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Maximal Neutrino Mixing from Discrete Symmetry in Extra Dimensions

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I review the construction of a model for lepton masses based on the flavour symmetry group $A_4\times$ U(1) reproducing the so-called tri-bimaximal lepton mixing, in eccelent agreement with current data. The model predicts a neutrino spectrum of normal hierarchy type, not far from degenerate. A testable relation between neutrino masses is obtained. I shortly discuss also general requirements for models based on spontaneously broken flavour symmetries, in order to get a maximal atmospheric mixing angle.

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The present allowed range of leptonic mixing angles is rather constrained by the available data on neutrino oscillations. At the 2σ level (95% C.L.) [1]:

$$\sin^2\theta_{23} = 0.44\times(1^{+0.41}_{-0.22}) \quad , \quad \sin^2\theta_{13} = 0.9^{+2.3}_{-0.9}\times 10^{-2} \quad , \quad \sin^2\theta_{12} = 0.314\times(1^{+0.18}_{-0.15}) \quad . \tag{1}$$

This range is fully compatible with the so-called Harrison-Perkins-Scott (HPS) mixing scheme [2]:

$$U_{HPS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad . \tag{2}$$

Today the errors on θ_{23} and θ_{13} are large and sizeable deviations from the HPS scheme are still allowed. *(Continue)*

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More impressive is the agreement on the θ_{12} angle, the most precisely measured. The HPS scheme predicts $\theta_{12} \approx 35.3^0$ while the experimental value is $\theta_{12} = (34.1^{+1.7}_{-1.6})^0$ (1σ errors). In the not so far future, probably less than 10 years from now [3], also θ_{23} and θ_{13} might be known or constrained with a comparable precision, thus confirming or excluding the HPS scheme at the λ^2 level, $\lambda \approx 0.22$ being the Cabibbo angle, here regarded as a typical expansion parameter for flavour mixing. Since the HPS scheme requires a maximal θ_{23} angle and a vanishing θ_{13}, two features that are far from generic in model building, it is interesting to see if it can be justified on the basis of some dynamical or symmetry principle.

Maximal and vanishing mixing angles are rather special and we might expect that they arise in the context of a flavour symmetry in the limit of exact symmetry, that is by neglecting all the symmetry breaking effects that are typically needed to reproduce detailed features of a realistic pattern of fermion masses. However this is not the case, at least for realistic flavour symmetries, where breaking terms are small compared to the leading ones [4]. Indeed, if the flavour symmetry is broken only by small effects, the mass matrices for charged leptons and neutrinos can be written as:

$$m_e = m_e^0 + ... \quad , \qquad m_\nu = m_\nu^0 + ... \tag{3}$$

where dots denote symmetry breaking effects and m_e^0 has rank less or equal than one. Rank greater than one, as for instance when both the tau and the muon have non-vanishing masses in the symmetry limit, is clearly an unacceptable starting point, since the difference between the two non-vanishing masses can only be explained by large breaking effects. If the rank of m_e^0 vanishes, than all mixing angles in the charged lepton sector are undetermined in the symmetry limit and θ_{23} is also completely undetermined. If m_e^0 has rank one, then by a unitary transformation we can always go to a field basis where

$$m_e^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau^0 \end{pmatrix} \quad . \tag{4}$$

Denoting by U_ν and U_e the unitary matrices that diagonalize m_ν^0 and $m_e^{0\dagger} m_e^0$, we have

$$U_e = R_{12}(\theta_{12}^e) \tag{5}$$

where the angle θ_{12}^e is completely undetermined (R_{ij} is the orthogonal matrix representing a rotation in the ij sector). Moreover, by neglecting phases and adopting the standard parametrization $U_\nu = R_{23}(\theta_{23}^\nu) R_{13}(\theta_{13}^\nu) R_{12}(\theta_{12}^\nu)$, we find that the angle θ_{23} of the physical mixing matrix $U_{PMNS} = U_e^\dagger U_\nu$ is given by:

$$\tan\theta_{23} = \cos\theta_{12}^e \tan\theta_{23}^\nu + \sin\theta_{12}^e \frac{\tan\theta_{13}^\nu}{\cos\theta_{23}^\nu} \quad . \tag{6}$$

Therefore, in general, the atmospheric mixing angle is always undetermined at the leading order (this conclusion is unchanged if phases are accounted for). When small symmetry breaking terms are added to m_e^0 and m_ν^0, it is possible to obtain $\theta_{23} = \pi/4$, provided these breaking terms have suitable orientations in the flavour space.

If the breaking terms originate from a spontaneous symmetry breaking, there are four requirements to satisfy in order to obtain the HPS scheme. 1) Two independent scalar sectors are needed. One of them communicates the breaking to charged fermions and the other one feeds the breaking to neutrinos. In such a framework a maximal atmospheric mixing angle is always the result of a special vacuum alignment between these two sectors. 2) This alignment should be natural. It should correspond to a local minimum of the potential energy of the theory, in a finite region of the parameter space, *i.e.* without enforcing any ad-hoc relation among parameters. 3) The alignment should not be spoiled by large sub-leading terms. In general the mixing angles are power series in the symmetry breaking order parameters. Calling $\langle\varphi\rangle$ the generic such parameter, even by enforcing HPS at the leading order, we expect:

$$\theta_{13} = 0 + a_1\frac{\langle\varphi\rangle}{\Lambda} + a_2\frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots \quad , \qquad \theta_{23} = \frac{\pi}{4} + b_1\frac{\langle\varphi\rangle}{\Lambda} + b_2\frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots \tag{7}$$

the higher-order corrections coming from higher-dimensional operators compatible with the flavour symmetry. It is not sufficient that the alignment produces the desired first term in the expansion. We should also be able to keep under control the remaining contributions. This can be done either by adopting, if possible, a small breaking parameter (for instance $\langle\varphi\rangle/\Lambda < \lambda$), or by building the model in such a way that the first corrections a_1 and b_1 vanish. 4) Finally, the alignment should be compatible with the mass hierarchies. In particular m_e/m_τ and m_μ/m_τ should vanish when $\langle\varphi\rangle/\Lambda$ is set to zero.

A close relation between the HPS scheme and the discrete symmetry group A_4 has been known for some time [5] and, indeed, all the requirements listed above can be satisfied in a model for lepton masses based on the flavour symmetry $A_4\times$ U(1) [6], where the U(1) factor controls the charged lepton mass hierarchies. The group A_4 is made of the twelve three-dimensional rotations leaving invariant a tetrahedron and possesses four representations: three singlets 1, 1$'$, 1$''$ and a triplet 3. The assignment of the relevant fields to representations of $A_4\times$ U(1) is given in the table,

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	θ
A_4	3	1	$1'$	$1''$	1	3	3	1	1
$U(1)$	0	4	2	0	0	0	0	0	-1

where the symmetry breaking sector is described in the last five columns. The VEV of θ breaks the U(1) symmetry and provides the correct hierarchy to charged leptons. The alignment needed to reproduce the HPS pattern is

$$\langle\varphi'\rangle = (v',0,0) \quad , \qquad \langle\varphi\rangle = (v,v,v) \quad , \qquad \langle\xi\rangle = u \quad , \tag{8}$$

where, in the absence of particular relations among the parameters of the model, $v \approx v' \approx u \approx \langle\theta\rangle$ is expected. A simple, not necessarily unique, set up that gives rise to (8) is depicted in the figure. The fields φ and (φ',ξ), giving masses respectively to charged leptons and to neutrinos, live at the opposite ends of an extra spatial dimension and it can be shown that this naturally leads to the desired alignment. Left-handed leptons live on the brane at $y = L$ and neutrinos acquire masses directly from the operators:

$$\frac{1}{\Lambda^2}\xi(ll)h_u h_u \quad , \qquad \frac{1}{\Lambda^2}(\varphi' ll)h_u h_u \quad . \tag{9}$$

Right-handed leptons live in $y = 0$ and e, μ and τ get their masses indirectly, by the exchange of an heavy bulk fermion F of mass M that interacts on the two branes through the operators:

$$\frac{(f^c\varphi F)}{\sqrt{\Lambda}}\delta(y) \quad , \qquad \frac{(F^c l)h_d}{\sqrt{\Lambda}}\delta(y-L) \quad . \tag{10}$$

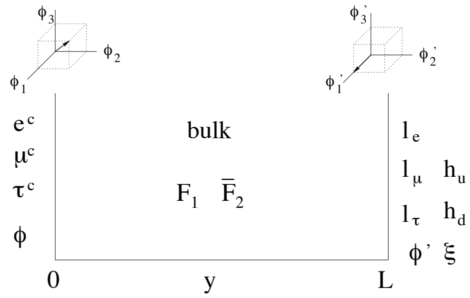

Figure 1: Fifth dimension and localization of scalar and fermion fields. The symmetry breaking sector includes the A_4 triplets φ and φ', localized at the opposite ends of the interval. Their VEVs are dynamically aligned along the directions shown at the top of the figure.

At energies much smaller than M, an effective Yukawa of the kind $(f^c \varphi l) h_d e^{-ML}/\Lambda$ is generated. At leading order, by neglecting possible higher dimensional operators, the HPS scheme and the correct hierarchies of charged fermion masses are obtained. A detailed analysis, including possible non-leading effects arising from higher dimensionality operators, reveals that the first corrections to the HPS mixing pattern only arise at the second order in the expansion parameter VEV/Λ. In this model, in order to accommodate the hierarchy of charged fermion masses, $VEV/\Lambda \approx \lambda \approx 0.22$ and thus the expected deviations from the HPS scheme are tiny. The neutrino spectrum is of normal hierarchy type and, in a large portion of the parameter space, we find $|m_3| \approx 0.053$ eV, $|m_1| \approx |m_2| \approx 0.017$ eV. An accidental quasi-degeneracy is not excluded. We also have $|m_{ee}| \approx 0.005$ eV, at the upper edge of the range allowed for normal hierarchy, but unfortunately too small to be detected in a near future. In the whole parameter space, barring small corrections, the following testable relation holds:

$$|m_3|^2 = |m_{ee}|^2 + \frac{10}{9} \Delta m^2_{atm} \left(1 - \frac{\Delta m^2_{sol}}{2 \Delta m^2_{atm}}\right) \quad . \tag{11}$$

This model provides an existence proof of a class of ‘special’ models where, at variance with most of the existing models [7], θ_{23} is maximal and θ_{13} is vanishing within tiny corrections which are probably below the sensitivity achievable in the near future.

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