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$K \to \pi\pi$ from electroweak penguins in $N_f = 2$ domain-wall QCD

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We present the calculation of $K \to \pi\pi$ matrix elements of electroweak penguin operators *i.e.* Q_7 and Q_8. In the numerical simulation, we use the gauge configurations generated by the combination of $N_f = 2$ domain-wall fermions and DBW2 gauge action. From $K \to \pi$ and $K \to 0$ matrix elements on the lattice, we construct $K \to \pi\pi$ matrix elements at next-to-leading order in the chiral expansion by using recent analytic results. Renormalization factor of these matrix elements are obtained by the non-perturbative renormalization technique in RI/MOM scheme. Brief discussion based on our results are made as well as a comparison with previous works.

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1. Introduction

In the efforts toward the theoretical treatment of the non-leptonic kaon decay, ε'/ε, the direct CP violation parameter of $K \to \pi\pi$, is one of the important focuses. This quantity is approximated to a linear combination of the $K \to \pi\pi$ matrix elements of the local operators Q_6 and Q_8 in the $\Delta S = 1$ effective hamiltonian $H_W^{(\Delta S=1)} = \sum_{i=1}^{10} W_i(\mu) Q_i(\mu)$ (where $W_i(\mu)$ is the Wilson coefficients obtained by perturbative calculations). However, calculations in lattice QCD (e.g. [1, 2]) resulted in unacceptable central values of ε'/ε due to a small magnitude of $\langle \pi\pi_{(I=0)} | Q_6 | K \rangle$ compared to $\langle \pi\pi_{(I=2)} | Q_8 | K \rangle$. In these works, $K \to \pi\pi$ matrix elements are obtained through the low energy constants (LECs) extracted from $K \to \pi$ and $K \to 0$ matrix elements on the lattice. Because this method is based on the lowest order (LO) chiral perturbation theory (ChPT), results could change significantly by taking the next-to-leading order (NLO) effects into account.

For electroweak penguin operators $Q_{\rm ewp} = Q_{7,8}$, there are limited number of NLO ChPT operators due to their lower mass dimension than operators in other classes. By recent efforts [3, 4, 5], there arose a prospection that the LO method can be extended to NLO for $Q_{\rm ewp}$. In particular, the authors of Ref. [5] have shown that it is possible to extract all LECs needed to construct $K \to \pi\pi$ at NLO from $K \to \pi$ and $K \to 0$ matrix elements in the framework of partially quenched ChPT (PqChPT). Following the line, we calculate $K \to \pi\pi$ matrix elements of electroweak penguins on dynamical gauge configurations.

2. Numerical simulation and analysis

We use $N_f = 2$ gauge configurations generated by the RBC Collaboration [6] using domain-wall fermions with $L_s = 12$ and $M_5 = 1.8$ and DBW2 gauge action with $\beta = 0.80$ in the $16^3 \times 32$ box. There are ensembles with $m_{\rm sea} = 0.02$, 0.03 and 0.04, each of which 94 configurations are available. $a^{-1} = 1.69(5)$ GeV and $am_{\rm res} = 0.00137(5)$ have been obtained in Ref. [6].

Electroweak penguin operators are defined as

$$Q_7 = (\bar{s}_a L_\mu d_a) \sum_{q=u,d,s} e_q (\bar{q}_b R_\mu q_b), \quad Q_8 = (\bar{s}_a L_\mu d_b) \sum_{q=u,d,s} e_q (\bar{q}_b R_\mu q_a), \tag{2.1}$$

where $L_\mu = (1-\gamma_5)\gamma_\mu$ and $R_\mu = (1+\gamma_5)\gamma_\mu$, and $(e_u, e_d, e_s) = (2/3, -1/3, -1/3)$. Roman indices represent color. Each operator is divided into two contributions with $\Delta I = 1/2$ and $3/2$ as $Q_{\rm ewp} = Q_{\rm ewp}^{(1/2)} + Q_{\rm ewp}^{(3/2)}$. We calculate correlation functions with the wall pseudo-scalar operator $P(t)$ and the point operator $Q_{\rm ewp}(\mathbf{x}, t)$ for each $m_{\rm sea}$. With valence quark masses $m_{\rm val} = 0.01, 0.02, 0.03, 0.04, 0.05$, matrix elements are computed as a plateau in the t-dependence of the ratios

$$\left\langle \pi^+ \left| Q_{\rm ewp}^{\rm (latt)} \right| K^+ \right\rangle = \frac{\sum_{\mathbf{x}} \langle 0 | P(t_f) Q_{\rm ewp}(\mathbf{x}, t) P^\dagger(t_i) | 0 \rangle}{\langle 0 | P(t_f) P^\dagger(t_i) | 0 \rangle} \times 2 m_{PS}, \tag{2.2}$$

$$\left\langle 0 \left| Q_{\rm ewp}^{\rm (latt)} \right| K^0 \right\rangle = \frac{\sum_{\mathbf{x}} \langle 0 | Q_{\rm ewp}(\mathbf{x}, t) P^\dagger(t_i) | 0 \rangle}{\langle 0 | P(t) P^\dagger(t_i) | 0 \rangle} \times \sqrt{\frac{2 m_{PS'}}{V} \cdot {\rm Amp}^{(PS')}}, \tag{2.3}$$

where m_{PS} and $m_{PS'}$ are pseudo-scalar meson masses for degenerate and non-degenerate quark, respectively. ${\rm Amp}^{(PS')}$ is the amplitude of the PP-correlator. Locations of sources are $(t_i, t_f) = (5, 27)$ and we use $t = 14$–17 for the fit range.

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One may write $Q_{\rm ewp}$ as an expansion of ChPT operators with $(8_L, 8_R)$ representation:

$$Q_{\rm ewp} = \alpha_{88}\left(\Sigma_{13}\Sigma^\dagger_{21}\right) + \sum_{i=1}^{6} c_i \mathscr{O}_i^{(\rm ewp)} + O(p^4), \tag{2.4}$$

where $\mathscr{O}_i^{(\rm ewp)}$ are the NLO operators. Functions to which we fit our numerical results are [5]

$$\begin{aligned}
\left\langle \pi^+ \left| Q^{(1/2)}_{\rm ewp} \right| K^+ \right\rangle &= \alpha_{88}\left[\frac{1}{2\pi^2 f^4}\left\{m^2_{PS}\ln\frac{m^2_{PS}}{\mu^2} - 4(m^2_{PS}+m^2_{SS})\ln\left(\frac{m^2_{PS}+m^2_{SS}}{2\mu^2}\right) + m^2_{PS}\right\}\right. \\
&\left. -\frac{16}{f^3}(f_{PS}-f) + \frac{8}{f^2}\right] + \xi^{(1/2)}\frac{4}{f^2}m^2_{PS} + c^r_6\frac{32}{f^2}m^2_{SS} + O(m^4_{PS}), \qquad (2.5) \\
\left\langle \pi^+ \left| Q^{(3/2)}_{\rm ewp} \right| K^+ \right\rangle &= \alpha_{88}\left[\frac{-1}{2\pi^2 f^4}\left\{m^2_{PS}\ln\frac{m^2_{PS}}{\mu^2} + 8(m^2_{PS}+m^2_{SS})\ln\left(\frac{m^2_{PS}+m^2_{SS}}{2\mu^2}\right) + m^2_{PS}\right\}\right. \\
&\left. -\frac{8}{f^3}(f_{PS}-f) + \frac{4}{f^2}\right] + \xi^{(3/2)}\frac{4}{f^2}m^2_{PS} + c^r_6\frac{16}{f^2}m^2_{SS} + O(m^4_{PS}), \qquad (2.6) \\
\left\langle 0 \left| Q^{(1/2)}_{\rm ewp} \right| K^0 \right\rangle &= \frac{\alpha_{88}}{4\pi^2 f^3}\left[2m^2_{PS'}\ln\frac{m^2_{PS'}}{\mu^2} - m^2_{PS}\ln\frac{m^2_{PS}}{\mu^2} - (2m^2_{PS'}-m^2_{PS})\ln\left(\frac{2m^2_{PS'}-m^2_{PS}}{\mu^2}\right)\right. \\
&\left. +2m^2_{sS}\ln\frac{m^2_{sS}}{\mu^2} + 2m^2_{uS}\ln\frac{m^2_{uS}}{\mu^2}\right] - \frac{8c^r_4}{f}(m^2_{PS'}-m^2_{PS}) + O(m^4_{PS}), \qquad (2.7)
\end{aligned}$$

where $m^2_{SS} = m^2_{PS(m_{\rm val}=m_{\rm sea})}$. For a set of non-degenerate valence quark masses such that $m_{\rm val1} < m_{\rm val2}$, we define $m^2_{sS} = m^2_{PS'(m_{\rm val1}=m_{\rm sea})}$ and $m^2_{uS} = m^2_{PS'(m_{\rm val2}=m_{\rm sea})}$. Pseudo-scalar decay constant f_{PS} computed with degenerate quark masses is extrapolated to f in the chiral limit. After cancelling the 1-loop divergences from the first term, there remain constant coefficients c^r_i as LECs for NLO. Note that, in above functions, the number of LECs reduced from seven to five: $\left\{\alpha_{88}, \xi^{(1/2)}, \xi^{(3/2)}, c^r_4, c^r_6\right\}$, where $\xi^{(1/2)} = -c^r_1 + c^r_2 + 2c^r_3 + 10c^r_4 + 8c^r_5$ and $\xi^{(3/2)} = -c^r_1 - c^r_2 + 4c^r_4 + 4c^r_5$. One can easily check that these LECs successfully construct $K \to \pi\pi$ matrix elements at NLO [5].

3. Non-perturbative renormalization

Before computing $K \to \pi\pi$ matrix elements, we renormalize lattice values non-perturbatively in RI/MOM scheme [7]. Numerical simulation for this step is carried out on 48,102 and 67 configurations for $m_{\rm sea} = 0.02, 0.03$ and 0.04, respectively. The main procedure is based on the method in Ref. [2]. We consider the renormalization factor of electroweak penguin Z as a 2×2 matrix to solve the mixing between Q_7 and Q_8. The renormalization condition is

$$Z_q^{-2} Z_{ij} \Gamma^{\rm latt}_{Q_j}(p_1, p_2) = \Gamma^{\rm tree}_{Q_i}, \tag{3.1}$$

where $\Gamma^{\rm latt}_{Q_i}$ is the amputated Green's function on the lattice in the momentum space.

In the case of $\Delta I = 1/2$, there is a contribution from the disconnected diagram in which an one loop contraction of Q_i exchanges gluons with the spectator. For its reasonable implementation, we assign two momenta p_1 and p_2 which satisfy $|p_1| = |p_2| = |p_1 - p_2|$ to each part of diagram

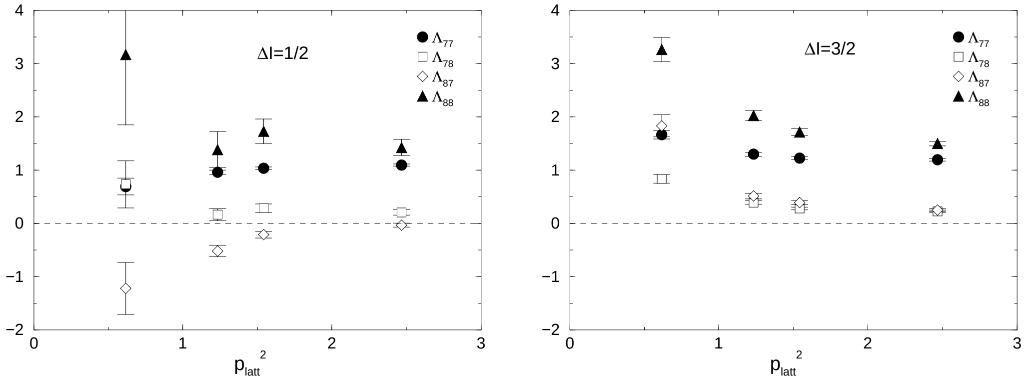

Figure 1: Λ_{ij} for $\Delta I = 1/2$ (left) and $3/2$ (right) in the chiral limit as a function of $p^2_{\rm latt}$. In each panel, diagonal (off-diagonal) elements are indicated by filled (open) symbols.

so that all relevant momenta have same magnitude. In terms of lattice momentum $p_{\rm latt} \cdot 16/(2\pi) = (n_x, n_y, n_z, n_t)$, four sets $(1,1,1,1), (0,2,2,0), (1,1,2,2), (2,2,2,2)$ and 4,8,8,4 equivalent partners to each are employed. We average results over the sets of momenta with same magnitude.

Another issue for $\Delta I = 1/2$ is the mixing with lower dimension operators. As discussed in Ref. [2], this mixing with the two major operators can be subtracted as

$$Q_i^{(1/2)\rm subt} = Q_i^{(1/2)} + c_i^1 \cdot (\bar{s}d) + c_i^2 \cdot \bar{s}(\overleftarrow{D}_s + \overrightarrow{D}_d)d \tag{3.2}$$

by determining the mixing coefficients $c^1_{7,8}$ and $c^2_{7,8}$. As a result of the procedure described in in Ref. [2], we find the effect of subtraction is $\lesssim 1\%$ for the diagonal elements and $\lesssim 6\%$ for the off-diagonal ones.

From (3.1), the renormalization factor $Z_q^{-2} Z_{ij}$ is the inverse of $\Lambda_{ij} \equiv {\rm Tr}[\Gamma^{\rm latt}_{Q_i} \Gamma^{\rm tree}_{Q_j}]$. After the chiral extrapolation using the data with $m_{\rm sea} = m_{\rm val}$, we show Λ_{ij} in Figure 1 as a function of $p^2_{\rm latt}$ for $\Delta I = 1/2$ (left panel) and $3/2$ (right panel). Momentum dependences for $p^2_{\rm latt} \lesssim 1.5$ imply the contamination of low energy effects. To avoid this, we employ the data at the largest momentum $(ap_{\rm latt})^2 = 2.467$ for the rest of analysis. Given differences between two largest momentum are $\lesssim 2\sigma$ except for the 87-elements for both ΔI, we expect this contamination is under control. This small difference also moderates the worry that the $O(a^2)$ error is too large for the NPR technique to work correctly.

After obtaining Z_{ij} by an alternative calculation of Z_q, we move to $\overline{\rm MS}$ with NDR through the perturbative procedure [8]. At $\mu = a^{-1}$, our preliminary results are

$$Z^{(\Delta I=1/2)}_{\overline{\rm MS},\,\rm NDR} = \begin{bmatrix} 0.599(21) & -0.059(38) \\ 0.010(30) & 0.557(93) \end{bmatrix}, \quad Z^{(\Delta I=3/2)}_{\overline{\rm MS},\,\rm NDR} = \begin{bmatrix} 0.564(22) & -0.060(15) \\ -0.121(21) & 0.550(30) \end{bmatrix}, \tag{3.3}$$

where only the statistical errors are considered. While the diagonal elements are $\approx 40\%$ smaller than the perturbative results [9], similar results have been obtained by RI/MOM in Ref. [10].

4. Results and discussion

We fit the renormalized matrix elements to (2.5), (2.6) and (2.7) simultaneously to ensure

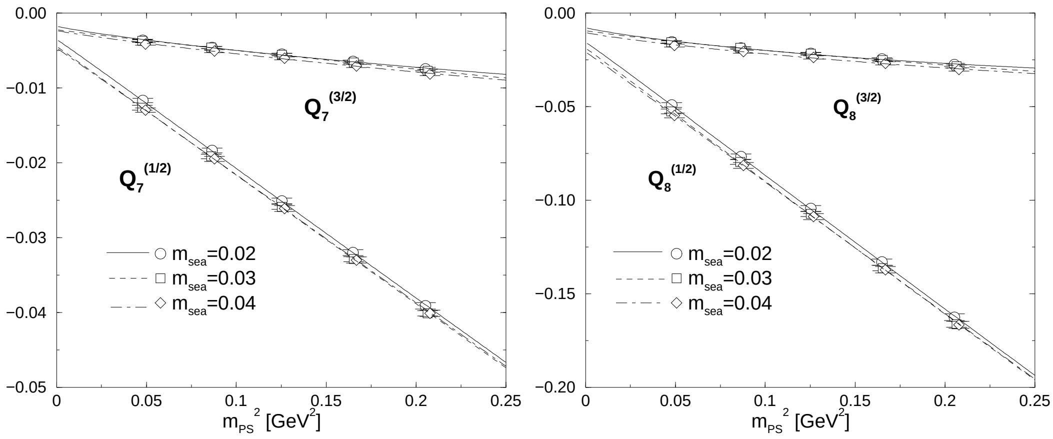

Figure 2: Renormalized $K \to \pi$ matrix elements of Q_7 (left) and Q_8 (right) as a function of m_{PS}^2 [GeV^2]. In each panel, matrix elements with $\Delta I = 1/2$ and $3/2$ are presented for $m_{\rm sea} = 0.02$ (circles), 0.03 (squares) and 0.04 (diamonds). Fit curves are obtained from simultaneous fits to the PqChPT results.

common values of α_{88} and c_6^r. This fit is repeated independently for each $m_{\rm sea}$. Using all data points (5 points for $K \to \pi$ and 10 points for $K \to 0$), we find reasonable quality of fit with $0.2 < \chi^2/\mathrm{dof} < 0.7$. In Figure 2, we plot $K \to \pi$ matrix elements of Q_7 (left panel) and Q_8 (right panel) and their fit curves for each $m_{\rm sea}$.

By using LECs from each $m_{\rm sea}$ and physical meson masses and decay constants, we compute $K \to \pi\pi$ matrix elements at NLO. For comparison, we also obtain the LO value by neglecting higher order terms with same α_{88}. Figure 3 summarises the results of $K \to \pi\pi$ matrix elements in the physical unit with $\Delta I = 3/2$ for Q_7 in left panel and Q_8 in right panel. The horizontal lines show the average over $m_{\rm sea}$'s with the weights of the errors. Results from previous quenched calculations Refs.[1, 2, 11, 12] are plotted in filled diamonds for comparison. We observe large enhancements from LO to NLO sandwiching all of quoted results both for Q_7 and Q_8.

Because NLO analysis of $\langle \pi\pi_{(I=0)} | Q_6 | K \rangle$ is not available currently, it is not realistic to estimate ε'/ε using our NLO result of $\langle \pi\pi_{(I=2)} | Q_8 | K \rangle$. Instead of that, we could use the LO result from the NLO analysis in the estimation in the framework of LO. It should be noted that we obtain consistent values to our NLO results by the LO method employed in Refs. [1, 2]. Therefore, assuming similar gap we have seen between NLO and LO in the quenched calculations, there should have been a large enhancements from higher order effects in Refs. [1, 2]. Then it is conceivable to obtain a fair value of ε'/ε by using $\langle \pi\pi_{(I=2)} | Q_8 | K \rangle$ after removing large higher order effects by the NLO analysis, which is only possible for unquenched calculation. However, more careful study of the systematics must be done for actual conclusion.

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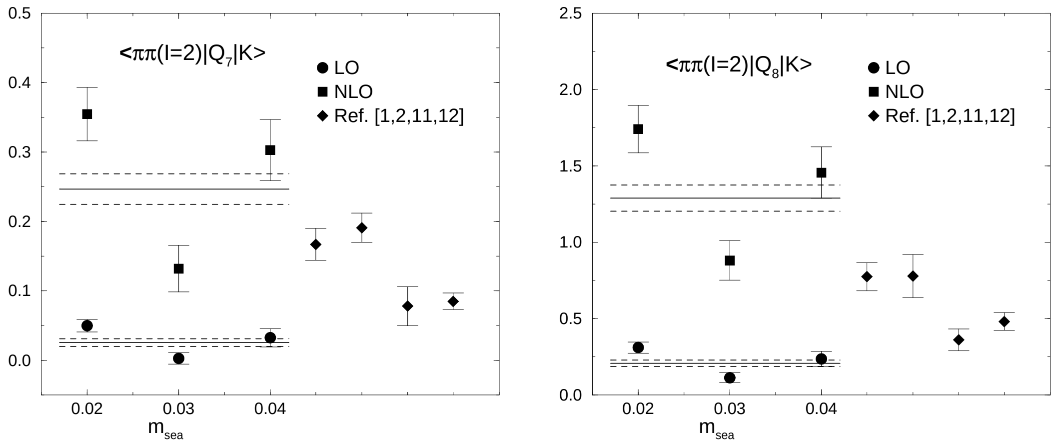

Figure 3: Results of $K \to \pi\pi$ matrix elements with $\Delta I = 3/2$ for Q_7 (left) and Q_8 (right) in the physical unit GeV^3. Averages over $m_{\rm sea}$'s in LO (circles) and NLO (squares) are indicated by horizontal lines. Filled diamonds are the results from previous works [1], [2], [11] and [12], from left to right.

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