PROCEEDINGS OF SCIENCE

Kaon semileptonic decay form factors in two-flavor QCD

JLQCD Collaboration:

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We present a calculation of the kaon form factors in two-flavor QCD with the non-perturbatively $O(a)$-improved Wilson quark action. In order to achieve a few percent accuracy in the study of SU(3) breaking effects, we use a set of double ratios of the matrix elements, with which the bulk of the statistical fluctuation and the multiplicative renormalization factors cancel.

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1. Introduction

One of the most important quantities in the low energy kaon physics is the form factor f_+ at zero momentum transfer, which provides a theoretical input for the determination of the Kobayashi-Maskawa matrix element $|V_{us}|$ through the K_{l3} decay.

For this form factor it is known that

1. The leading contribution is unity, which is exact in the SU(3) limit,

2. There is no contribution of $O(m_s - \overline{m})$ due to the Ademollo-Gatto theorem where $\overline{m} = (m_u + m_d)/2$.

In 1984, Leutwyler and Roos estimated this form factor analytically [1] and their result $f_+(0) = 0.961 \pm 0.008$ has been used as the standard in the phenomenological analysis. Though the leading correction was determined unambiguously using chiral perturbation theory, the next-to-leading order correction was estimated using a model of the wave function of the pseudoscalar meson, with which only a crude error estimate is available. The numerical simulation of lattice QCD could help this situation. Indeed, a quenched result appears recently [2] and several groups carry out the unquenched calculation with different lattice setups [3].

Our calculation is performed on two-flavor QCD gauge configurations generated with the non-perturbatively $O(a)$-improved Wilson fermion by the JLQCD Collaboration [4]. The lattice size is $20^3 \times 48$ at $\beta = 5.2$ and 12,000 HMC trajectories are accumulated. Five sea quark masses corresponding to $m_{PS}/m_V \sim 0.8 - 0.6$ enable us to make a careful study of the quark mass dependence of the form factors. Especially, we focus on the consistency between the lattice data and the ChPT predictions. The pion form factors are obtained as a by-product of the calculation. The results are presented in the poster session by S. Hashimoto [5].

2. Kaon form factors

The kaon semileptonic decay form factors $f_+(q^2)$ and $f_-(q^2)$ are defined as

$$\langle \pi(\vec{p}) | V_\mu | K(\vec{k}) \rangle = f_+(q^2)(k+p)_\mu + f_-(q^2)(k-p)_\mu, \tag{2.1}$$

where $q^2 = (k-p)^2$ and V_μ is the vector part of the weak current. Reliable theoretical estimate of $f_+(0)$ is necessary for the precise determination of the Kobayashi-Maskawa matrix element $|V_{us}|$ from the K_{l3} decay. Our goal is to calculate the form factor in a few percent accuracy from the first principle of QCD. To do that, careful study of SU(3) breaking effects and the chiral extrapolation are vital.

The scalar form factor $f_0(q^2)$ can be expressed as a linear combination of $f_+(q^2)$ and $f_-(q^2)$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2), \tag{2.2}$$

and the ratio of $f_-(q^2)$ and $f_+(q^2)$ is denoted as $\xi(q^2)$

$$\xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}. \tag{2.3}$$

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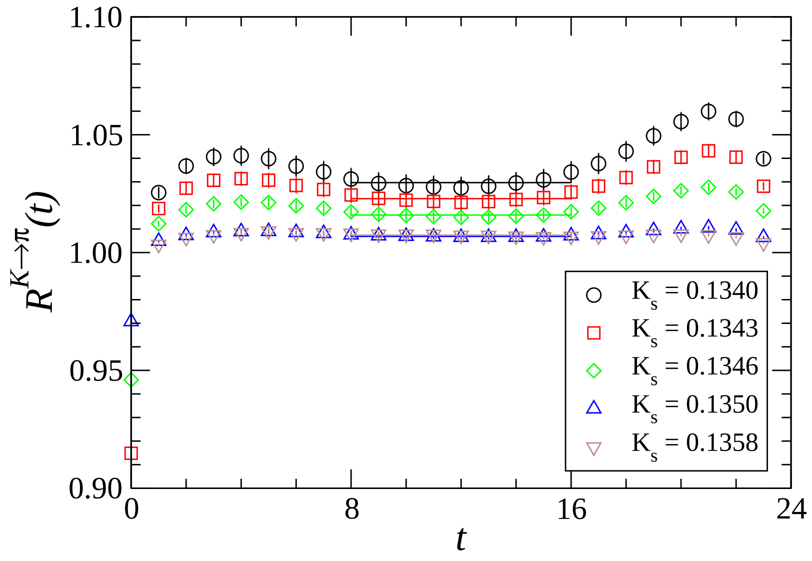


Figure 1: Double ratio $R^{K\to\pi}(t)$ for the scalar form factor $f_0(q^2_{max})$ at $K_{sea} = 0.1355$.

3. Lattice calculation

Our calculation proceeds in three steps, and the form factor $f_+(0)$ is expressed by a product of three factors

$$f_+(0) = f_0(q^2_{max}) \times \frac{f_+(0)\left[1+\xi(0)\dfrac{m_K-m_\pi}{m_K+m_\pi}\right]}{f_0(q^2_{max})} \times \frac{1}{1+\xi(0)\dfrac{m_K-m_\pi}{m_K+m_\pi}}. \tag{3.1}$$

In the first step, the scalar form factor at the maximum momentum transfer squared $q^2_{max} = (m_K - m_\pi)^2$ is estimated and the interpolation to $q^2 = 0$ is done in the second step. The unnecessary contribution from $\xi(0)$ is subtracted in the last step. A double ratio of the three-point functions is used to obtain each factor. We explain the details of each step in the following subsections.

3.1 Double ratio I

The scalar form factor at the maximum momentum transfer squared $f_0(q^2_{max})$ is obtained from the double ratio of three-point functions

$$R^{K\to\pi}(t) = \frac{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})C_{K V_4 \pi}(t,T/2;\vec{0},\vec{0})}{C_{\pi V_4 \pi}(t,T/2;\vec{0},\vec{0})C_{K V_4 K}(t,T/2;\vec{0},\vec{0})} \to \left[\frac{m_K+m_\pi}{2\sqrt{m_K m_\pi}} f_0(q^2_{max})\right]^2, \quad 0 \ll t \ll T/2, \tag{3.2}$$

where the three-point function is given by

$$C_{\pi V_\mu K}(t_x,t_y;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle O_\pi(t_y,\vec{y}) V_\mu(t_x,\vec{x}) O_K(0)\rangle e^{+i\vec{q}\cdot\vec{x}} e^{-i\vec{p}\cdot\vec{y}}, \tag{3.3}$$

and O_π, O_K is the interpolating operator of pion and kaon. This double ratio is essentially the same as that used before in the calculation of the $B \to D l\nu$ form factor by the Fermilab group [6].

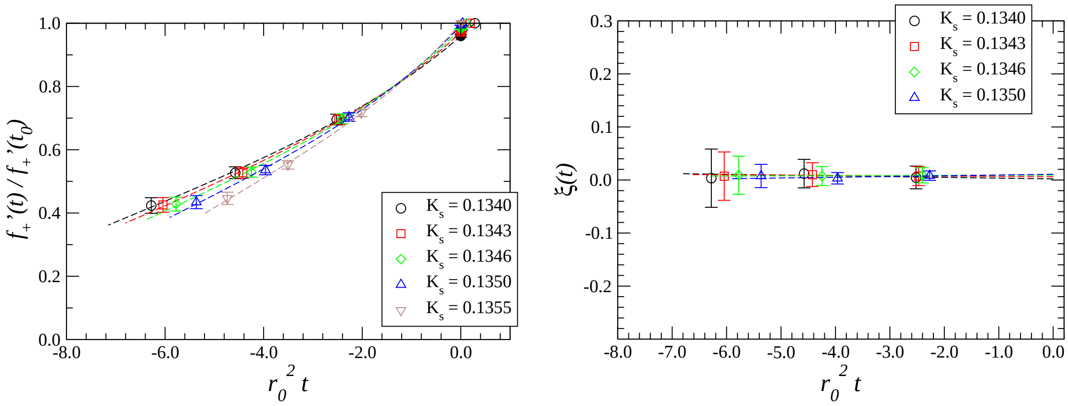


Figure 2: The ratio of form factor $f'_+(t)/f'_+(t_0)$ (left) and $\xi(t)$ (right) as a function of momentum transfer squared $t = q^2$ at $K_{sea} = 0.1355$. A notation $f'_+(t) = f_+(t)\left[1+\xi(t)\frac{m_K - E_\pi(\vec{p})}{m_K + E_\pi(\vec{p})}\right]$ is used here.

A typical time dependence of the double ratio $R^{K\to\pi}(t)$ at the lightest sea quark mass $K_{sea} = 0.1355$ is plotted in Figure 1. Smeared kaon and pion sources are set at time $t = 0, T/2$, respectively. The vector current is inserted at t. Because not only the bulk of the statistical fluctuation but the multiplicative renormalization factors and other systematic errors cancel, we can determine $f_0(q^2_{max})$ with an accuracy better than one percent.

3.2 Double ratio II

The second double ratio with the pion two-point function $C_{\pi\pi}(t;\vec{p})$

$$\frac{\dfrac{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})}}{\dfrac{C_{\pi\pi}(t;\vec{p})}{C_{\pi\pi}(t;\vec{0})}} \to \frac{m_K + E_\pi(\vec{p})}{m_K + m_\pi}\,\frac{f_+(q^2)\left[1+\xi(q^2)\dfrac{m_K - E_\pi(\vec{p})}{m_K + E_\pi(\vec{p})}\right]}{f_+(q^2_{max})\left[1+\xi(q^2_{max})\dfrac{m_K - m_\pi}{m_K + m_\pi}\right]}, \quad 0 \ll t \ll T/2, \tag{3.4}$$

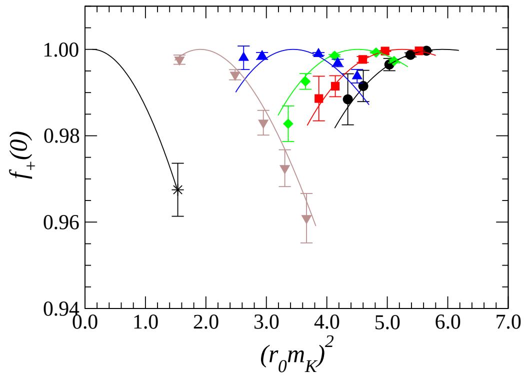
is used to interpolate the form factor to $q^2 = 0$. This is the second factor in (3.1), because $f_0(q^2_{max}) = f_+(q^2_{max})\left[1+\xi(q^2_{max})\frac{m_K - m_\pi}{m_K + m_\pi}\right]$. In this case, the momentum is inserted at t and $T/2$ so that the pion has a finite momentum and the kaon is at rest. The double ratio (3.4) is statistically noisier than the double ratio (3.2) in the previous subsection. The data corresponding to the pion momenta $\vec{p} = 2\pi/20 \times (\pm 1, 0, 0)$, $(\pm 1, \pm 1, 0)$ and $(\pm 1, \pm 1, \pm 1)$ are obtained and the quadratic function is used to interpolate to $q^2 = 0$, which is shown in Figure 2 (left).

3.3 Double ratio III

The last double ratio is for the determination of $\xi(0)$

$$\frac{C_{\pi V_i K}(t,T/2;\vec{p},\vec{p})C_{\pi V_4 \pi}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})C_{\pi V_i \pi}(t,T/2;\vec{p},\vec{p})} \to \frac{1-\xi(q^2)}{\dfrac{m_K + E_\pi(\vec{p})}{m_\pi + E_\pi(\vec{p})} + \xi(q^2)\dfrac{m_K - E_\pi(\vec{p})}{m_\pi + E_\pi(\vec{p})}}, \quad 0 \ll t \ll T/2, \tag{3.5}$$

where we have to measure the spatial component of the matrix element. The extrapolation to $q^2 = 0$ is done by assuming the linear dependence on q^2, which is shown in Figure 2 (right). We see that the q^2 dependence of the ξ is very weak and independent of the strange quark mass.



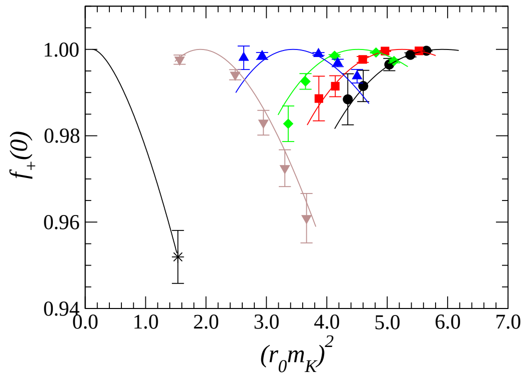


Figure 3: Chiral extrapolation of the form factor $f_+(0)$ with quadratic function (left) and the one-loop ChPT formula plus a quadratic function (right).

4. Result for $f_+(0)$

Following the method explained in the previous subsections, the form factor $f_+(0)$ is estimated as a function of the up/down and strange quark masses. In order to obtain the form factor at the physical pion and kaon masses, we first use a simple fitting function

$$f_+(0) = 1 - (c_0 + c_1[(r_0 m_K)^2 + (r_0 m_\pi)^2])[(r_0 m_K)^2 - (r_0 m_\pi)^2]^2, \tag{4.1}$$

where the meson masses are normalized by the Sommer scale r_0. The result is shown in Figure 3. Our data are well represented by function (4.1) and the preliminary result $f_+(0) = 0.967(6)$ is consistent with Leutwyler-Roos's value $0.961(8)$ [1], and the quenched result $0.960(9)$ by Bećirević *et al.* [2]

For small pion and kaon masses the ChPT predicts the mass dependence of the form factor

$$f_+(0) = 1 + \frac{3}{2}H_{K\pi}(0) + \frac{3}{2}H_{K\eta}(0), \quad H_{PQ}(0) = -\frac{1}{128\pi^2 f^2}\left[m_P^2 + m_Q^2 + \frac{2m_P^2 m_Q^2}{m_P^2 - m_Q^2}\ln\frac{m_Q^2}{m_P^2}\right]. \tag{4.2}$$

We try to fit the data with

$$f_+(0) = 1 + \frac{3}{2}H_{K\pi}(0) + \frac{3}{2}H_{K\eta}(0) - (c_0 + c_1[(r_0 m_K)^2 + (r_0 m_\pi)^2])[(r_0 m_K)^2 - (r_0 m_\pi)^2]^2. \tag{4.3}$$

At this order the one-loop ChPT formula has no tunable parameters. The chiral logarithm is significant only in the region where $m_\pi^2 \ll m_K^2$, while the data in the region $1/2 \le m_\pi^2/m_K^2 \le 2$ are well approximated by the quadratic form (4.1). The extrapolated value at physical pion and kaon mass is affected by the chiral logarithm. The result is $f_+(0) = 0.952(6)$.

5. Charge radius

The charge radius $\langle r^2 \rangle$ parametrizes the slope of the form factor near $q^2 = 0$

$$f(q^2) = f(0)\left[1 + \frac{1}{6}\langle r^2 \rangle q^2 + \cdots\right]. \tag{5.1}$$

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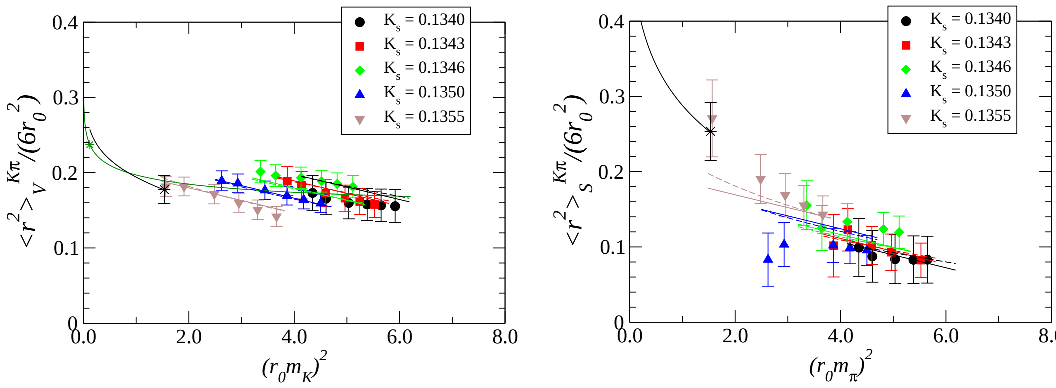

Figure 4: Chiral extrapolation of the vector (left) and scalar (right) charge radius.

The parameter $\lambda = \langle r^2 \rangle m_\pi^2/6$ is often used in the literature.

For the vector form factor, our result extrapolated with the one-loop ChPT plus a quadratic function shown in Figure 4 is $\langle r^2 \rangle_V^{K\pi} = 0.26(3)$ fm^2. It corresponds to $\lambda_+ = 0.021(2)$, which is significantly smaller than the experimental value 0.0278(7) or the quenched lattice result 0.026(2) by Bećirević *et al.* [2] Our result for the charge radius of the scalar form factor (Figure 4) is $\langle r^2 \rangle_S^{K\pi} = 0.37(6)$ fm^2, which corresponds to $\lambda_0 = 0.031(5)$ that overshoots the experimental value 0.0174(22) or the quenched lattice result 0.012(2).

Our calculation shows that the kaon form factors can be calculated on the lattice with good precision using the double ratio method. Further study on systematic errors, especially scaling violation effects, is needed to obtain the definitive value.

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