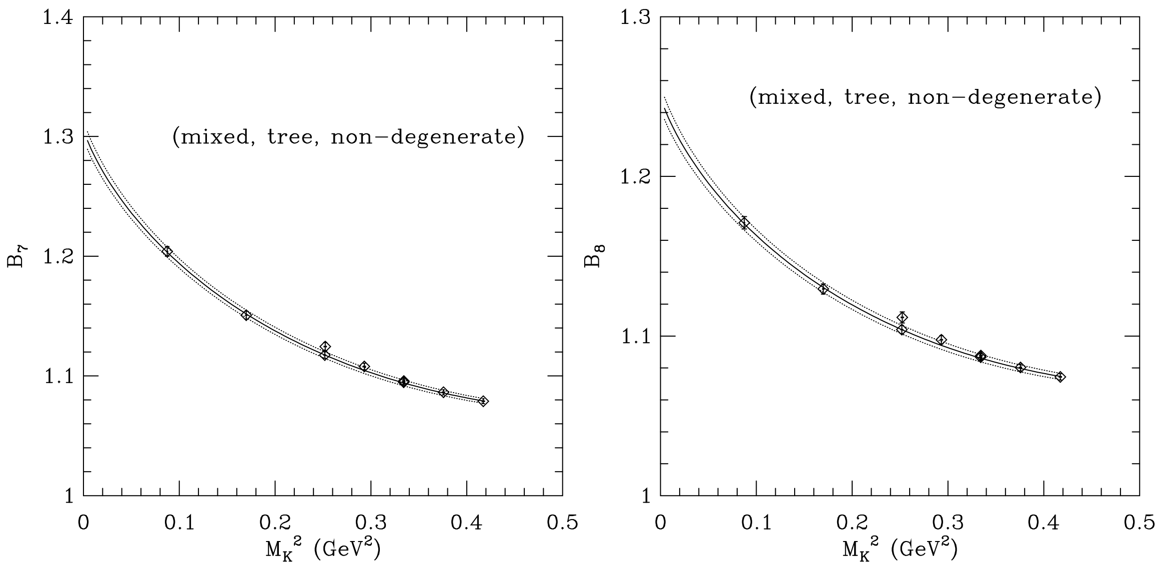

Figure 3: $B_7^{(3/2)}$ vs. M_K^2 (left) and $B_8^{(3/2)}$ vs. M_K^2 (right)

M_K^2. We show data of both degenerate and non-degenerate quark combinations. The difference in $B_7^{(3/2)}$ ($B_8^{(3/2)}$) between the quark mass pairs of (0.03, 0.03) and (0.01, 0.05) is about 0.6% (0.7%), which is much smaller than that in B_K (about 2%). Hence, the effect of non-degenerate quarks on $B_{7,8}^{(3/2)}$ is of less interest as much.

There are some calculation on the chiral behavior of $\langle \pi^+ | Q_{(8,8)}^{(3/2)} | K^+ \rangle$ available in Ref. [7]. This result applies to the $N_f = 3$ case with $m_u = m_d = m_s$. It, however, does not apply to the case of $N_f = 2+1$ with $m_u = m_d \neq m_s$. Therefore, in order to understand the data in Fig. 3 further, it is necessary to calculate the chiral behavior for $N_f = 2+1$ using staggered chiral perturbation theory.

References

[1] Jongjeong Kim, Taegil Bae and Weonjong Lee, *Calculating B_K using a mixed action*, in proceedings of *XXIIIrd International Symposium on Lattice Field Theory*, PoS (LAT2005) 338.

[2] Weonjong Lee *et al.*, *Testing improved staggered fermions with m_s and B_K*, *Phys. Rev.* D**71** (2005) 094501, [`hep-lat/0409047`].

[3] Weonjong Lee and Stephen Sharpe, *Perturbative matching of staggered four-fermion operators with hypercubic fat links*, *Phys. Rev.* D**68** (2003) 054510, [`hep-lat/0306016`].

[4] Weonjong Lee and Stephen Sharpe, *One-loop matching coefficients for improved staggered bilinears* , *Phys. Rev.* D**66** (2002) 114501, [`hep-lat/0208018`].

[5] C. Aubin *et al.*, *Light hadrons with improved staggered quarks: approaching the continuum limit* , *Phys. Rev.* D**70** (2004) 094505, [`hep-lat/0402030`].

[6] Ruth Van de Water and Stephen Sharpe, *B_K in Staggered Chiral Perturbation Theory* , [`hep-lat/0507012`].

[7] J. Laiho and A. Soni, *Lattice extraction of $K \to \pi\pi$ amplitudes to NLO in partially quenched and in full chiral perturbation theory*, *Phys. Rev.* D**71** (2005) 014021, [`hep-lat/0306035`].