Dynamical determination of B_K from improved staggered quarks

Elvira Gámiz[*], **Sara Collins, Christine T.H. Davies**

Department of Physics & Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK
E-mail: e.gamiz@physics.gla.ac.uk, c.davies@physics.gla.ac.uk

Junko Shigemitsu

Physics Department, The Ohio State University, Columbus, OH 43210, USA
E-mail: shige@mps.ohio-state.edu

Matthew Wingate

Institute for Nuclear Theory, University of Washington, Seattle, WA 98115, USA
E-mail: wingate@phys.washington.edu

The scaling corrections that affected previous staggered calculations of B_K have been proved to be reduced by using improved actions (HYP, Asqtad) in the quenched approximation. This improved behaviour allows us to perform a reliable dynamical calculation of B_K including quark vacuum polarization effects using the MILC (2+1) flavour dynamical configurations. We report here on the results from such dynamical calculation. We also discuss the renormalization effects with the Asqtad action.

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[*]Speaker.

1. Introduction

In the last few years lattice calculations have started achieving the level of precision and the control of uncertainties necessary to extract phenomenologically relevant results [1]. Simulations with dynamical quarks are required for this task, since the uncontrolled errors associated with the quenched approximation are usually the main source of uncertainty in these calculations. An example of this fact can be seen in the study of indirect CP violation in the neutral kaon system.

The CP violating effects in $K^0 - \bar{K}^0$ mixing are parametrized by ε_K. Experimentally, this quantity is known with a few per-cent level precision. On the other hand, theoretically it is given by the hadronic matrix element between K^0 and $\bar{K}^0$ of the $\Delta S = 2$ effective hamiltonian

$$H_{eff}^{\Delta S=2} = C_{\Delta S=2}(\mu) \int d^4 x \, Q_{\Delta S=2}(x) \tag{1.1}$$

with

$$Q_{\Delta S=2}(x) = \left[\bar{s}_\alpha \gamma_\mu d_\alpha \right]_{V-A}(x) \left[\bar{s}_\beta \gamma^\mu d_\beta \right]_{V-A}(x). \tag{1.2}$$

The Wilson coefficient $C_{\Delta S=2}(\mu)$ is a perturbative quantity known to NLO in α_s in both NDR and HV schemes. It depends on several CKM matrix elements about which we would like to obtain information. The matrix element $\langle \bar{K}^0 | Q_{\Delta S=2} | K^0 \rangle$, that encodes the non-perturbative physics of the problem, is usually normalized by its VIA value, defining B_K as the ratio

$$B_K(\mu) \equiv \frac{\langle \bar{K}^0 | Q_{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K^0 \rangle}. \tag{1.3}$$

The main source of uncertainty when one tries to constrain the value of the combination of CKM matrix elements involved in the theoretical calculation of ε_K using its experimental value is the error associated to the determination of B_K [2]. Improvement in the calculation of B_K is thus crucial in order to get information about the unitarity triangle.

The value of B_K that the phenomenologists are using at present in their studies of the unitarity triangle [2], that is considered as the benchmark of the lattice calculations of this parameter, was obtained by the JLQCD collaboration [3] using unimproved staggered quarks in the quenched approximation. The value given in [3] is $B_K^{NDR}(2\text{ GeV}) = 0.628(42)$.

The main source of uncertainty in this calculation is the unknown error from quenching, that could be as large as a 15% according to the ChPT estimate performed in [4]. In order to have a prediction at a few per-cent level it is thus necessary to perform a dynamical calculation of B_K that eliminates the quenched uncertainties. Another drawback of the calculation in [3] is the fact that it is affected by large scaling uncertainties. We will see that the scaling behaviour is going to be much better using improved[1] staggered actions instead of the standard unimproved staggered action used by the JLQCD collaboration. A third way of improving the JLQCD calculation would be incorporating $SU(3)$ breaking effects by using kaons made up of non-degenerate quarks, instead of degenerate quarks with $m_s/2$.

The goal of this work is to perform a dynamical calculation of B_K that eliminates the quenched uncertainty, using improved staggered fermions that have been proved to reduce the large $\mathcal{O}(a^2)$ discretization errors generated by the taste-changing interactions.

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[1]In the improved actions the thin links are substituted by fattened links and the quark-gluon interactions that violate the taste symmetry are reduced.

β	Volume	n_{confs}	a^{-1}(GeV)	$m_s/2$
5.7	$12^3 \times 24$	150	0.837(6)	0.086/0.064
5.93	$16^3 \times 32$	50	1.59(3)	0.039/0.030

Table 1: Parameters in the quenched simulations. The values of $m_s/2$ are for the HYP and Asqtad staggered actions respectively.
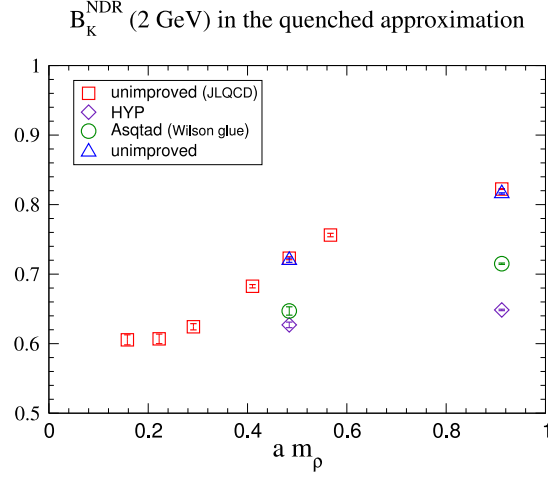

Figure 1: Scaling of $B_K^{NDR}(2\text{GeV})$ with a for improved staggered actions compared to the JLQCD unimproved staggered results.

Preliminary results for that study were presented in [5]. The matching coefficients needed in the calculation of the renormalized B_K with the action used in our dynamical simulations were not available at that moment, so an approximate renormalization was performed in order to get those preliminary results. The correct renormalization coefficients were calculated later [6] and have been used to obtain the results reported in these proceedings.

2. Improved versus unimproved staggered actions: scaling violations

The first thing we analyse is the impact of using improved staggered actions in comparison with the unimproved staggered action used in [3], that suffers from large scaling violations. We do this analysis for two different improved actions: the HYP [7] and the Asqtad[2] [8].

The values of the parameters used in the simulations are shown in Table 1. We choose these parameters to be the same as those used by the JLQCD collaboration in order to make a clear comparison with their results. In particular, we match kaon masses at a given β to those of the JLQCD collaboration. The matching of the lattice operators to the continuum ones in the $\overline{MS}$ scheme have been performed perturbatively using the one-loop coefficients calculated in [6].

The results we obtain for $B_K^{NDR}(2\text{GeV})$ as a function of the lattice spacing are shown in Figure 1. In that Figure a clear improvement in the scaling can be seen when using improved actions,

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[2]In our quenched analysis we are using a variation of the Asqtad action with no improvement in the gauge action.

β	n_{confs}	m_{sea}	$m_s/2$	$\alpha_V(1/a)$
6.76	560	0.01/0.05	0.02	0.4723
6.79	414	0.02/0.05	0.02	0.4699

Table 2: Parameters in the dynamical simulations.

in particular in the HYP case. We expect such improved scaling to survive unquenching and this therefore allows us to perform reliable dynamical calculations with only a few values of the lattice spacing and even obtain valuable information from a single point simulation.

3. Dynamical value of B_K

We now incorporate dynamical effects in the calculation of B_K using one of the improved staggered actions analyzed in the quenched approximation, Asqtad, since the final goal is to eliminate the irreducible systematic error associated to quenching that dominates the final uncertainty in previous determinations of B_K.

We performed a dynamical calculation of B_K with the Asqtad action, using the configurations from the MILC collaboration with $n_f = 2 + 1$ dynamical flavours [9]. The results reported in these proceedings correspond to the analysis at one lattice spacing with $a = 0.125$ fm and two different values of the sea light quark masses. The parameters used in the dynamical simulations are collected in Table 2.

The conversion of the values of the bare lattice operators to a value for $B_K^{NDR}(2\text{GeV})$ has been done perturbatively using the $\mathscr{O}(\alpha_s)$ lattice to continuum matching coefficients in [6]. In the matching process we take α_s in the V scheme at the scale $1/a$ where a is the lattice spacing -see values in Table 2. The value for α_V with $N_f = 3$ has been taken from the recent 4-loops lattice determination in [10].

The results we obtain for $B_K^{NDR}(2\text{GeV})$ including only statistical errors as a function of the light sea quark masses are shownin Figure 2. A decrease of the value of B_K with the reduction of the dynamical quark mass can be appreciated in that Figure. After linearly extrapolating these results to the physical s and $u(d)$ masses following the staggered chiral perturbation study in [11], the value of $B_K^{NDR}(2\text{GeV})$ we obtain is

$$B_K^{NDR}(2\text{GeV}) = 0.618(18)(19)(30)(130), \qquad (3.1)$$

where the first error is statistical, the second is from chiral fits, the third one is from discretizations errors and the final one is from the perturbative conversion to the $\overline{MS}$ scheme. The value in (3.1) is equivalent to $\hat{B}_K = 0.83 \pm 0.18$, with $\hat{B}_K$ defined as the product of $B_K^{NDR}(2\text{GeV})$ and the Wilson coefficient in the effective hamilton in (1.1) in the $\overline{MS} - NDR$ scheme and at 2GeV. This quantity is scheme and scale independent at $\mathscr{O}(\alpha_s)$. Note that this value of $\hat{B}_K$ is very similar to the previous quenched staggered result in [3] ($\hat{B}_K = 0.86(6)(14)$), so any final conclusion about the enhancement or decreasing due to the inclusion of dynamical effect in its calculation would need a drastic reduction of the error quoted in (3.1). That error is dominated by the uncertainty associated to the possible $\mathscr{O}(\alpha_s^2)$ corrections in the lattice to continuum matching process, that we estimate to be of
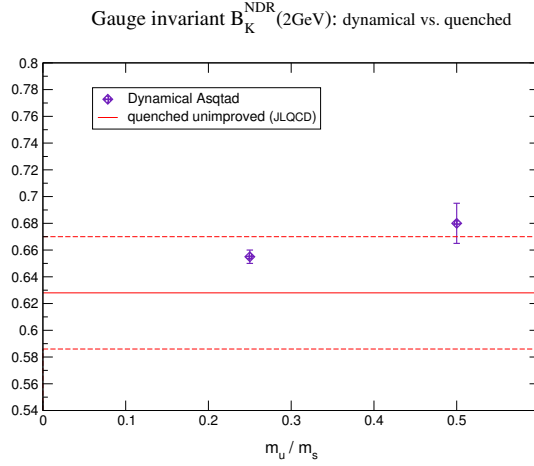
Figure 2: Dynamical value of $B_K^{NDR}(2\text{GeV})$ as a function of the ratio between the light sea quark mass and the (real) strange quark mass. The lines represent the quenched results from [3].

the order of $(\alpha_V(1/a))^2$. At the lattice spacing we are working, the unquenched values of α_V are given in Table 2 and translate into a $\sim 20\%$ error in the result for B_K.

4. Summary and discussion

Most of the previous lattice calculations of B_K, in particular that used in the unitarity triangle analysis, were performed in the quenched approximation. This induces a large, essentially unknown and irreducible systematic error into the result. Precise simulations with dynamical fermions are necessary in order to be able to make full use of the experimental data on ε_K to constrain the CKM matrix. These dynamical simulations are feasible with present computers using staggered fermions at light dynamical quark masses. However, the unimproved staggered action suffers from large taste-changing interactions that generate important scaling corrections, as those found by the JLQCD results. We have shown in Section 2 that the scaling behaviour is much better when using improved staggered actions, that have been designed to reduce the non-physical taste-changing interactions as well as other cut-off effects.

As a first step in the dynamical study of B_K on the MILC configurations we have calculated this quantity in two ensembles at $a = 0.125$ fm and with two different light sea quark masses. In doing that, we have used the recent results for the matching coefficients corresponding to the Asqtad action [6], the one used in the numerical simulations. The value of the renormalized B_K in the $\overline{MS}$ scheme we obtain is $B_K^{NDR}(2\text{GeV}) = 0.618 \pm 0.136$ or, equivalently, $\hat{B}_K = 0.83 \pm 0.18$. No sizeable deviation from quenched results can be inferred from this preliminary value. The uncertainty in this determination is dominated by the perturbative error associated to the one-loop matching process, so, in order to have the precision of a few per-cent needed by phenomenology, a two-loop matching or a numerical matching method is required.

The next step will be to redo our calculation on a finer lattice [12]. This new result for a

different lattice spacing will allow us to perform an appropriate continuum extrapolation[3], reducing the final discretization errors. The perturbative error will also be reduced since $\alpha_V(1/a)$ is smaller for finer lattices. And we will be able to check whether the good scaling behaviour observed in the quenched approximation using improved staggered actions (Asqtad) is in fact present in the dynamical results.

We also want to investigate other issues in the study of B_K, like the differences between using invariant and non-invariant gauge operators, the application of the staggered chiral perturbation theory results in [11], the impact of $SU(3)$ breaking effects or the chiral limit value of this quantity that could be compared to recent continuum calculations [13].

5. Acknowledgments

References

[1] C. T. H. Davies *et al.* [HPQCD Collaboration], Phys. Rev. Lett. **92** (2004) 022001 [hep-lat/0304004].

[2] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41** (2005) 1 [hep-ph/0406184]; M. Bona *et al.* [UTfit Collaboration], J. High Energy Phys. **0507** (2005) 028 [hep-ph/0501199].

[3] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **80** (1998) 5271 [hep-lat/9710073].

[4] S. R. Sharpe, Nucl. Phys. Proc. Suppl. **53** (1997) 181 [hep-lat/9609029].

[5] E. Gámiz, S. Collins, C. T. H. Davies, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Nucl. Phys. Proc. Suppl. **140** (2005) 353 [hep-lat/0409049].

[6] T. Becher, E. Gámiz and K. Melnikov, hep-lat/0507033.

[7] A. Hasenfratz and F. Knechtli, Phys. Rev. D **64** (2001) 034504 [hep-lat/0103029].

[8] S. Naik, Nucl. Phys. B **316** (1989) 238; G. P. Lepage, Phys. Rev. D **59** (1999) 074502 [hep-lat/9809157]; K. Orginos, D. Toussaint and R. L. Sugar [MILC Collaboration], Phys. Rev. D **60** (1999) 054503 [hep-lat/9903032].

[9] C. W. Bernard *et al.*, Phys. Rev. D **64** (2001) 054506 [hep-lat/0104002].

[10] Q. Mason *et al.* [HPQCD Collaboration], Phys. Rev. Lett. **95** (2005) 052002 [hep-lat/0503005].

[11] R. S. Van de Water and S. R. Sharpe, hep-lat/0507012, see also these proceedings.

[12] S. Collins, C. T. H. Davies, E. Gámiz, J. Shigemitsu and M. Wingate [HPQCD Collaboration], in preparation.

[13] J. Prades, J. Bijnens and E. Gámiz, hep-ph/0501177; O. Catà and S. Peris, J. High Energy Phys. **03** (2003) 060 [hep-ph/0303162]; S. Peris and E. de Rafael, Phys. Lett. B **490** (2000) 213 [Erratum, hep-ph/0006146]; J. Bijnens and J. Prades, J. High Energy Phys. **01** (2000) 002 [hep-ph/9911392].

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[3]In view of the improved scaling behaviour we obtained in the quenched approximation within the Asqtad action described in Section 2, results for two lattice spacings will be enough to perform a reliable continuum limit.