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Aspects of twist-two matrix elements*

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We present analytical results concerning lattice calculations of the moments of the twist-two matrix elements. First, we discuss the determination of higher-moments of the deep-inelastic hadron structure functions. By using a fictitious heavy quark, direct calculations of the Compton scattering tensor can be performed in Euclidean space that allow the extraction of the moments of structure functions. This overcomes issues of operator mixing and renormalisation that have so far prohibited lattice computations of higher moments. This approach is especially suitable for the study of the twist-two contributions to isovector quark distributions, which is practical with current computing resources. This method is equally applicable to other quark distributions and to generalised parton distributions. By looking at matrix elements such as $\langle \pi^{\pm} | T[V^{\mu}(x) A^{\nu}(0)] | 0 \rangle$ (where V^{μ} and A^{ν} are vector and axial-vector heavy-light currents) within the same formalism, moments of meson distribution amplitudes can also be extracted. Second, we comment on finite-volume effects in the lattice calculation of the twist-two matrix elements.

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1. Introduction

Lattice QCD offers the prospect of exploring the structure functions probed in deeply inelastic scattering (DIS) and other high-energy experiments from first principles. The structure functions describe the hadronic part of the DIS process, *viz.*, the hadronic tensor

$$W_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\, [J^\mu(x), J^\nu(0)]\, |p,S\rangle, \tag{1.1}$$

where p and S are the momentum and spin of the external state, q is the momentum transfer between the lepton and the hadron, and J^μ is the electromagnetic current. Using the optical theorem, $W_S^{\mu\nu}$ can be related to the imaginary part of the forward Compton scattering tensor

$$T_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\, T\, [J^\mu(x) J^\nu(0)]\, |p,S\rangle\, . \tag{1.2}$$

Since lattice QCD is necessarily formulated in Euclidean space, direct calculation of the structure functions is challenging because of the analytical continuation to Minkowski space that is required. In addition, such a calculation would involve all-to-all light-quark propagators, and is therefore numerically demanding. For this reason, beginning with pioneering works, lattice studies of the deep-inelastic structure of hadrons have focused on calculations of matrix elements of local operators that arise from the light-cone operator product expansion (OPE) of the currents

$$T[J^\mu(x) J^\nu(0)] = \sum_{i,n} \mathscr{C}_i\left(x^2,\mu^2\right)\, x_{\mu_1}\ldots x_{\mu_n} \mathscr{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu), \tag{1.3}$$

where the $\mathscr{C}_i$ are the perturbatively calculable Wilson coefficients that incorporate the short-distance physics, and the sum is over all local operators, $\mathscr{O}_i^{\mu\nu\mu_1\ldots\mu_n}$ with the correct symmetries. μ is the renormalisation scale. This expansion enables the investigation of $T_S^{\mu\nu}$ via the knowledge of hadronic matrix elements of local operators. The analytical continuation of these matrix elements from Euclidean space to Minkowski space is straightforward. However, a number of difficulties arise in this approach due to the lattice regularisation. Firstly, the non-zero lattice spacing breaks the symmetry group of Euclidean space-time from $O(4)$ to the discrete hyper-cubic subgroup $H(4)$, consequently modifying the transformation properties of the local operators in the OPE. In general, operators belonging to different irreducible representations of $O(4)$, which span the right-hand side of the OPE in Eq. (1.3), mix unavoidably in the lattice theory since $H(4)$ has only a finite set of irreducible representations. For twist-two (twist = dimension - spin) contributions, this becomes particularly severe for operators of spin $n > 4$ as they mix with lower dimensional operators and the mixing coefficients contain power divergences. Currently this restricts the available lattice calculations to operators of spin $n = 1,2,3,4$. For higher-twist operators, such power divergences are generally unavoidable. A second issue is that the matching of the lattice regularisation to continuum renormalisation schemes, in which the Wilson coefficients are calculated, becomes more involved as n increases.

In this talk, we present a strategy, as proposed in Ref. [1] for obtaining higher matrix elements of higher-spin, twist-two operators in Eq. (1.3). We also discuss finite-volume effects in the calculation of these matrix elements [2].

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2. Euclidean operator product expansion in lepton-hadron deep-inelastic scattering

Our approach is based upon directly studying the OPE on the lattice, as was first investigated in kaon physics in Ref. [3]. A similar technique has also been applied to determine Wilson coefficients non-perturbatively [4] and extract the lowest moment of the isovector twist-two quark distribution [5] (our method is related to this latter work but improves on it in a number of ways). In our proposal, one simulates the Compton scattering tensor using lattice QCD, with currents coupling the physical light quarks, $\psi(x)$, present in the hadron to a non-dynamical (purely valence), unphysically heavy quark, $\Psi(x)$. The introduction of this heavy quark significantly simplifies the calculation of isovector matrix elements because it removes the requirement of all-to-all propagators. After performing an extrapolation to the continuum limit, the lattice data for the Compton tensor are compared to the predictions of the OPE in Euclidean space to extract the matrix elements of local operators in Eq. (1.3), directly in the continuum renormalisation scheme in which the Wilson coefficients are calculated. This approach also removes the power divergences, thereby enabling extraction of matrix elements of higher spin ($n > 4$) operators for twist-two operators with a simple renormalisation procedure. These matrix elements determine the Mellin moments of the structure functions which are identical in Euclidean space and Minkowski space and their analytical continuation is trivial. Finally, the chiral and infinite volume extrapolations can now be performed at the level of the local matrix elements using chiral perturbation theory.

We consider fictitious currents that couple light up and down quarks to unphysical heavy quarks of mass m_Ψ. We focus on a purely vector coupling, leaving the discussion of other possible currents to the end of the section. We define

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)\,, \tag{2.1}$$

and construct the Euclidean Compton scattering tensor

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\, \mathrm{e}^{iq\cdot x}\langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle\,, \tag{2.2}$$

(henceforth all momenta are Euclidean), with the constraint

$$\Lambda_{\mathrm{QCD}} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{\hat{a}}, \tag{2.3}$$

where $\hat{a}$ is the coarsest lattice spacing used in the calculation. Secondly, the non-dynamical nature of the heavy quark automatically removes many contributions (for example, so-called "cat's ears" diagrams – see Fig. 1(d) below) that are higher-twist contaminations in traditional DIS.

In the limit $q^2 \to \infty$ or $m_\Psi \to \infty$, $T^{\mu\nu}_{\Psi,\psi}$ is given by the leading-twist contribution, the "handbag diagrams" in Figs. 1 (a) and (b). The "box diagram", Fig. 1 (c)[1], which involves purely gluonic operators after the OPE, is strongly suppressed in our approach and is completely absent in the study of the OPE of the isovector Compton scattering tensor

$$T^{\mu\nu}_{\Psi,v} = T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}. \tag{2.4}$$

[1] In Fig. 1 (c), we specify that the large momentum, q^2, flows through the three light-quark lines; the contributions in which these quarks have soft momenta are already included in Figs. 1 (a) and (b). In principle, these gluonic contributions can be disentangled from their different q^2 behaviour.

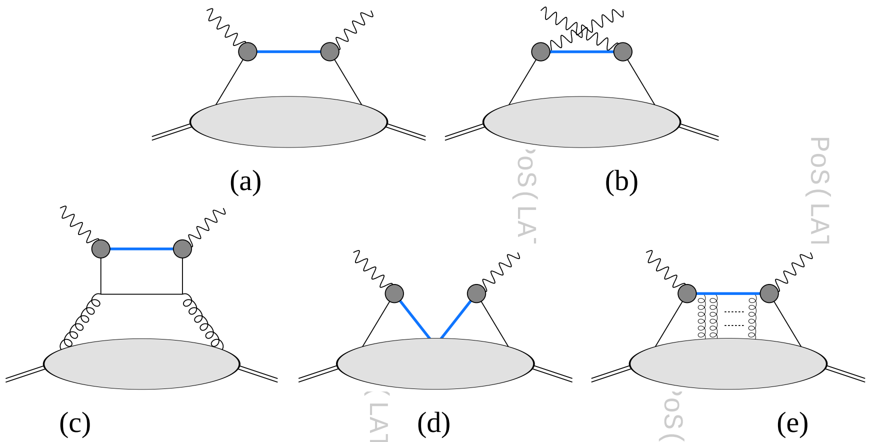

Figure 1: Contributions to the Compton scattering tensor. Diagrams (a), (b) and (c) correspond to the leading twist contributions. Diagram (c) (the "box diagram") involves gluonic operators and vanishes for the isovector combination, Eq. (2.4). Diagram (d) (the "cat's ears diagram") is higher-twist and absent in our analysis. Diagram (e) includes leading- and higher-twist terms and is discussed in the main text. The thick lines correspond to the heavy-quark propagators, the shaded circles to the heavy-light currents and the large shaded regions to the various parton distributions.

This makes the extraction of moments of the isovector quark distributions practical, and we focus on this case here.

At moderate q^2 and m_Ψ, higher-twist terms also contribute. However, the non-dynamical nature of the fictitious heavy quark entirely eliminates the higher-twist contributions involving more than one quark propagator between the currents, *e.g.*, the "cat's ears diagram" in Fig. 1 (d). The diagrams in Fig. 1 (e) contain pieces that contribute to the twist-two operators, and also higher-twist terms which are small and can be treated as fit parameters in the procedure. Also, we replace the heavy-quark mass by the heavy-light meson mass which is entirely free of the renormalon ambiguity. This introduces an additional unknow parameter, the binding energy, into the procedure. In this talk, we only give a specific example, in which we choose the rest frame of the proton, $p = (0,0,0,i\,M)$ and parameterise $q = (0,0,\sqrt{q_0^2 - Q^2}, i\,q_0)$. In this case, the symmetric combination of $\{\mu,\nu\} = \{3,4\}$ is

$$T^{\{34\}}_{\Psi,\psi}(p,q) \;=\; \sum_{n=2,\text{even}}^{\infty} A^n_\psi(\mu^2)\, f(n)\,, \tag{2.5}$$

where

$$\begin{aligned} f(n) \;=\; & -\sqrt{q_0^2 - Q^2}\,\zeta^n \Bigg\{ \frac{2}{q_0}\left[\mathscr{C}_n \frac{\widetilde{Q}^2}{Q^2} \frac{(n-1)\eta C^{(2)}_{n-1}(\eta) - 4\eta^2 C^{(3)}_{n-2}(\eta)}{n(n-1)} + \mathscr{C}''_n \frac{\eta}{n} C^{(2)}_{n-1}(\eta) \right] \\ & + \frac{q_0}{Q^2}\Bigg[\mathscr{C}_n \frac{\widetilde{Q}^2}{Q^2} \frac{n(n-2)C^{(1)}_n(\eta) - 2\eta(2n-3)C^{(2)}_{n-1}(\eta) + 8\eta^2 C^{(3)}_{n-2}(\eta)}{n(n-1)} + \\ & 2\mathscr{C}''_n \left(C^{(1)}_n(\eta) - 2\frac{\eta}{n} C^{(2)}_{n-1}(\eta) \right) \Bigg] \Bigg\}\,, \end{aligned} \tag{2.6}$$

and $A^n_\psi(\mu^2)$ is defined as

$$\sum_S \langle p,S|\overline{\psi}\gamma^{\{\mu_1}(iD^{\mu_2})\dots(iD^{\mu_n\}})\psi - \text{traces}|p,S\rangle \;=\; A^n_\psi(\mu^2)\,[p^{\mu_1}\dots p^{\mu_n} - \text{traces}]\,. \qquad (2.7)$$

The $C_n^{(\lambda)}$ are the Gegenbauer polynomials arising from summing the target mass effects, and

$$\zeta = \frac{\sqrt{p^2q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p\cdot q}{\sqrt{p^2q^2}}, \tilde{Q}^2 = Q^2 + M_\Psi^2 + \alpha M_\Psi + \beta\,, \qquad (2.8)$$

where α is the binding energy and β parameterises the higher-twist contributions. As shown in Ref. [1], by varying Q^2, q_0 and M_Ψ, one can hope to extract several moments from Eq. (2.5).

3. Meson distribution amplitudes from current-current matrix elements

A further application of the approach we have outlined is in computing moments of meson distribution amplitudes, ϕ_M. In the lattice approach, we can extract moments of distribution amplitudes in the same way as DIS determines moments of parton distributions; for example, we may study the matrix element $\langle\pi^\pm|T[V^\mu_{\Psi,\psi}(x)A^\nu_{\Psi,\psi}(0)]|0\rangle$, where $V^\mu_{\Psi,\psi}$ and $A^\mu_{\Psi,\psi}$ are fictitious vector and axial vector heavy-light currents. This process is described by the tensor

$$S^{\mu\nu}_{\Psi,\psi}(p,q) = \int d^4x\, e^{iq\cdot x}\langle\pi^+(p)|T[V^\mu_{\Psi,\psi}(x)A^\nu_{\Psi,\psi}(0)]|0\rangle\,. \qquad (3.1)$$

The OPE of the two currents leads to the matrix elements of twist-two operators that determine the moments of the pion distribution amplitude:

$$\langle\pi^+(p)|\overline{\psi}\gamma^{\{\mu_1}\gamma_5(iD)^{\mu_2}\dots(iD)^{\mu_n\}}\psi|0\rangle \;=\; f_\pi\langle\xi^{n-1}\rangle_\pi\,[p^{\mu_1}\dots p^{\mu_n} - \text{traces}]\,, \qquad (3.2)$$

where

$$\langle\xi^n\rangle_\pi \equiv \int_0^1 d\xi\,\xi^n\phi_\pi(\xi)\,. \qquad (3.3)$$

These matrix elements can be determined by studying the various components of $S^{\mu\nu}_{\Psi,\psi}$ for varying m_Ψ and q^μ. As in the DIS case, many higher-twist contributions are absent because of the valence nature heavy quark and the problems that plague direct evaluation of higher moments due to the lattice cutoff are eliminated. Since only the zeroth (decay constant) and second moments of the pion distribution amplitude have been investigated in the direct approach, any information on higher moments will be useful in constraining the distribution amplitude from QCD. For flavour non-diagonal mesons (*e.g.* $\pi^\pm$, $K^{\pm,0}$), extraction of the tensor $S^{\mu\nu}_{\Psi,\psi}$ on the lattice only requires the computation of the Wick contraction shown in Fig. 2.

4. Finite-voume effects of the twist-two matrix elements

In Ref. [2], finite-volume effects in lattice calculations of the twist-two matrix elements are estimated using heavy-baryon chiral perturbation theoy, in quenched, partially-quenched ($N_f = 2,3$) and full QCD. Here we only present a specific example of this calculation, namly, the $N_f = 2$ full

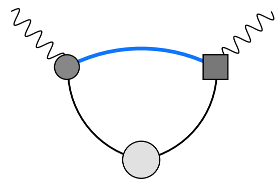

Figure 2: Extraction of moments of meson distribution amplitudes. Here, the light-shaded circle denotes the pion interpolating operator and the dark circle and dark square indicate the vector and axial-vector currents, respectively.

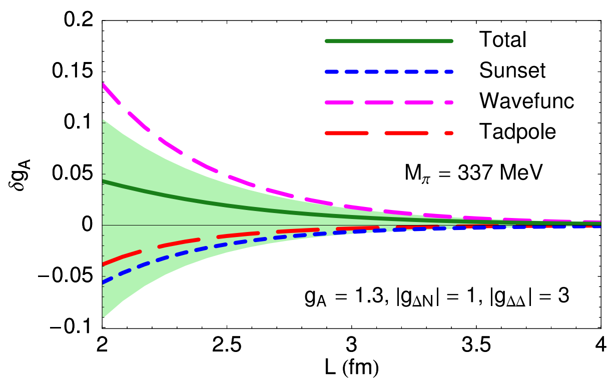

Figure 3: Finite volume effects in QCD calculations of g_A at $M_\pi = 337$ MeV (as appropriate for the recent LHP simulations [6]). $L = 2$ fm corresponds to $M_\pi L = 3.4$. The shaded region corresponds to varying $1.0 \le g_A \le 1.5$, $0 \le |g_{\Delta N}| \le 2$. "Total" means the volume effects predicted by the effective theory with a specific choice of couplings, and the other curves indicate contributions from various one-loop diagrams.

QCD results for the nucleon axial coupling g_A. Figure 3 shows finite-volume effect in this quantity at a fixed pion mass relevant to the numerical simulation performed by the LHP Collaboration [6]. The shaded region in this plot corresponds to the variation of unknown couplings in the effective theory. It is clear that volume effects can be as large as 10%, depending on the values of these couplings. However, as reported at this conference [6], lattice data seem to suggest that volume effects are small for this quantity.

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