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Neutrino mass matrices, texture zeros, and family symmetries

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We demonstrate that Abelian family symmetries allow one to enforce texture zeros in arbitrary entries of the fermion mass matrices. Placing zeros in any number of elements of all occurring mass matrices can be done with two alternative methods; one of them utilizes the group $\mathbb{Z}_n$ with n sufficiently high. Concentrating on the lepton sector and on neutrino masses, we discuss the methods in the case of seesaw models and scalar triplet models. As an illustration, we present an example for each type of model.

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Texture zeros in fermion mass matrices [1] present the simplest procedure to reduce the number of parameters and to induce relations among the physical quantities (masses, mixing angles, and CP phases). At first sight this procedure is quite arbitrary and in general it will not lead to renormalizable models. However, we have shown that schemes with texture zeros can be promoted to renormalizable models by an enlargement of the scalar sector [2]: *For every set of fermion mass matrices with texture zeros in arbitrary entries, there exists a scalar sector such that the texture zeros are enforced by means of Abelian symmetries.* In this talk we confine ourselves to the lepton sector with Majorana neutrinos and extensions of the Standard Model (SM) below the GUT scale. However, we emphasize that our method is completely general and also applies to the quark sector.

We will discuss the lepton sector with the seesaw mechanism and show that there are two methods for the symmetry implementation of texture zeros. For this purpose, we consider the Yukawa Lagrangian [2]

$$\mathscr{L}_Y = -\sum_{a,b=1}^{3}\left(\Gamma_{ab}\bar{\ell}_{Ra}\phi^\dagger_{ab}D_{Lb} + \tilde{\Gamma}_{ab}\,\bar{\nu}_{Ra}\tilde{\phi}^\dagger_{ab}D_{Lb} + \frac{1}{2}Y_{ab}\,\chi_{ab}\bar{\nu}_{Ra}C\bar{\nu}^T_{Rb}\right) + \text{H.c.}, \tag{1}$$

where D_L denotes the left-handed doublets, ℓ_R the right-handed charged singlets, and ν_R the right-handed neutrino singlets. Note that there is one scalar multiplet for every fermion bilinear! Thus we have nine Higgs doublets ϕ_{ab} with hypercharge $+1$, nine Higgs doublets $\tilde{\phi}_{ab}$ with hypercharge -1, and six gauge singlet scalars $\chi_{ab} \equiv \chi_{ba}$. The corresponding vacuum expectation values (VEVs) are denoted by v_{ab}, w^*_{ab}, and X_{ab}, respectively. The charged-lepton mass matrix is given by $\left(M_\ell\right)_{ab} = v^*_{ab}\Gamma_{ab}$, the neutrino Dirac mass matrix by $\left(M_D\right)_{ab} = w_{ab}\tilde{\Gamma}_{ab}$, and the mass matrix of the heavy neutrino singlets by $\left(M_R\right)_{ab} = X_{ab}Y_{ab}$.

Method 1: We introduce the Abelian symmetry group $\mathscr{G} = \times_f \mathscr{G}(f)$ for $f = \ell_{Ra}, D_{La}, \nu_{Ra}$ ($a = 1,2,3$), which has thus nine factors. Then, in order to allow the couplings in (1), the scalar multiplets transform as

$$\phi_{ab} : \mathscr{G}^*(\ell_{Ra}) \otimes \mathscr{G}(D_{Lb}), \quad \tilde{\phi}_{ab} : \mathscr{G}^*(\nu_{Ra}) \otimes \mathscr{G}(D_{Lb}), \quad \chi_{ab} : \mathscr{G}(\nu_{Ra}) \otimes \mathscr{G}(\nu_{Rb}). \tag{2}$$

There is one scalar multiplet for every entry in all three mass matrices. Now it is easy to place zeros in arbitrary entries of M_ℓ, M_D, and M_R. Consider for instance M_ℓ. If there exists a ϕ_{ab} transforming as $\mathscr{G}^*(\ell_{Ra}) \otimes \mathscr{G}(D_{Lb})$, then $\Gamma_{ab} \neq 0$; if such a ϕ_{ab} does not occur, then $\Gamma_{ab} = 0$.

Method 2: We consider the symmetry group $\mathscr{G} = \mathbb{Z}_n$ or $\mathscr{G} = \mathbb{Z}_n \times \mathbb{Z}_2$ with n sufficiently high. It turns out that in the multi-Higgs SM with the seesaw mechanism one never needs a larger group [2] than $\mathbb{Z}_{12} \times \mathbb{Z}_2$. This is easily demonstrated by assuming, e.g., that $\bar{\ell}_R$ and $\bar{\nu}_R$ transform as $(\omega, \omega^2, \omega^5)$ and D_L transform as $(\omega, \omega^3, \omega^8)$ with $\omega = \exp(i\pi/6)$. Consequently, fermionic bilinears transform as

$$\bar{\ell}_{Ra}D_{Lb}, \bar{\nu}_{Ra}D_{Lb} \sim \begin{pmatrix} \omega^2 & \omega^4 & \omega^9 \\ \omega^3 & \omega^5 & \omega^{10} \\ \omega^6 & \omega^8 & \omega \end{pmatrix}, \quad \bar{\nu}_{Ra}C\bar{\nu}^T_{Rb} \sim \begin{pmatrix} \omega^2 & \omega^3 & \omega^6 \\ \omega^3 & \omega^4 & \omega^7 \\ \omega^6 & \omega^7 & \omega^{10} \end{pmatrix}. \tag{3}$$

The additional $\mathbb{Z}_2 : \tilde{\phi}_{ab} \to -\tilde{\phi}_{ab}$, $\nu_R \to -\nu_R$ couples the ϕ_{ab} solely to ℓ_R and the $\tilde{\phi}_{ab}$ solely to ν_R. Now the argument for placing zeros goes as before. Consider Eq. (3); if for instance ϕ_{13} is present transforming as ω^9 under $\mathbb{Z}_{12}$, then $\left(M_\ell\right)_{13} \neq 0$, and so on.

Some remarks to both methods are at order. In practice, in predictive models there are many texture zeros, thus rather few scalars are necessary and a proliferation of scalars is avoided. Moreover, Methods 1 and 2 often merge more or less. The symmetry group $\mathscr{G}$ is large, therefore usually soft breaking of $\mathscr{G}$ in the scalar potential is necessary to avoid Goldstone bosons.

Let us now consider texture zeros in neutrino mass matrices. We assume a diagonal M_ℓ, which amounts to six texture zeros in this matrix. Then, as shown in [3], for the Majorana mass matrix $\mathscr{M}_\nu$ of the light neutrinos, there are seven viable textures with two zeros. Modulo phase redefinitions, such mass matrices have five physical parameters. Since there are nine physical quantities in $\mathscr{M}_\nu$ (three neutrino masses, three mixing angles, one CKM-like phase, and two Majorana phases), in such a scenario one has four relations among the physical quantities [3, 4].

As an illustration we consider two cases of [3]:

$$\text{Case A}_2\text{:} \quad \mathscr{M}_\nu \sim \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \text{Case C:} \quad \mathscr{M}_\nu \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}. \tag{4}$$

There are several possible type I seesaw realizations of $\mathscr{M}_\nu = -M_D^T M_R^{-1} M_D$ for Case A_2, see [5]. As an example we take

$$M_D \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad M_R \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}. \tag{5}$$

Applying Method 1, we observe that M_ℓ being diagonal allows to make the identification $\mathscr{G}(\ell_{Ra}) \equiv \mathscr{G}(D_{La})$. We choose $\mathscr{G}(D_{La}) = \mathbb{Z}_2(D_{La})$, $\mathscr{G}(\nu_{Ra}) = \mathbb{Z}_4(\nu_{Ra})$. Then we straightforwardly arrive at the scalar sector and its transformation properties [2]:

$$\begin{array}{ll} \tilde{\phi}_{11} : \mathbb{Z}_4^*(\nu_{R1}) \otimes \mathbb{Z}_2(D_{L1}), & \chi_{11} : \mathbb{Z}_4(\nu_{R1}) \otimes \mathbb{Z}_4(\nu_{R1}), \\ \tilde{\phi}_{13} : \mathbb{Z}_4^*(\nu_{R1}) \otimes \mathbb{Z}_2(D_{L3}), & \chi_{22} : \mathbb{Z}_4(\nu_{R2}) \otimes \mathbb{Z}_4(\nu_{R2}), \\ \tilde{\phi}_{23} : \mathbb{Z}_4^*(\nu_{R2}) \otimes \mathbb{Z}_2(D_{L3}), & \chi_{13} : \mathbb{Z}_4(\nu_{R1}) \otimes \mathbb{Z}_4(\nu_{R3}), \\ \tilde{\phi}_{32} : \mathbb{Z}_4^*(\nu_{R3}) \otimes \mathbb{Z}_2(D_{L2}). & \end{array} \tag{6}$$

In addition, in the charged-lepton sector one Higgs doublet transforming trivially is needed. Thus we end up with five Higgs doublets and three scalar singlets. In this example we have seen the typical simplification of Method 1 if one mass matrix happens to be diagonal: $\mathscr{G}$ reduces to a direct product of only six groups. Applying Method 2, we find a more economical symmetry realization of Case A_2: $\mathscr{G} = \mathbb{Z}_8$ with two Higgs doublets and two scalar singlets [2].

All cases found in [3] can also be realized via Abelian symmetries and scalar triplets [6]; no right-handed neutrino singlets are needed and there is a single Higgs doublet ϕ responsible for the charged lepton masses. We exemplify this with Case C.[1] The Yukawa couplings of the scalar triplets are given by

$$\mathscr{L}_{Y\Delta} = \frac{1}{2} \sum_j \sum_{a,b=1}^{3} h_{ab}^j D_{La}^{\mathrm{T}} C^{-1} \left(i\tau_2 \Delta_j \right) D_{Lb} + \mathrm{H.c.} \tag{7}$$

[1] In [7], this case is realized via the non-Abelian symmetry group $\mathbb{Q}_8$.

The VEVs w_j of the neutral components of the scalar triplets Δ_j generate the neutrino mass matrix $\mathcal{M}_\nu = \sum_j w_j h^j$. We confine ourselves to Method 2 and make the ansatz

$$D_{La} \to p_a D_{La}, \quad \ell_{Ra} \to p_a \ell_{Ra}, \quad \phi \to \phi \quad \text{with} \quad |p_a| = 1 \tag{8}$$

for the symmetry transformation. With all phase factors p_a different from each other, M_ℓ is automatically diagonal. A suitable choice is $p_e = 1,\ p_\mu = i,\ p_\tau = -i$. Then the transformation properties of the bilinears in leptonic doublets determine the number and the transformation properties of the scalar triplets:

$$D_{La}^T C^{-1} D_{Lb} \sim \begin{pmatrix} 1 & i & -i \\ i & -1 & 1 \\ -i & 1 & -1 \end{pmatrix} \Rightarrow \begin{cases} \Delta_1 \to \ \ \Delta_1, \\ \Delta_2 \to -i\Delta_2, \\ \Delta_3 \to \ \ i\Delta_3. \end{cases} \tag{9}$$

Thus we have found a symmetry realization of Case C with the family symmetry $\mathbb{Z}_4$ which needs only one Higgs doublet and three scalar triplets. For the symmetry realization of all cases of [3] with scalar triplet models see [6]. Texture zeros in $\mathcal{M}_\nu$ with triplet realizations are stable under the renormalization group running because only one Higgs doublet is present.

In summary, the two methods presented in [2] allow to embed all kinds of fermion mass matrix schemes with texture zeros in renormalizable models, possibly at the cost of a proliferation of the scalar sector; such an embedding is not unique. We emphasize the versatility of the methods which would equally well apply in the quark sector or in Grand Unified Theories. Finally we note that not only texture zeros in $\mathcal{M}_\nu$ but also in $\mathcal{M}_\nu^{-1}$ can lead to interesting models [8].

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