PROCEEDINGS OF SCIENCE

Preliminary Study of B_K on 2+1 flavor DWF lattices from QCDOC

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I present some preliminary calculations of B_K on 2+1 flavor domain-wall fermion lattices from the QCDOC, including a set of $16^3 \times 32 \times 8$ lattices with a^{-1} near 1.6 GeV. Although a final result awaits the production of a much longer run, I will compare this preliminary value to previous results.

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1. Introduction

The study of hadrons allows experimental access to a wide variety of phenomena. However, at low energies QCD does not provide easy theoretical tools for parsing experimental results into measurements of fundamental parameters. Here, experimentalists may turn to lattice QCD to provide first-principles calculations of the factors required to connect experiment with theory.

In this proceeding, I will describe preliminary calculations of B_K on 2+1 flavor domain-wall fermion lattices.

2. The Kaon Bag Parameter

In the Standard Model, there are two sources of CP violation: direct and indirect. In indirect CP violation, decay into a disallowed number of pions is due a small amount of mixing between the K^0 and the $\overline{K^0}$. In order to use kaon data to constrain the unitarity triangle, we must disentangle the magnitude of kaon mixing, parametrized by the kaon bag parameter B_K .

It is defined by

$$B_K = \frac{\langle \overline{K^0} | \mathcal{O}_{LL}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}, \tag{2.1}$$

where M_K is the mass of the neutral kaon, f_K is the decay constant of the kaon (given by its coupling to the axial current), and $O_{LL}^{\Delta S=2} = (\bar{s}d)_L(\bar{s}d)_L$ is a four-quark operator coupling to left-handed quarks that changes strangeness by 2.

3. Challenges

In the chiral limit, B_K contains only the operator of the form $VV + AA$:

$$\mathcal{O}_{VV+AA}^{\Delta S=2} = (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu \gamma_5 d)(\bar{s}\gamma_\mu \gamma_5 d), \tag{3.1}$$

which may be derived from $L = (V - A)$ and parity symmetry, which forbids VA and AV terms. However, on the lattice we typically do not have the luxury of chiral symmetry; in this case, we must consider mixing with other chiral structures.

Even worse, it may be shown in chiral perturbation theory that these operators are not proportional to mass in the chiral limit, while the desired operator is. If significant mixing occurs, we will see almost nothing of the continuum operator we desire.

Domain-wall fermions offer a solution by allowing us to control the amount of chiral symmetry breaking by varying the size of the fifth dimension. The results of symmetry breaking are easily treated by the adding a residual mass. It is crucial that our residual mass be kept small, since the unwanted off-chirality terms will contribute as $O(m_{\text{res}}^2)$.

4. Our Lattices

Our action must be chosen to minimize chiral symmetry breaking while remaining computationally fast enough to produce thousands of lattices. Since this means our lattices will be rather

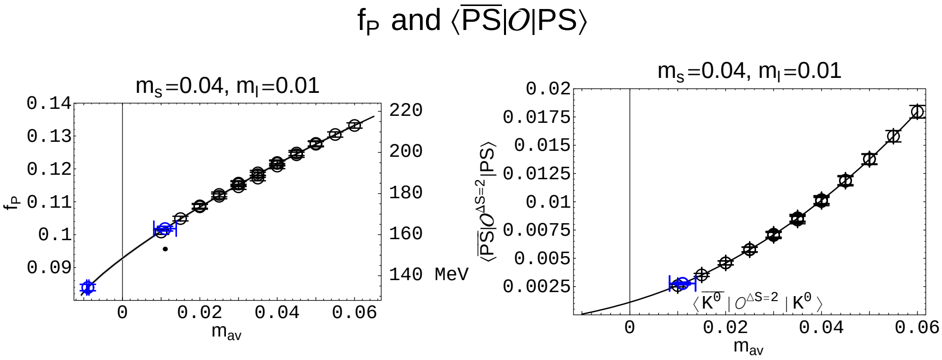

Figure 1: Left: Pseudoscalar decay constant as a function of average valence quark mass (blue points mark physical f_π and f_K); **Right**: $\langle\overline{\mathrm{PS}}|\mathscr{O}^{\Delta S=2}|\mathrm{PS}\rangle$ (blue point marks physical kaon mass)

coarse and our fifth dimension relatively small, we turn to improved actions to help lower the residual mass. Adding a rectangle term to the action accomplishes this by smoothing the gauge field at short distances. We use the DBW2 rectangle coefficient ($c_1 = -1.4069$) and gauge coupling $\beta = 0.72$ in a gauge action of the form $S_g = -\frac{\beta}{3}\sum_x\left((1-8c_1)\,\mathrm{Tr}\,U_{\mathrm{plaq}} + c_1\mathrm{Tr}\,U_{\mathrm{rect}}\right)$.

In summary, our lattices are $16^3\times 32\times 8$ and have bare sea quark masses $m_s = 0.04$ and $m_l \in (0.01, 0.02)$. We used the RHMC to generate 6000 trajectories and discarded 1000 for thermalization. We select configurations separated by 50 for analysis, giving us a total of 100 sets of weak matrix elements for each light sea quark mass. Each set combines nondegenerate pairs from valence quark masses $m_V \in (0.01, 0.02, \ldots, 0.06)$.

5. Preliminary Results

The rho mass determines our lattice scale in the physical sea and valence quark mass limit ; it is consistent with the scale determined from the static quark potential. We find $a^{-1} = 1.6(1)$GeV.

The residual mass is determined from the midpoint correlator, and agrees with the (negative) quark mass at which the pseudoscalar mass extrapolates to zero. We find $m_{\mathrm{res}} = 0.0106(1)$, which is fairly large, being larger than our lowest light quark mass (0.01).

The pseudoscalar mass then allows us to determine the light and strange quark masses. We find $m_l = 0.00171(21)$ and $m_s = 0.042(5)$. It is important that our strange quark mass be near our sea input $(0.04 + m_{\mathrm{res}})$, since we do not intend to extrapolate its value.

The pseudoscalar decay constant enters in the normalization of B_K . It may be determined from a ratio of the wall-point pseudoscalar correlator to the wall-wall pseudoscalar correlator.

$$f_P = \frac{2(m_q + m_{\mathrm{res}})}{M_P^2}\frac{\mathscr{C}^{PP}_{wp}(0,t)\sqrt{2M_P}}{\mathscr{C}^{PP}_{ww}(0,t)e^{-M_P t}V}, \tag{5.1}$$

where m_q and m_{res} are quark and residual mass, M_P is pseudoscalar meson mass, V is volume, and the $\mathscr{C}$'s are correlators with superscripts denoting the source and sink operators (both *P*seudoscalar in this case) and subscripts denoting source and sink shapes (*p*oint or *w*all).

We fit f_P to the chiral form

$$f_P = a + bm + cm\log m, \tag{5.2}$$

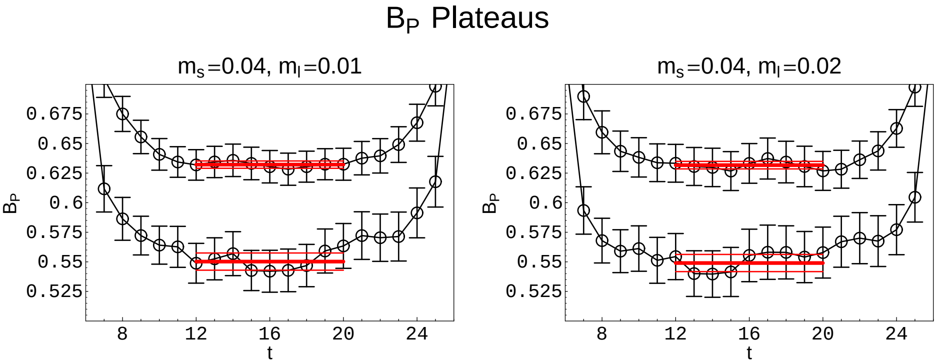

Figure 2: Plateaus of B_P for highest (0.06) and lowest (0.01) average valence quark mass

where $m = m_q + m_{res}$. See Fig. 1 (left).

The most naive way to derive the matrix element associated with B_K is simply to take the three-point correlator and divide by the wall-wall two-point correlator to remove the matrix elements associated with the wall sources:

$$\langle \overline{\mathrm{PS}} | \mathscr{O} | \mathrm{PS} \rangle = 4M_P \frac{\mathscr{C}^{P\mathscr{O}P}_{wpw}(t_{\mathrm{src}}, t, t_{\mathrm{snk}})}{\mathscr{C}^{PP}_{ww}(t_{\mathrm{src}}, t_{\mathrm{snk}})} \tag{5.3}$$

See Fig. 1 (right).

However, B_K may be derived at once without using the noisy wall-wall correlator by a clever use of wall-point correlators:

$$B_P = \frac{M_P^2 V}{2\frac{8}{3}(m_q + m_{\mathrm{res}})^2} \frac{\mathscr{C}^{P\mathscr{O}P}_{wpw}(t_{\mathrm{src}}, t, t_{\mathrm{snk}})}{\mathscr{C}^{PP}_{wp}(t_{\mathrm{src}}, t)\mathscr{C}^{PP}_{wp}(t, t_{\mathrm{snk}})} \frac{Z_{\mathscr{O}}}{Z_A^2} Z_A^2 Z_{\overline{\mathrm{MS}}}, \tag{5.4}$$

where $Z_A = 0.732$ is the renormalization factor of the axial current computed on our lattices, $Z_{\mathscr{O}}/Z_A^2 = 0.93$ is the renormalization factor of the three-point operator $\mathscr{O}^{\Delta S=2}_{LL}$, and $Z_{\overline{\mathrm{MS}}} = 1.02$ is the conversion factor from RI-MOM to MS-bar scheme. We take $Z_{\mathscr{O}}/Z_A^2$ from previous calculations on 2 flavor lattices and $Z_{\overline{\mathrm{MS}}}$ from previous quenched calculations.

We expect B_K to approach its asymptotic value far from the source and sink. Depicted in Fig. 2 are the heaviest and lightest valence quark masses ($m_V = 0.01$ and 0.06). The plateaus do not appear to have any unusual wiggles or trends.

B_K is fit to a form including the chiral fits to f_P and M_P:

$$\langle \overline{\mathrm{PS}} | \mathscr{O} | \mathrm{PS} \rangle = a + b \frac{M_P^2}{4\pi f_P} \log M_P^2 + c M_P^2, \tag{5.5}$$

where M_P is a function of quark mass, but f_P is taken in the chiral limit. See Fig. 3.

Taking the valence=sea point (red on Fig. 3) for each set of lattices, we may extrapolate to the chiral limit for the light quarks (leaving the strange quarks at the physical strange quark mass). This yields a final value (See Fig. 4.):

$$B_K = 0.571(17) \tag{5.6}$$

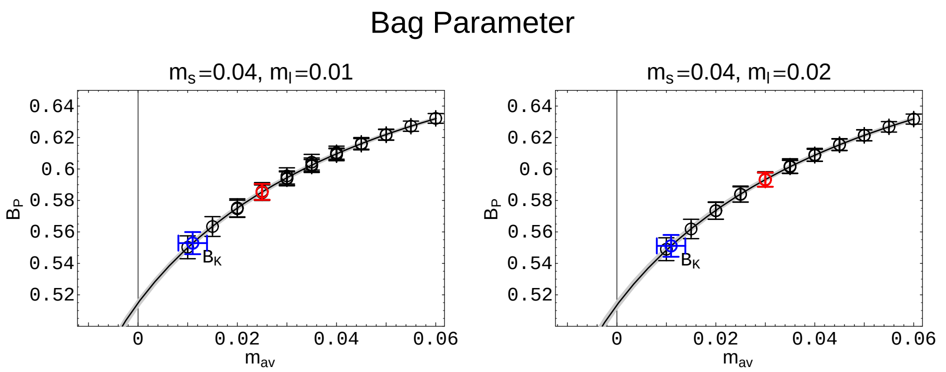

Figure 3: Pseudoscalar bag parameter as a function of average valence quark mass (blue point marks valence extrapolation to physical B_K, red point marks the valence=sea point)

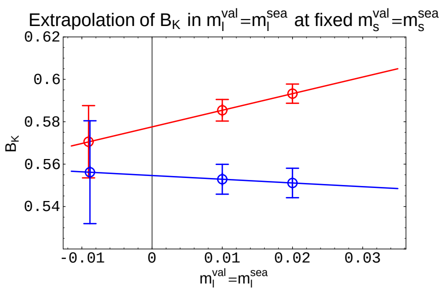

Figure 4: Two extrapolations of B_K to the physical point: **red:** extrapolates valence=sea to the physical point; **blue:** takes valence=physical, then extrapolates sea to the physical point

Alternatively, we may take the valence=physical limit first (blue on Fig. 3), taking advantage of the chiral fitting form, and then extrapolate linearly to the sea=physical limit. This alternate method yields a somewhat lower value of

$$B_K = 0.556(24) \tag{5.7}$$

6. Conclusions

Since neither extrapolation correctly accounts for nonlinear chiral terms, we take the average of the two and add a systematic error associated with the difference:

$$B_K = 0.563(15)(21), \tag{6.1}$$

where the first error is systematic and the second statistical. In either case, the preliminary 2+1 value of B_K is somewhat higher than the scale extrapolation from 2 flavor DWF values. See Fig. 5.

In the future, we wish to improve this calculation with larger volumes, smaller residual masses and a full nonperturbative treatment of renormalization. Larger volumes will allow us to use longer plateaus to reduce statistical error and also diminish any finite-volume effects. Smaller residual masses will further diminish mixing with wrong-chirality operators. A correct treatment of the factors $Z_{\mathscr{O}}$ and $Z_{\overline{\mathrm{MS}}}$ on 2+1 flavor lattices is necessary to complete this calculation, which will require NPR techniques.

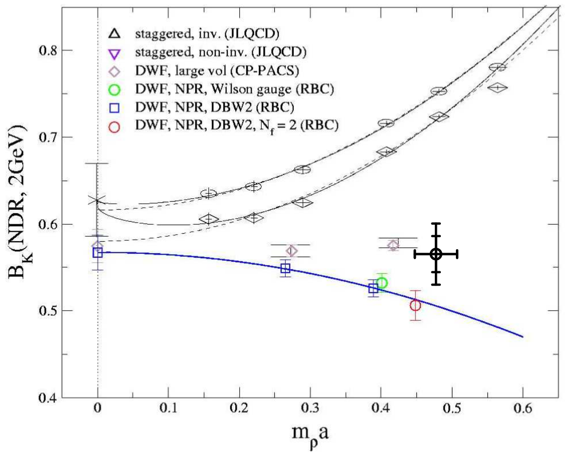

Figure 5: Dynamical extrapolation of B_K to the physical point of 2+1 flavor lattices at 1.6 GeV (marked in bold black) compared to other recent values

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