

Kaon B-parameter with $N_F = 2$ dynamical Wilson fermions

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We present a preliminary study of the neutral kaon mixing bag parameter B_K using two flavors of dynamical Wilson fermions. We determine the matrix element of the relevant $\Delta S = 2$ operator by using both the conventional approach and the so called “non-subtraction method”, and find that the latter leads to results with smaller uncertainties. After having implemented non-perturbative renormalization, we study the dependence of B_K on the sea quark mass. At our relatively heavy values of quark masses ($M_P/M_V \simeq 0.60 \div 0.75$) such a dependence is found to be negligible and the results, within the statistical accuracy, are consistent with a quenched determination. As a preliminary result for the renormalization group invariant parameter we quote $\hat{B}_K = 1.02(20)$.

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1. Preamble

The parameter $B_K(\mu)$, defined through

$$\langle \overline{K^0} | Q^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 m_K^2 B_K(\mu), \quad (1.1)$$

with

$$Q^{\Delta S=2} = \bar{s}\gamma_\mu(1 - \gamma_5)d \bar{s}\gamma^\mu(1 - \gamma_5)d \quad (1.2)$$

includes the long distance QCD effects in the evaluation of the indirect CP-violating parameter ϵ_K . The theoretical uncertainty in the determination of B_K represents at present one of the main sources of error in the unitarity triangle analysis [1]. Within the quenched approximation the lattice estimates of B_K have reached an accuracy better than 10% [2]-[6], so that the quenching effect, which has been estimated to be as high as $\pm 15\%$ [7], remains the primary source of systematic error to be investigated. Preliminary unquenched studies are presented in [8].

In this talk we report on an exploratory calculation using $N_f = 2$ degenerate flavors of dynamical fermions. With respect to previous calculations, we simulate lighter sea quark masses on a finer lattice, but we work with unimproved Wilson fermions. We use the Wilson plaquette gauge action and the Wilson quark action at $\beta = 5.8$, which corresponds to a lattice spacing $a^{-1} \simeq 3.2 \text{ GeV}$. The lattice volume is $24^3 \times 48$. The numerical simulation is performed by using the Hybrid Monte Carlo (HMC) algorithm [9]. A sample of 50 configurations has been generated at four different values of sea quark masses, for which the ratio of the pseudoscalar over vector meson masses lies in the range $M_P/M_V \simeq 0.60 \div 0.75$. Further details on the numerical simulation can be found in [10]. Our kaons are made with degenerate valence quarks and we considered four values of valence quark masses for each dynamical quark.

To extract B_K , the following three steps are needed:

- renormalization of the relevant $\Delta S = 2$ operators;
- extraction of the matrix elements from the appropriate correlation functions;
- determination of B_K and extrapolation to the physical quark masses. Since we use Wilson fermions, which break chiral symmetry at $\mathcal{O}(a)$, cautions are required in this latter step [11]. In particular, in addition to the finite mixing with $dim = 6$ operators, we have to subtract the effects of $dim \geq 7$ operators at finite lattice spacing.

2. Renormalization and matrix elements

The first ingredient in the evaluation of B_K is the renormalization of the $Q^{\Delta S=2}$ operator of eq. (1.2). From eq. (1.1), we see that only the parity even component of $Q^{\Delta S=2}$, namely $Q_1 = \bar{s}\gamma_\mu d \bar{s}\gamma^\mu d + \bar{s}\gamma_\mu \gamma_5 d \bar{s}\gamma^\mu \gamma_5 d$, gives a non-vanishing contribution to B_K .

In regularizations with exact chiral symmetry, such as continuum naïve dimensional regularization or Ginsparg-Wilson fermions on the lattice, the operator Q_1 renormalizes multiplicatively. For Wilson fermions, instead, the renormalization pattern is more involved, and can be expressed as

$$\hat{Q}_1(\mu) = Z_{VV+AA}(a\mu) \left[Q_1(a) + \sum_{i=2}^5 \Delta_i(a) Q_i(a) \right]. \quad (2.1)$$

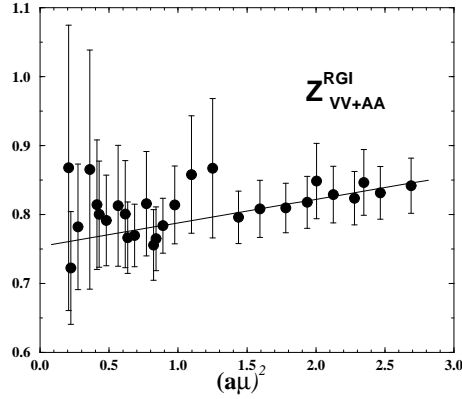


Figure 1: The renormalization constant Z_{VV+AA}^{RGI} obtained with the RI-MOM method. A residual $\mathcal{O}((a\mu)^2)$ dependence is observed. Our value of Z_{VV+AA}^{RGI} in the text is given by the intercept of a linear fit in $(a\mu)^2$ (black line).

Here $Z_{VV+AA}(a\mu)$ is the multiplicative renormalization constant, present also in formulations where chiral symmetry is preserved, while $\Delta_{2-5}(a)$ are mixing coefficients peculiar for the Wilson regularization. The corresponding four-fermion operators are

$$\begin{aligned} Q_2(\mu) &= \bar{s}\gamma_\mu d \bar{s}\gamma^\mu d - \bar{s}\gamma_\mu \gamma_5 d \bar{s}\gamma^\mu \gamma_5 d, & Q_3(\mu) &= \bar{s}d \bar{s}d + \bar{s}\gamma_5 d \bar{s}\gamma_5 d, \\ Q_4(\mu) &= \bar{s}d \bar{s}d - \bar{s}\gamma_5 d \bar{s}\gamma_5 d, & Q_5(\mu) &= \bar{s}\sigma_{\mu\nu} d \bar{s}\sigma_{\mu\nu} d. \end{aligned} \quad (2.2)$$

In this study the renormalization constants $Z_{VV+AA}(a\mu)$ and the mixing coefficients $\Delta_{2-5}(a)$ have been computed non-perturbatively with the RI-MOM method [12]. The procedure is illustrated in details in [13] for the quenched case. In the unquenched case, the situation is similar. The only additional constraint is that, in order to get these constants in a mass-independent renormalization scheme, a chiral extrapolation in both the valence and sea quark masses has to be performed. At this stage, the Goldstone pole effects have been non-perturbatively subtracted [13], even though their contributions turn out to be negligible. Moreover, in order to get rid of potential $\mathcal{O}((a\mu)^2)$ lattice artifacts, we have performed a linear fit in $(a\mu)^2$ in the range $[0.8, 2.5]$. As an example, we show this procedure in fig. 1 for the renormalization group invariant (RGI) constant Z_{VV+AA}^{RGI} . The scheme and scale dependent $Z_{VV+AA}(a\mu)$ and/or $B_K(\mu)$ are related to their RGI expressions through

$$\hat{B}_K = [\alpha_s(\mu)]^{-\frac{\gamma_0}{2\beta_0}} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J \right] B_K(\mu), \quad (2.3)$$

where $\gamma_0 = 4$ and $\beta_0 = 11 - 2N_f/3$. We use the RI-MOM scheme for which the NLO coefficient $J = 2.83551$ with $N_f = 2$. Our results at $\beta = 5.8$ for the renormalization constant and mixing coefficients are $Z_{VV+AA}^{RGI} = 0.75(6)$, $\Delta_2 = -0.08(10)$, $\Delta_3 = -0.05(3)$, $\Delta_4 = 0.02(4)$, $\Delta_5 = 0.03(2)$. As known from quenched studies [12], the practical difficulty in the calculation of B_K with Wilson fermions is not only the non-perturbative evaluation of the subtraction constants $\Delta_{2-5}(a)$ but also the necessity to perform it with a high level of accuracy, since the lattice bare matrix elements $\langle Q_{2-5} \rangle$ are orders of magnitude larger than $\langle Q_1 \rangle$. Therefore, even though the subtraction constants are numerically small, the net effect of the subtraction is large.

An alternative approach that allows one to compute the matrix element (1.1) without the necessity of performing the subtraction has been developed in [14]. In this procedure “without subtractions” one uses the hadronic chiral axial Ward identity to relate the matrix element of the operator

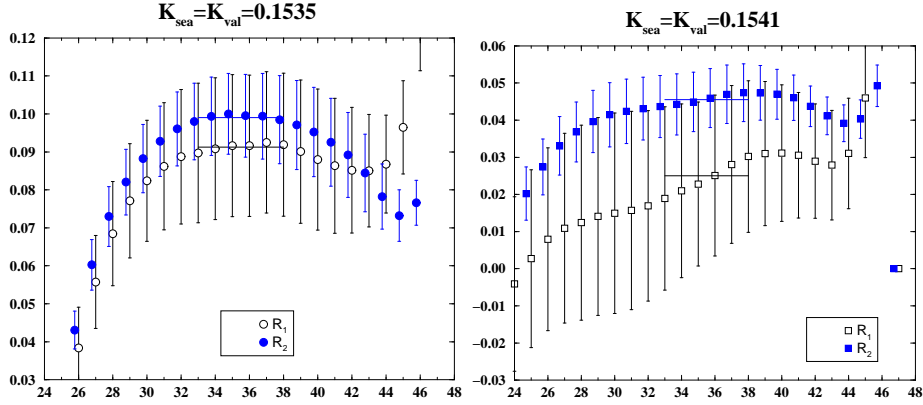


Figure 2: The ratios R_1 and R_2 computed with mesons at rest.

Q_1 to its parity violating counterpart, $\mathcal{Q}_1 = \bar{s}\gamma_\mu\gamma_5 d \bar{s}\gamma^\mu d + \bar{s}\gamma_\mu d \bar{s}\gamma^\mu\gamma_5 d$. The latter only renormalizes multiplicatively with the constant $Z_{VA}(a\mu)$, which we have calculated with the RI-MOM method, obtaining $Z_{VA}^{RGI} = 0.80(2)$. In this way, the problem of mixing with the other dimension-six operators is circumvented. The price to pay is that one has to compute a four-point correlation function where one pion operator is integrated over all lattice space-time coordinates.

In practice, we consider the following two ratios of correlation functions

$$R_1^{\mathbf{p},\mathbf{q}}(t) = \frac{\sum_{\mathbf{xy}} \langle P(\mathbf{x}, t) \hat{Q}_1(0) P(\mathbf{y}, t_f) \rangle e^{i(\mathbf{xp} + \mathbf{yq})}}{\sum_{\mathbf{x}} \langle P(\mathbf{x}, t) P(0) \rangle e^{i\mathbf{xp}} \sum_{\mathbf{y}} \langle P(\mathbf{y}, t_f) P(0) \rangle e^{i\mathbf{yq}}} \quad (2.4)$$

$$R_2^{\mathbf{p},\mathbf{q}}(t) = \frac{\sum_{\mathbf{xyz}} \langle P(\mathbf{x}, t) (m^{AWI} \Pi(z)) \hat{Q}_1(0) P(\mathbf{y}, t_f) \rangle e^{i(\mathbf{xp} + \mathbf{yq})}}{\sum_{\mathbf{x}} \langle P(\mathbf{x}, t) P(0) \rangle e^{i\mathbf{xp}} \sum_{\mathbf{y}} \langle P(\mathbf{y}, t_f) P(0) \rangle e^{i\mathbf{yq}}} \quad (2.5)$$

for the method with and without subtraction respectively (see details in [5]). Thanks to the Ward identity introduced in [14], $R_1^{\mathbf{p},\mathbf{q}}$ and $R_2^{\mathbf{p},\mathbf{q}}$ only differ by $\mathcal{O}(a)$ effects. At large times $t_f, T - t \gg 0$, the ratios $R_i(t)$ are both proportional to the desired matrix element $\langle \bar{P}^0 | Q^{\Delta S=2}(\mu) | P^0 \rangle$. In our simulation, we have chosen $t_f = 14$ and t in the range $[23, 37]$, whereas the momentum configurations $\{\mathbf{p}, \mathbf{q}\}$ (in units of $2\pi/La$) are given by $\{(0, 0, 0), (0, 0, 0)\}, \{(0, 0, 0), (1, 0, 0)\}$ and $\{(1, 0, 0), (0, 0, 0)\}$. An average over momentum configurations equivalent under hypercubic rotations is also performed. In fig. 2 we compare the results for R_1 and R_2 in the cases of the heaviest and the lightest quark masses ($m_{\text{val}} = m_{\text{sea}}$). We notice that R_2 suffers from rather smaller statistical fluctuations.

3. Chiral behavior at finite lattice spacing

At finite lattice spacing, the matrix elements $\langle \bar{P}^0 | Q^{\Delta S=2}(\mu) | P^0 \rangle$ extracted from the ratios R_i are no longer proportional to B_K . The best approach to get rid of the $\mathcal{O}(a)$ contributions coming from the mixing with $\dim \geq 7$ operators would be to perform a continuum extrapolation. In the present simulation, however, we have data only at a single value of the lattice spacing. Therefore, we rely on the approach proposed in [11]. By calculating matrix elements of external kaons with non-zero momentum, we introduce an additional degree of freedom which allows us to partially remove the leading lattice artifacts. By writing

$$\langle \bar{P}^0 | \hat{Q}_1 | P^0, \mathbf{q} \rangle_{m_{\text{sea}}} = \alpha_{m_{\text{sea}}} + \beta_{m_{\text{sea}}} M_P^2 + \frac{8}{3} F_P^2 M_P^2 \gamma_{m_{\text{sea}}}, \quad (3.1)$$

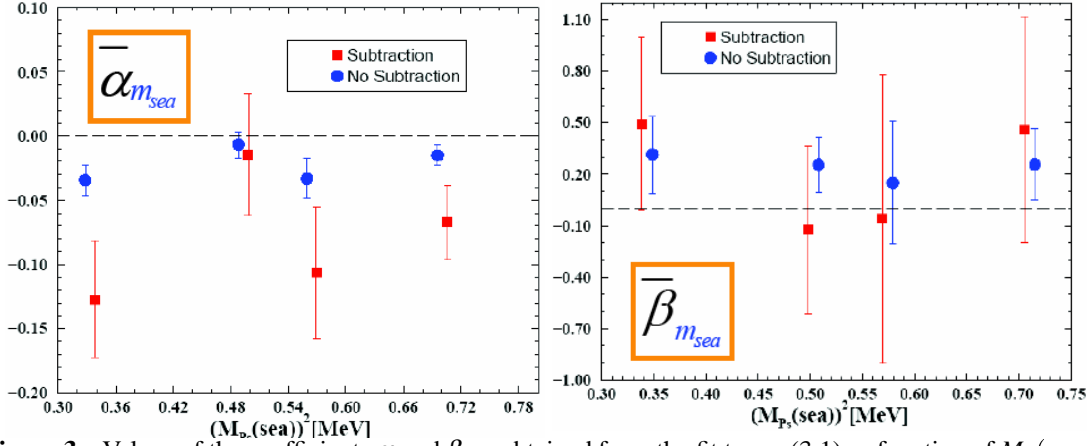


Figure 3: Values of the coefficients α and β as obtained from the fit to eq. (3.1) as function of $M_P(m_{sea})$.

one finds that the physical contribution to \hat{B}_K , at each m_{sea} , is given by the coefficient γ , while α and β parameterizes pure lattice artifacts. The results of the fit for the coefficients α and β are shown in fig. 3. These coefficients turn out to be sizable and their contribution in the extraction of the parameter γ cannot be neglected. It is also interesting to notice that the effect of these coefficients is less relevant in the method without subtraction (blue points in fig. 3).

We finally plot in fig. 4 $\hat{B}_K(m_{sea}) = \gamma m_{sea}$, as a function of the square of the pseudoscalar meson mass $M_P(m_{sea})$. We observe that the method without-subtractions (squared points) looks more promising, by suffering from smaller uncertainties. We also compare in the plot our results with those obtained by the UKQCD Collaboration [8] by using $\mathcal{O}(a)$ -improved Clover fermions but at a larger value of the lattice spacing. The two sets of results are very well compatible. In order to obtain the physical value of \hat{B}_K , an extrapolation to $M_P(m_{sea}) = 0$ should be performed. Within the errors, however, we do not see any significant dependence on the sea quark mass and our results, at the simulated values of quark masses, are compatible with the quenched estimate (yellow band in fig. 4), $\hat{B}_K = 0.97(9)$ obtained by performing a quenched simulation on a lattice

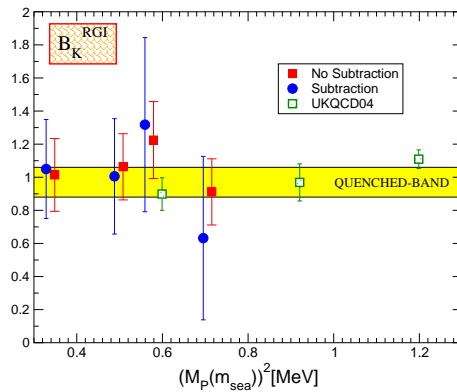


Figure 4: Values of \hat{B}_K , as obtained from the two methods with and without subtraction, as a function of the pseudoscalar meson mass squared. The results obtained by the UKQCD Collaboration [8] and the band corresponding to the quenched estimate are also shown for comparison.

with similar size and resolution. As a preliminary result, we quote the value of \hat{B}_K obtained from a constant fit to the points obtained with the method without subtraction, namely

$$\hat{B}_K = 1.02(20) \quad (3.2)$$

In order to reduce the uncertainties, we plan to include in the analysis more external momenta by exploiting the θ -boundary condition method of ref. [15].

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