PROCEEDINGS OF SCIENCE

Calculating B_K using a mixed action

Jongjeong Kim*
Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: rvanguard@phya.snu.ac.kr

Taegil Bae
Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: esrevinu@phya.snu.ac.kr

Weonjong Lee †
Center for Theoretical Physics, Department of Physics, Seoul National University, Seoul, 151-747, South Korea
E-mail: wlee@phya.snu.ac.kr

We present preliminary results of B_K calculated using improved staggered fermions with the mixed action (valence quarks = HYP staggered fermions and sea quarks = AsqTad staggered fermions). We analyze the data based upon the prediction by Van de Water and Sharpe. A hint of consistency with the prediction is observed. We also present preliminary results of $B_8^{(3/2)}$ and $B_7^{(3/2)}$.

XXIIIrd International Symposium on Lattice Field Theory
25-30 July 2005
Trinity College, Dublin, Ireland

*Speaker.

†This research is supported in part by the KOSEF grant (R01-2003-000-10229-0), by the MOST/KISTEP grant of international collaboration, and by the BK21 program.

1. B_K

The size of indirect CP violation in the neutral Kaon system is, in experiment, parameterized by ε, which is proportional to the kaon bag parameter B_K defined as

$$B_K = \frac{\langle \bar{K_0}|[\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d]|K_0\rangle}{\frac{8}{3}\langle \bar{K_0}|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K_0\rangle} \tag{1.1}$$

A precise determination of B_K constrains the CKM matrix, which might lead us to a window of new physics beyond the standard model. Hence, there have been a number of efforts to calculate B_K with higher precision. As pointed out in Ref. [1], the large scaling violation observed in the calculation using unimproved staggered fermions can be reduced remarkably by improving staggered fermions with HYP fat links. Even though the scaling violation is taken care of by improving staggered fermions, there has been an uncertainty originating from the quenched approximation. Hence, it has been crucial to perform a numerical study in unquenched QCD so that we can remove the uncertainty due to quenched approximation. In this paper, we describe our first attempt to perform a numerical study on B_K in partially quenched QCD, while minimizing the scaling violation using improved staggered fermions. We use HYP staggered fermions as valence quarks and AsqTad staggered fermions as sea quarks (we call this a “mixed action”). Details of the simulation parameters are summarized in Table 1.

parameter	value
sea quark	AsqTad Staggered
valence quark	HYP Staggered
β	6.76 ($N_f = 2+1$ QCD)
# of confs	593
lattice	$20^3 \times 64$
sea quark masses	$m_{u,d} = 0.01$, $m_s = 0.05$
valence quark mass	0.01, 0.02, 0.03, 0.04, 0.05

Table 1: Parameters for the numerical study

We measure weak matrix elements and hadron spectrum over a subset of the MILC gauge configurations [2]. We set up U(1) noise sources at $T = 0$ and $T = 26$, which project out only pseudo-Goldstone pions ($\gamma_5 \otimes \xi_5$) and exclude all the other non-Goldstone pions. The kaon signal as a function of Euclidean time T is shown in the lefthand side of Fig. 1. Here, we observe that there is a noticeable contamination from excited states in the range of $0 \le T < 10$. Hence, the best fitting range for B_K is $10 \le t \le 15$ in order to exclude the contamination from excited states, as you can see in the righthand side of Fig. 1.

Ref. [3] presents a chiral behavior of B_K and its finite volume effect in the case of $N_f = 2$ partially quenched QCD. Although this result is interesting, it does not apply directly to our numerical study mainly because it is in $N_f = 2+1$ partially quenched QCD. Recently, Van de Water and Sharpe have calculated the chiral behavior of B_K in $N_f = 2+1$ partially quenched QCD using staggered chiral perturbation theory [4]. The result for the degenerate valence quark mass

PoS(LAT2005)338

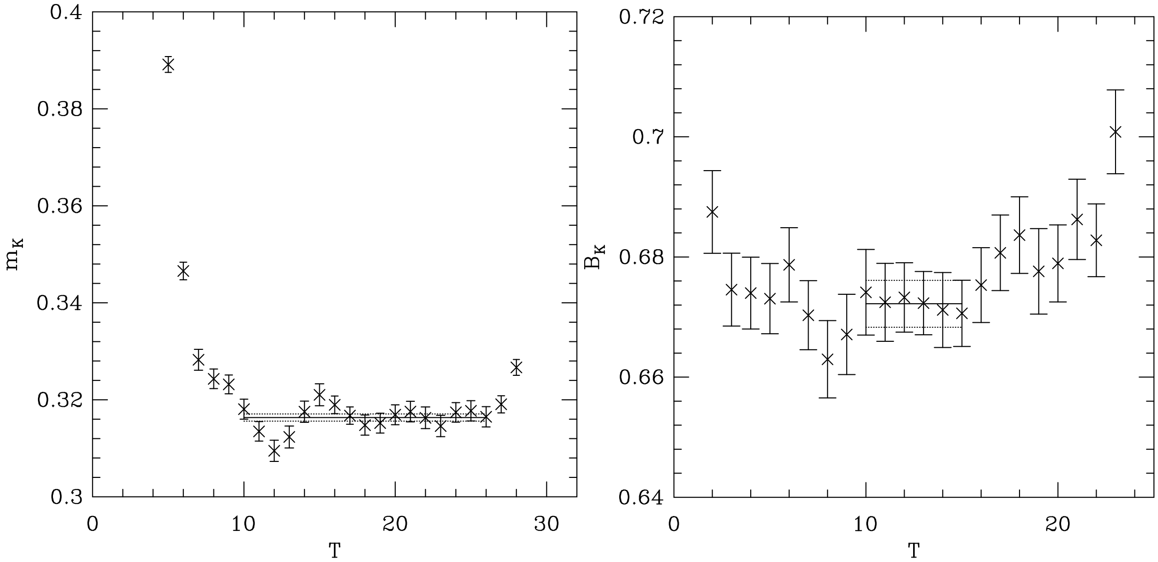

Figure 1: m_K vs. T (left) and B_K vs. T (right) at $m_s = m_d = 0.03$

combination ($m_x = m_y$) is

$$B_K = \tilde{c}_0\left(1+\frac{1}{48\pi^2 f^2}\left[I_{\text{conn}} + I_{\text{disc}} + \tilde{c}_1 m_{xy}^2 + \tilde{c}_3(2m_U^2 + m_S^2)\right]\right) \tag{1.2}$$

$$I_{\text{conn}} = 6m_{xy}^2 \tilde{l}(m_{xy}^2) - 12 l(m_{xy}^2) \tag{1.3}$$

$$I_{\text{disc}} = 0 \tag{1.4}$$

$$l(X) = X\log(X/\Lambda^2) + \text{F.V.} \tag{1.5}$$

$$\tilde{l}(X) = -[\log(X/\Lambda^2)+1] + \text{F.V.} \tag{1.6}$$

where $f = 132$ MeV, $\tilde{c}_i$ are unknown dimensionless low-energy constants and F.V. represents a finite volume correction. The notations for the various meson masses are as follows for those composed of sea quarks:

$$m_U^2 = 2\mu m_d, \qquad m_S^2 = 2\mu m_s, \qquad m_\eta^2 = (m_U^2 + 2m_S^2)/3 \tag{1.7}$$

and as follows for those composed of valence quarks

$$m_X^2 = 2\mu m_x, \qquad m_Y^2 = 2\mu m_y, \qquad m_{xy}^2 = \mu(m_x + m_y) \tag{1.8}$$

Here, we set sea quark masses to $m_u = m_d \neq m_s$ and the two valence quark masses are m_x and m_y. In this paper, we consider only the case of $m_x = m_y$ (*i.e.* $m_X^2 = m_Y^2 = m_{xy}^2$).

In Fig. 2, we plot B_K data as a function of kaon mass squared M_K^2. Here, all of them have degenerate valence quarks ($m_x = m_y$). We fit the data to the form of Eq. (1.2) suggested by chiral perturbation theory:

$$B_K = c_1\left(1+\frac{1}{48\pi^2 f^2}\left[I_{\text{conn}} + I_{\text{disc}}\right]\right) + c_2 m_{xy}^2 + c_4 m_{xy}^4 \tag{1.9}$$

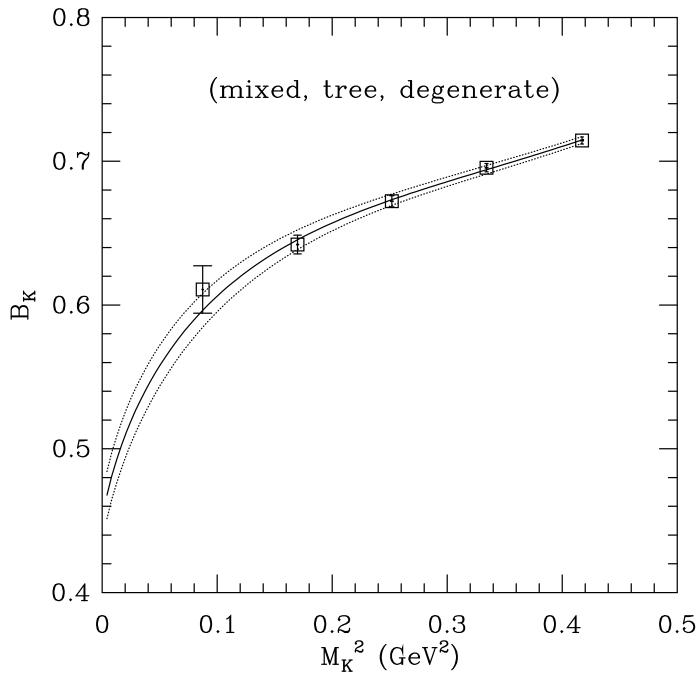

Figure 2: B_K vs. M_K^2

where the cut-off scale Λ is set to $\Lambda = 4\pi f$ and the remaining scale dependence is absorbed into c_2. The fitting results are summarized into Table 2.

In Table 2, the χ^2 is rather high when we take into account the fact that all the data are correlated. In fact, one can observe that the fitting curve does not fit the lightest two data points very well in Fig. 2. In other words, the fitting curve miss the lightest two data points in the opposite direction. In Ref. [4], it is pointed out that the contribution from the non-Goldstone pions are so significant that the curvature of the fitting curve becomes smoother, which is consistent with what we observe in Fig. 2. However, the full prediction from staggered chiral perturbation contains 21 unknown low-energy constants for a single lattice spacing and 37 unknown low-energy constants for the full analysis [4]. In order to determine all of them, it is required to carry out a significantly more extensive numerical work including data with non-degenerate quarks [5].

parameters	average	error
c_1	0.4488	0.0162
c_2	-1.4883	0.1883
c_3	$-$	$-$
c_4	1.4533	0.2207
χ^2/d.o.f.	0.6812	0.6

Table 2: Fitting results for B_K

In the current analysis of B_K data, we match the lattice results to the continuum values at the tree level. In this respect, the results are preliminary. Hence, we plan to calculate the one-loop corrections to the four fermion operators in near future. In addition, note that there exists a different approach to B_K using staggered fermions [6].

2. $B_7^{(3/2)}$ and $B_8^{(3/2)}$

In the case of direct CP violation ε'/ε, there are two contributions which interfere with each other destructively: the $\Delta I = 1/2$ part and $\Delta I = 3/2$ part. The $\Delta I = 3/2$ part is dominated by the electroweak penguin contribution from Q_8 and Q_7. In this section, we focus on the bag parameters of Q_8 and Q_7.

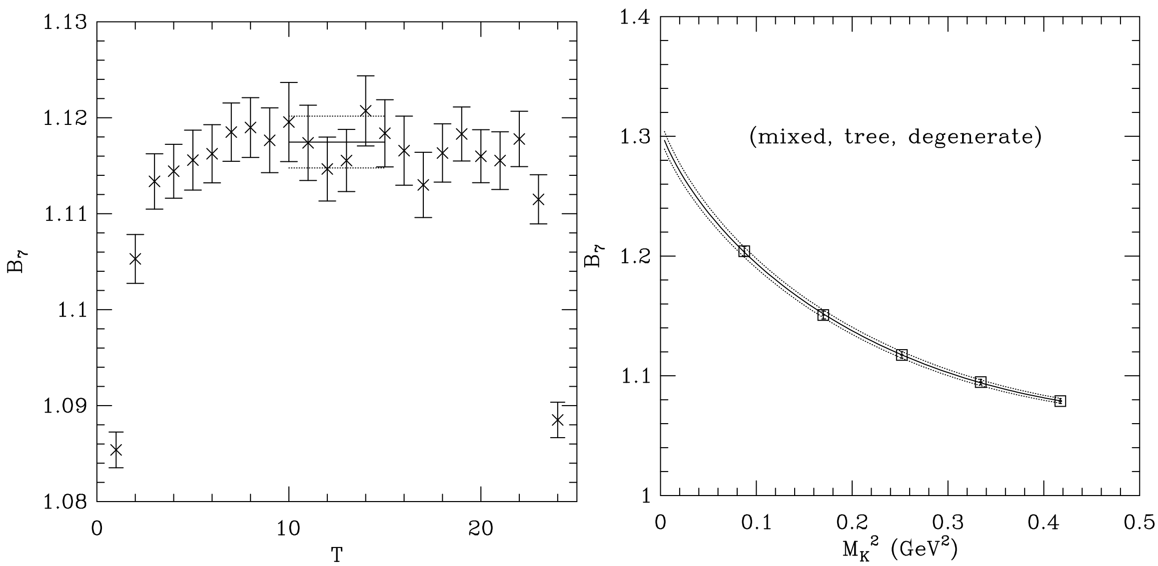

Figure 3: B_7 vs. T at $m_s = m_d = 0.03$ (left) and B_7 vs. M_K^2 (right)

The $B_7^{(3/2)}$ data as a function of Euclidean time T at quark mass $m_x = 0.03$ is presented in the lefthand side of Fig. 3. The fitting range is the same as for B_K. In the righthand side of Fig. 3, $B_7^{(3/2)}$ data is plotted as a function of M_K^2. We fit the data to the form suggested by chiral perturbation theory:

$$B_{7,8}^{(3/2)} = c_1 + c_2 m_{xy}^2 + c_3 m_{xy}^2 \log(m_{xy}^2/\Lambda^2) \tag{2.1}$$

The results are summarized in Table 3. Here, note that we match the lattice results to the continuum values at the tree level. In this respect, the results are preliminary.

parameters	$B_7^{(3/2)}$	$B_8^{(3/2)}$
c_1	1.3068 ± 0.0078	1.2505 ± 0.0074
c_2	-0.1889 ± 0.0093	-0.1439 ± 0.0091
c_3	0.4080 ± 0.0228	0.3175 ± 0.0222

Table 3: Fitting results for $B_7^{(3/2)}$ and $B_8^{(3/2)}$

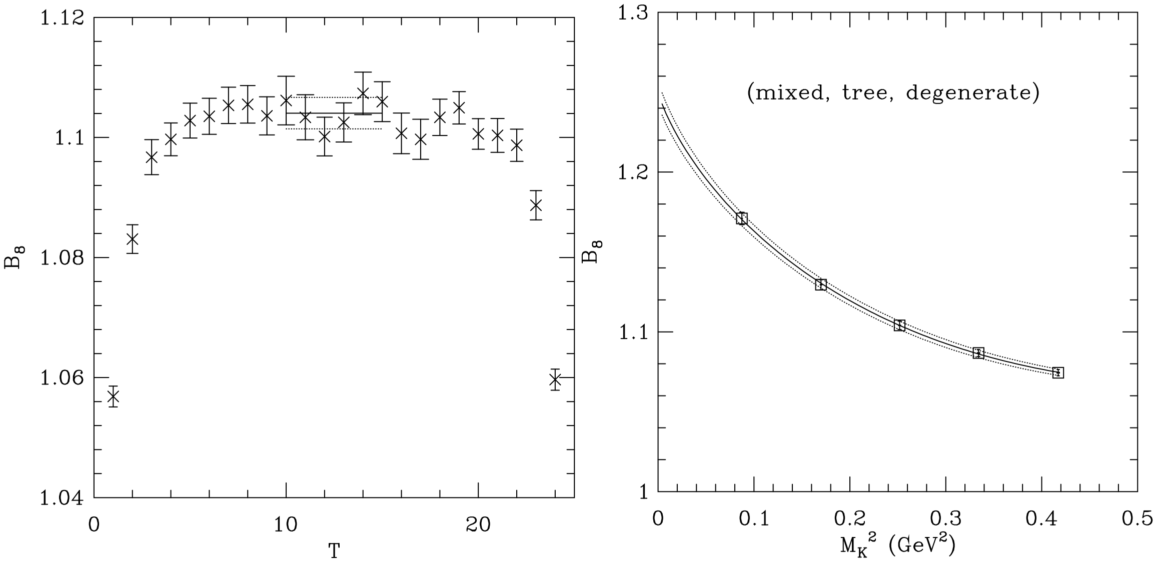

Figure 4: B_8 vs. T at $m_s = m_d = 0.03$ (left) and B_8 vs. M_K^2 (right)

We present $B_8^{(3/2)}$ data as a function of Euclidean time in the lefthand side of Fig. 4. We plot $B_8^{(3/2)}$ as a function of M_K^2 in the righthand side of Fig. 4. We fit the data to the form of Eq. (2.1) and the preliminary results are summarized in Table 3.

In order to perform a data analysis with higher precision, we need to know the chiral behavior of $B_8^{(3/2)}$ and $B_7^{(3/2)}$. Hence, we plan to calculate this using staggered chiral perturbation theory.

References

[1] Weonjong Lee *et al.*, *Testing improved staggered fermions with m_s and B_K*, *Phys. Rev.* D**71** (2005) 094501, [hep-lat/0409047].

[2] C. Aubin *et al.*, *Light hadrons with improved staggered quarks: approaching the continuum limit* , *Phys. Rev.* D**70** (2004) 094505, [hep-lat/0402030].

[3] Damir Becirevic and Giovanni Villadoro, *Impact of the finite volume effects on the chiral behavior of f_K and B_K* , *Phys. Rev.* D**69** (2004) 054010, [hep-lat/0311028].

[4] Ruth Van de Water and Stephen Sharpe, *B_K in Staggered Chiral Perturbation Theory* , [hep-lat/0507012].

[5] Taegil Bae, Jongjeong Kim and Weonjong Lee, *Non-degenerate quark mass effect on B_K with a mixed action*, PoS (LAT2005) 335.

[6] Elvira Gamiz, these proceedings.