Unifying the Fixed Order Evolution of Fragmentation Functions with the Modified Leading Logarithm Approximation

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An approach which unifies the Double Logarithmic Approximation at small x and the leading order DGLAP evolution of fragmentation functions at large x is presented. This approach reproduces exactly the Modified Leading Logarithm Approximation, but is more complete due to the degrees of freedom given to the quark sector and the inclusion of the fixed order terms. We find that data from the largest x values to the peak region can be better fitted than with other approaches.

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The current optimum description of single hadron inclusive production is provided by the QCD parton model, which requires fragmentation functions (FFs) $D_a^h(x,Q^2)$ describing the probability for a parton a to emit a hadron h carrying a fraction x of its momentum. Omitting the h and a labels, the evolution of the FFs in the factorization scale Q^2 at large and intermediate x is well described [1] by the leading order (LO) DGLAP equation [2]

$$\frac{d}{d\ln Q^2}D(x,Q^2) = \int_x^1 \frac{dz}{z}a_s(Q^2)P^{(0)}(z)D\left(\frac{x}{z},Q^2\right), \tag{1}$$

where $P^{(0)}(z)$ are the LO splitting functions calculated from fixed order (FO) pQCD, and $a_s(Q^2) = \alpha_s(Q^2)/(2\pi)$. As $z \to 0$, the LO splitting function $a_s P^{(0)}(z)$ diverges due to terms of the form a_s/z. These double logarithms (DLs) occur at all orders in the FO splitting function, being generally of the form $(1/z)(a_s \ln z)^2(a_s \ln^2 z)^r$ for $r = -1,...,\infty$. As x decreases, Eq. (1) will therefore become a poor approximation once $\ln(1/x) = O(a_s^{-1/2})$. A small x description is obtained by resumming these DLs using the Double Logarithmic Approximation (DLA) [3], given by

$$\frac{d}{d\ln Q^2}D(x,Q^2) = \int_x^1 \frac{dz}{z}\frac{2C_A}{z}Az^{2\frac{d}{d\ln Q^2}}\left[a_s(Q^2)D\left(\frac{x}{z},Q^2\right)\right], \qquad A = \begin{pmatrix} 0 & \frac{2C_F}{C_A} \\ 0 & 1 \end{pmatrix} \tag{2}$$

for $D = (D_\Sigma, D_g)$, where $D_\Sigma = \frac{1}{n_f}\sum_{q=1}^{n_f}(D_q + D_{\overline{q}})$ is the singlet FF. The evolution of the valence quark and non-singlet FFs vanishes in the DLA.

We now construct an approach suitable for large and small x simply and consistently by using the DLA to resum the DLs in the DGLAP evolution, as described in more detail in Ref. [4]. We will use Eq. (2) to modify $a_s P^{(0)}$ in Eq. (1) to

$$a_s P^{(0)}(z) \to P^{\text{DL}}(z,a_s) + a_s\overline{P}^{(0)}(z), \tag{3}$$

where $P^{\text{DL}}(z,a_s)$ contains the complete DL contribution, while $a_s\overline{P}^{(0)}(z)$ is obtained by subtracting the LO DLs, already accounted for in P^{DL}, from $a_s P^{(0)}(z)$ to prevent double counting. To obtain P^{DL}, we work in Mellin space, $f(\omega) = \int_0^1 dx x^\omega f(x)$. Upon Mellin transformation, Eq. (2) becomes

$$\left[\left(\omega + 2\frac{d}{d\ln Q^2}\right)\frac{d}{d\ln Q^2} - 2C_A a_s(Q^2)A - \left(\omega + 2\frac{d}{d\ln Q^2}\right)a_s(Q^2)\overline{P}^{(0)}(\omega)\right]D(\omega,Q^2) = 0, \tag{4}$$

where for completeness we have also accounted for $a_s\overline{P}^{(0)}$, which can be neglected in the following calculation of P^{DL}. Making the replacement in Eq. (3) in Eq. (1), taking its Mellin transform, $\frac{d}{d\ln Q^2}D(\omega,Q^2) = P^{\text{DL}}(\omega,a_s(Q^2))D(\omega,Q^2)$, and then substituting this into Eq. (4) gives $2(P^{\text{DL}})^2 + \omega P^{\text{DL}} - 2C_A a_s A = 0$. We choose the solution $P^{\text{DL}}(\omega,a_s) = \frac{A}{4}\left(-\omega + \sqrt{\omega^2 + 16C_A a_s}\right)$ since its expansion in a_s yields at LO the result $a_s P^{\text{DL}(0)}(\omega,a_s) = 2C_A A\frac{a_s}{\omega}$, which agrees with the LO DLs from the literature [5]. The resummed result in x space is then $P^{\text{DL}}(z,a_s) = \frac{A\sqrt{C_A a_s}}{z\ln\frac{1}{z}}J_1\left(4\sqrt{C_A a_s}\ln\frac{1}{z}\right)$, with J_1 being the Bessel function of the first kind.

If we approximate $a_s\overline{P}^{(0)}(\omega)$ in Eq. (4) by its SLs, defined at LO to be the coefficients of ω^0, $P_{qq}^{\text{SL}(0)} = 0$, $P_{qg}^{\text{SL}(0)} = -3C_F$, $P_{gq}^{\text{SL}(0)} = \frac{2}{3}T_R n_f$ and $P_{gg}^{\text{SL}(0)} = -\frac{11}{6}C_A - \frac{2}{3}T_R n_f$, then if we apply the approximate result that follows from the DLA at large Q,

$$D_{q,\overline{q}} = \frac{C_F}{C_A}D_g, \tag{5}$$

the gluon component of Eq. (4) becomes the MLLA differential equation [6]. Therefore we conclude that, since we do not use these two approximations, our approach is more complete and accurate than the MLLA.

We now compare our approach to normalized differential cross section data for light charged hadron production from $e^+e^- \to (\gamma, Z) \to h + X$. We fit the gluon $D_g(x, Q_0^2)$ and the quark FFs

$$D_{uc}(x, Q_0^2) = \frac{1}{2}\left(D_u(x, Q_0^2) + D_c(x, Q_0^2)\right), \quad D_{dsb}(x, Q_0^2) = \frac{1}{3}\left(D_d(x, Q_0^2) + D_s(x, Q_0^2) + D_b(x, Q_0^2)\right),$$
(6)

where $Q_0 = 14$ GeV. Since the hadron charge is summed over, we set $D_{\bar{q}} = D_q$. For each of these three FFs, we choose the parameterization $D(x, Q_0^2) = N\exp(-c\ln^2 x)x^\alpha(1-x)^\beta$. To prevent too many free parameters, we use Eq. (5) to fix $c_{uc} = c_{dsb} = c_g$ and $\alpha_{uc} = \alpha_{dsb} = \alpha_g$. Since all scales are above the bottom quark mass, we set $n_f = 5$.

Performing a fit using the FO approach to LO, we obtain $\chi^2_{\mathrm{DF}} = 3.0$ and the results in Fig. 1. The fitted result of $\Lambda_{\mathrm{QCD}} = 388$ MeV is quite consistent with that of other analyses, at least within the theoretical error of a factor of $O(1)$. It is clear that FO DGLAP evolution fails in the description of the peak region and shows a different trend outside the fit range.
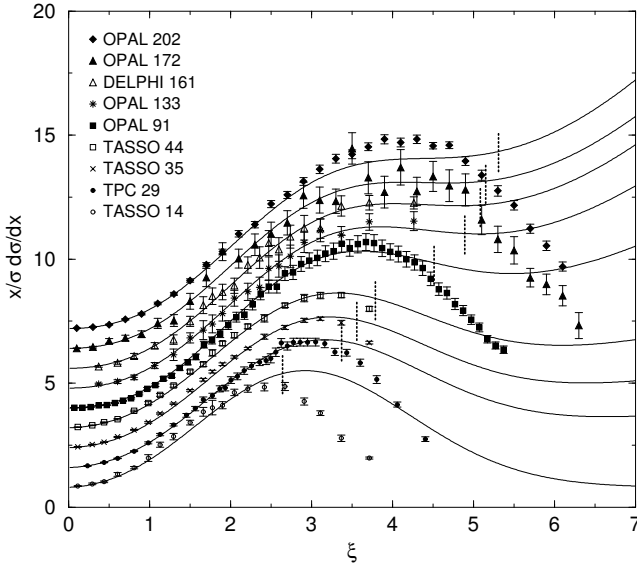

Figure 1: Fit to data in the FO approach to LO. Some of the data sets used for the fit are shown, together with their theoretical predictions from the results of the fit. Only data for which $\xi = \ln(1/x) < \ln\sqrt{s}$ [7] were used, indicated by the vertical lines. Each curve is shifted up by 0.8 for clarity.

Now we perform the same fit again, but using our approach, i.e. Eq. (1) with the replacement in Eq. (3), for the evolution. The results are shown Fig. 2. We obtain $\chi^2_{\mathrm{DF}} = 2.1$, a significant improvement to the fit above with FO DGLAP evolution. The data around the peak is now much better described. The energy dependence is well reproduced up to the largest $\sqrt{s}$ value, $\sqrt{s} = 202$ GeV. We obtain a rather large $\Lambda_{\mathrm{QCD}} = 801$ MeV. We note that had we made the usual DLA (MLLA) choice $Q = \sqrt{s}/2$ instead of our choice $Q = \sqrt{s}$ as is done in analyses using the DGLAP equation, we would have obtained half this value for Λ_{QCD}. A treament to NLO is required to understand this problem, as well as treatment of hadron mass effects which are important at small x.
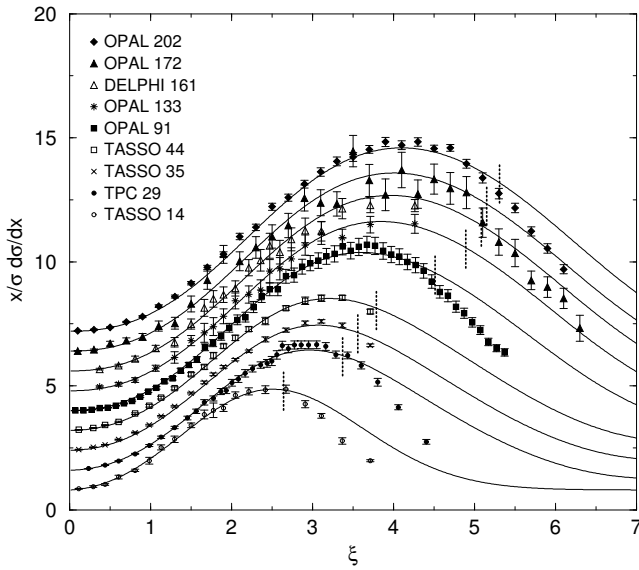
Figure 2: As in Fig. 1, but using our approach for the evolution.

In conclusion, we have proposed a single unified scheme which can describe a larger range in x than either FO DGLAP evolution or the DLA. Our scheme allows a determination of quark and gluon FFs over a wider range of data than previously achieved, and should be incorporated into global fits of FFs such as that in Ref. [8] since the current range of $0.1 < x < 1$ is very limited. Our approach should be expected to improve the description of other inclusive hadron production processes, e.g. those involving protons in the initial state. This work was supported in part by DFG through Grant No. KN 365/3-1 and by BMBF through Grant No. 05 HT4GUA/4.

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