# A QCD analysis of $\bar{p} N \rightarrow \gamma^{*} \pi$ and $\bar{p} N \rightarrow \gamma^{*} \gamma$ Where is the pion in the proton? 

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We study the scaling regime of nucleon - anti-nucleon annihilation into a deeply virtual photon and a photon or meson, $\bar{p} N \rightarrow \gamma^{*} \pi, \bar{p} N \rightarrow \gamma^{*} \gamma$, in the forward direction. The leading twist amplitude factorizes into an antiproton distribution amplitude, a short-distance matrix element and a long-distance dominated transition distribution amplitude (TDA) which describes the nucleon to meson or photon transition. The impact representation of this TDA maps out the transverse locations of the small size core and the meson or photon cloud inside the proton.

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## 1. A new factorization

The understanding of the hadronic structure needs appropriate tools to be manufactured[1]. It recently appeared that a fruitful approach could be accessed through exclusive hard quasi forward scattering, the prototype reaction being deep virtual Compton scattering in the forward region. We have generalized $[2,3]$ this analysis to the reactions

$$
\bar{p} N \rightarrow \gamma^{*} \pi \quad \bar{p} N \rightarrow \gamma^{*} \gamma
$$

which will be accessible at future intense antiproton facilities[4]. Our arguments for the factorization of the short distance hard subprocess from the usual distribution amplitude and a new transition distribution amplitude, defined below, are a succession of logical steps generalizing the factorization proof [5] of deep exclusive meson electroproduction on a meson $\gamma^{*} M 1 \rightarrow M 2 M 3$ in the forward direction, to its time reversed [6] $M 2 M 1 \rightarrow \gamma^{*} M 3$, to meson-meson annihilation $M 2 M 1 \rightarrow \gamma^{*} \gamma$ with the meson-photon analogy proven by the studies of the photon structure functions. The ultimate generalization from the meson case to the baryon case, implying three quark exchanges is advocated to be safe on the basis of the QCD analysis of baryon form factors.

We thus propose to write the $\bar{p} N \rightarrow \gamma^{*} \pi$ amplitude as

$$
\begin{equation*}
\mathscr{M}\left(Q^{2}, \xi, t\right)=\int d x d y \phi\left(y_{i}, Q^{2}\right) T_{H}\left(x_{i}, y_{i}, Q^{2}\right) T\left(x_{i}, \xi, t, Q^{2}\right), \tag{1.1}
\end{equation*}
$$

where $\phi\left(y_{i}, Q^{2}\right)$ is the antiproton distribution amplitude, $T_{H}$ the hard scattering amplitude, calculated in the colinear approximation and $T\left(x_{i}, \xi, t, Q^{2}\right)$ the new TDAs.


Figure 1: The factorization of the annihilation process $\bar{p} p \rightarrow \gamma^{*} \pi$ into the antiproton distribution amplitude $(D A)$, the hard subprocess amplitude $\left(T_{H}\right)$ and a baryon $\rightarrow$ meson transition distribution amplitude (TDA).

## 2. Transition Distribution Amplitudes

To define the transition distribution amplitudes from a nucleon to a pseudoscalar meson, we introduce light-cone coordinates $v^{ \pm}=\left(v^{0} \pm v^{3}\right) / \sqrt{2}$ and transverse components $v_{T}=\left(v^{1}, v^{2}\right)$ for any four-vector $v$. The skewedness variable $\xi=-\Delta^{+} / 2 P^{+}$with $\Delta=p^{\prime}-p$ and $P=\left(p+p^{\prime}\right) / 2$ describes the loss of plus-momentum of the incident hadron in the proton $\rightarrow$ meson transition. We parametrize the quark momenta as shown on Fig. 1. The fractions of + momenta are labelled $x_{1}, x_{2}$ and $x_{3}$, and their supports are within $[-1+\xi, 1+\xi]$. Momentum conservation implies : $\sum_{i} x_{i}=2 \xi$.

The fields with positive momentum fractions, $x_{i} \geq 0$, describe creation of quarks, whereas those with negative momentum fractions, $x_{i} \leq 0$, the absorption of antiquarks. The eight leading twist TDAs for the $p \rightarrow \pi^{0}$ (which can be expressed in terms of eight independent helicity amplitudes for $p \rightarrow u u d \pi$ transition) then reads :

$$
\begin{align*}
& \left.4\left\langle\pi^{0}\left(p^{\prime}\right)\right| \varepsilon^{i j k} u_{\alpha}^{i}\left(z_{1} n\right) u_{\beta}^{j}\left(z_{2} n\right) d_{\gamma}^{k}\left(z_{3} n\right)|p(p, s)\rangle\right|_{z^{+}=0, z_{T}=0}  \tag{2.1}\\
& =-\frac{f_{N}}{2 f_{\pi}}\left[V_{1}^{0}(\hat{P} C)_{\alpha \beta}(B)_{\gamma}+A_{1}^{0}\left(\hat{P} \gamma^{5} C\right)_{\alpha \beta}\left(\gamma^{5} B\right)_{\gamma}-3 T_{1}^{0}\left(P^{v} i \sigma_{\mu \nu} C\right)_{\alpha \beta}\left(\gamma^{\mu} B\right)_{\gamma}\right] \\
& +V_{2}^{0}(\hat{P} C)_{\alpha \beta}\left(\hat{\Delta}_{T} B\right)_{\gamma}+A_{2}^{0}\left(\hat{P} \gamma^{5} C\right)_{\alpha \beta}\left(\hat{\Delta}_{T} \gamma^{5} B\right)_{\gamma}+T_{2}^{0}\left(\Delta_{T}^{\mu} P^{v} \sigma_{\mu \nu} C\right)_{\alpha \beta}(B)_{\gamma} \\
& +T_{3}^{0}\left(P^{v} \sigma_{\mu \nu} C\right)_{\alpha \beta}\left(\sigma^{\mu \rho} \Delta_{T}^{\rho} B\right)_{\gamma}+\frac{T_{4}^{0}}{M}\left(\Delta_{T}^{\mu} P^{v} \sigma_{\mu \nu} C\right)_{\alpha \beta}\left(\hat{\Delta}_{T} B\right)_{\gamma}
\end{align*}
$$

where $\sigma^{\mu \nu}=i / 2\left[\gamma^{\mu}, \gamma^{v}\right], C$ is the charge conjugation matrix and $B$ the nucleon spinor. $f_{\pi}$ is the pion decay constant $\left(f_{\pi}=93 \mathrm{MeV}\right)$ and $f_{N}$ is the constant which determines the value of the nucleon wave function at the origin, and which has been estimated through QCD sum rules to be of order $5.3 \cdot 10^{-3} \mathrm{GeV}^{2}$. Each TDA is then Fourier transformed to get the usual representation in terms of the momentum fractions, through the relation

$$
\begin{equation*}
F\left(z_{i} P \cdot n\right)=\int_{-1+\xi}^{1+\xi} d^{3} x \delta\left(x_{1}+x_{2}+x_{3}-2 \xi\right) e^{-i P n \Sigma x_{i} z_{i}} F\left(x_{i}, \xi\right) \tag{2.2}
\end{equation*}
$$

where $F$ stands for $V_{i}, A_{i}, T_{i}$ and $\int d^{3} x \equiv \int d x_{1} d x_{2} d x_{3} \delta\left(2 \xi-x_{1}-x_{2}-x_{3}\right)$. The first three terms in (2.1) are the only ones surviving the forward limit $\Delta_{T} \rightarrow 0$. The constants in front of these three terms have been chosen in reference to the soft pion $(\xi \rightarrow 1)$ limit results :

$$
\begin{align*}
V_{1}^{0}\left(x_{1}, x_{2}, x_{3}\right) & \rightarrow \frac{1}{2}\left(\phi_{N}\left(x_{1}, x_{2}, x_{3}\right)+\phi_{N}\left(x_{2}, x_{1}, x_{3}\right)\right) \\
A_{1}^{0}\left(x_{1}, x_{2}, x_{3}\right) & \rightarrow \frac{1}{2}\left(\phi_{N}\left(x_{1}, x_{2}, x_{3}\right)-\phi_{N}\left(x_{2}, x_{1}, x_{3}\right)\right)  \tag{2.3}\\
T_{1}^{0}\left(x_{1}, x_{2}, x_{3}\right) & \rightarrow \frac{1}{2}\left(\phi_{N}\left(x_{1}, x_{3}, x_{2}\right)+\phi_{N}\left(x_{2}, x_{3}, x_{1}\right)\right)
\end{align*}
$$

where $\phi_{N}\left(x_{1}, x_{2}, x_{3}\right)$ is the standard leading twist DA.

## 3. Impact Parameter Picture

As in the case of generalized parton distributions [7] and distribution amplitudes [8] the simultaneous presence of two transverse scales $Q^{2}$ and $-t$, allows through a Fourier transform to map the impact parameter dependence of the scattering amplitude. In the case under study, the $t$ - dependence of the $N \rightarrow \pi$ transition distribution amplitude allows in its ERBL region (namely, when all $x_{i}>0$ ) a transverse scan of the location of the small sized (of the order of $1 / Q$ ) hard core made of three quarks when a pion carries the rest of the momentum of the nucleon. This may be phrased alternatively as detecting the transverse mean position of a pion inside the proton, when the proton state is of the "next to leading Fock " order, namely $\mid q q q \pi>$. This is shown on Fig. 2. The other regions have slightly different interpretations.

The study of different TDAs such as the ones for $p \rightarrow \pi^{0}$ and $n \rightarrow \pi^{-}$, related to the $\bar{p} p$ and $\bar{p} n$ reactions, may shed light on the $\mid u u d \pi^{0}>$ versus $\mid u d d \pi^{+}>$components of the proton.


Figure 2: Impact parameter space representation of the $p \rightarrow \pi^{0}$ TDA in the ERBL region $x_{i}>0$.

## 4. Conclusion

The formalism developed for the proton antiproton exclusive annihilation may as well be used for related channels such as backward virtual Compton scattering or backward electroproduction of a meson, where data exist for moderate values of $Q^{2}$. These spacelike analogs of the processes discussed here share the same virtues and their studies should allow a first look at the internal structure of the $\mid q q q \pi>$ states inside the nucleon. Mesonic channels may also be studied as $\gamma^{*} \gamma \rightarrow \pi \pi, \gamma^{*} \gamma \rightarrow \pi \rho$ or $\gamma^{*} \gamma \rightarrow \rho \rho$ in the near forward region. The TDAs are then not much different from the mesonic GPDs.

Work of L.Sz. is supported by the Polish Grant 1 P03B 028 28. He is a Visiting Fellow of the FNRS (Belgium).

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