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Hybrid Textures of Neutrinos

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We present numerical and comprehensive analyses of the sixty hybrid textures of neutrinos, which have an equality of matrix elements and one zero. These textures are possibly derived in the models with discrete flavor symmetry. Only six textures among sixty ones are excluded by the present experimental data. Since there are many textures which give similar predictions, the textures are classified based on the numerical results. The neutrinoless double beta decay is also examined in these textures. Our results suggest that there remain still rich structures of the neutrino mass matrix in the phenomenological point of view.

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The results of neutrino oscillation experiments indicate the neutrino masses and mixings, especially, the bi-large flavor mixing. It is therefore important to investigate how the textures of lepton mass matrices can link up with the observables of the flavor mixings. Many authors studied the texture zeros [1, 2, 3], which may follow from the flavor symmetry. On the other hand, one finds some relations among the non-zero mass matrix elements in the discrete symmetry of the flavor. This fact suggests that the texture zero analyses is not enough to reveal some underlying flavor symmetry.

For example, in the e, μ, τ basis, one finds the following symmetric mass matrices of neutrinos, where there are two entries with same values and one zero,

$$\begin{pmatrix} a & c & d \\ & b & 0 \\ & & b \end{pmatrix} , \qquad \begin{pmatrix} 0 & c & d \\ & a & b \\ & & a \end{pmatrix} , \tag{1}$$

both are presented in the quaternion family symmetry, Q_8 [4] and the latter is given in the S_3 symmetry [5]. The variant of these textures is also discussed in ref. [6]. We call this type texture as the “Hybrid” texture, which has an equality of matrix elements and one zero.

The analytical study of various structures of the neutrino mass matrix was presented systematically by Frigerio and Smirnov [7], who also discussed the case of equalities of matrix elements. The textures for the Dirac neutrinos also discussed in [8]. However, numerical and comprehensive analyses have not been given. In this work, we present numerical analyses of the sixty hybrid textures, which have equal two neutrino mass matrix elements with one zero [9]. Our analyses include textures in the previous studies in [4, 5]. Our results are consistent with their ones.

Let us construct the neutrino mass matrix in terms of neutrino mass eigenvalues m_1, m_2, m_3, mixing angles and CP violating phases. Neutrino mass matrix M_ν in the basis where the charged lepton mass matrix is diagonal (flavor basis) is given as follows:

$$M_\nu = P\, U^*\, M_{\text{diagonal}}\, U^\dagger\, P\ , \tag{2}$$

where M_{diagonal} and P are the diagonal mass matrix and the diagonal phase matrix, respectively:

$$M_{\text{diagonal}} = \text{diag}(\lambda_1, \lambda_2, \lambda_3)\ , \ \ P = \text{diag}(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})\ , \tag{3}$$

where λ_i $(i = 1, 2, 3)$ are the complex mass eigenvalues including Majorana phases, so three neutrino masses m_i are given as the absolute values of λ_i. U is the MNS mixing matrix. On the other hand, ϕ_i $(i = e, \mu, \tau)$ are unphysical phases depending on the phase convention. Then, the neutrino mass matrix elements $(M_\nu)_{\alpha\beta}$ are given in the flavor basis as

$$(M_\nu)_{\alpha\beta} = e^{i(\phi_\alpha+\phi_\beta)} \sum_f^3 U^*_{\alpha f} U^*_{\beta f} \lambda_f = e^{i(\phi_\alpha+\phi_\beta)}\ (U^*_{\alpha 1} U^*_{\beta 1} \lambda_1 + U^*_{\alpha 2} U^*_{\beta 2} \lambda_2 + U^*_{\alpha 3} U^*_{\beta 3} \lambda_3)\ . \tag{4}$$

By use of these mass matrix elements, we can analyze the sixty cases in terms of mass eigenvalues and mixings. These cases are combinations of the one zero textures and the equal elements: (i) fifteen cases of the equal elements (Type A $\sim$ O) $(M_\nu)_{\alpha\beta} = (M_\nu)_{\gamma\delta}$, (ii) six cases of one zero (Type I $\sim$ VI) $(M_\nu)_{\alpha\beta} = 0$. These combinations give ninety textures, however, among them, thirty textures have two zeros, which have been studied in details [1, 2, 3]. Therefore, we study numerically in the sixty textures, which are summarized in [9].

Since the textures have the conditions $(M_\nu)_{\alpha\beta} = 0$ and $(M_\nu)_{ij} = (M_\nu)_{k\ell}$, we can get the ratios of mass eigenvalues by solving two equations as follows [3]:

$$\begin{aligned} \frac{\lambda_1}{\lambda_2} &= \frac{(U^*_{i3}U^*_{j3} - Q\,U^*_{k3}U^*_{\ell 3})U^*_{\alpha 2}U^*_{\beta 2} - (U^*_{i2}U^*_{j2} - Q\,U^*_{k2}U^*_{\ell 2})U^*_{\alpha 3}U^*_{\beta 3}}{(U^*_{i1}U^*_{j1} - Q\,U^*_{k1}U^*_{\ell 1})U^*_{\alpha 3}U^*_{\beta 3} - (U^*_{i3}U^*_{j3} - Q\,U^*_{k3}U^*_{\ell 3})U^*_{\alpha 1}U^*_{\beta 1}} \,, \\ \frac{\lambda_3}{\lambda_2} &= \frac{(U^*_{i1}U^*_{j1} - Q\,U^*_{k1}U^*_{\ell 1})U^*_{\alpha 2}U^*_{\beta 2} - (U^*_{i2}U^*_{j2} - Q\,U^*_{k2}U^*_{\ell 2})U^*_{\alpha 1}U^*_{\beta 1}}{(U^*_{i1}U^*_{j1} - Q\,U^*_{k1}U^*_{\ell 1})U^*_{\alpha 3}U^*_{\beta 3} - (U^*_{i3}U^*_{j3} - Q\,U^*_{k3}U^*_{\ell 3})U^*_{\alpha 1}U^*_{\beta 1}} \,, \end{aligned} \qquad (5)$$

where $Q \equiv e^{i\varphi} = e^{i(\phi_k+\phi_\ell-\phi_i-\phi_j)}$. Taking absolute values of these ratios, we get the neutrino mass ratios, m_1/m_2 and m_3/m_2. Therefore mass ratios are given in terms of θ_{12}, θ_{23}, θ_{13}, CKM-like phase δ in U and the unknown phase φ. Absolute values of neutrino masses are fixed by putting the experimental data $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sun}$.

We classify the textures based on the predicted mixings θ_{23} and $|U_{e3}|$. We cannot distinguish the textures by the mixing θ_{12} at the present stage of the experimental data: [1]

Case 1 : The predicted mixings of the eighteen textures cover whole experimental allowed region : A-I, A-II, A-III, B-I, B-II, B-III, D-V, D-VI, E-II, F-I, I-I, I-II, L-I, L-II, L-IV, O-I, O-II and O-IV.

Case 2 : The allowed points of the twenty-two textures are very few : C-I, C-II, C-III, D-III, E-V, E-VI, F-V, F-VI, G-III, G-V, G-VI, H-III, H-V, H-VI, J-III, J-VI, K-III, K-VI, M-III, M-V, N-III and N-V.

Case 3 : The $\sin^2 2\theta_{23}$ has the lower bound 0.99 : C-IV.

Case 4 : The $|U_{e3}|$ of the six textures has the lower bound, which increases as $\sin^2 2\theta_{23}$ increases : A-VI, B-V, G-II, H-I, L-VI and O-V. The lower bound $|U_{e3}| \geq 0.03$ is obtained in A-VI and B-V, and $|U_{e3}| \geq 0.04$ is clearly predicted in G-II and H-I. The bound $|U_{e3}| \geq 0.05$ is roughly obtained although the allowed points are few in L-VI and O-V.

Case 5 : The lower bound of $|U_{e3}|$ decreases as $\sin^2 2\theta_{23}$ increases in the seven textures : D-IV, I-V, I-VI, J-II, K-I, M-II and N-I, in which $|U_{e3}| = 0$ is allowed at $\sin^2 2\theta_{23} = 1$ except D-IV. The texture D-IV has the lower bound $|U_{e3}| \geq 0.001$.

Case 6 : The six textures are excluded by the experimental data : E-IV, F-IV, J-IV, K-IV, M-IV and N-IV.

We will discuss a typical texture for each cases in the above classification :

$$M_\nu = \begin{pmatrix} X & 0 & e \\ & X & f \\ & & c \end{pmatrix} : \text{A-I} \,, \begin{pmatrix} a & d & e \\ & X & 0 \\ & & X \end{pmatrix} : \text{C-III} \,, \begin{pmatrix} 0 & d & e \\ & X & f \\ & & X \end{pmatrix} : \text{C-IV} \,, \begin{pmatrix} X & X & 0 \\ & b & f \\ & & c \end{pmatrix} : \text{G-II} \,, \begin{pmatrix} X & d & e \\ & 0 & X \\ & & c \end{pmatrix} : \text{I-V} \,. \qquad (6)$$

[1] Please see [9] for the concrete textures of each type.

The texture A-I is a typical one, which leads to the normal hierarchy of the neutrino masses mainly, but the quasi-degenerate spectrum is also allowed. The predicted mixings cover all experimental allowed region on the $\sin^2 2\theta_{12} - |U_{e3}|$ plane as well as on the $\sin^2 2\theta_{23} - |U_{e3}|$ plane. The texture C-III is a typical one, which leads to the inverted hierarchy of the neutrino masses mainly. The texture C-IV gives a specific mass hierarchy and mixing angle θ_{23}, on the other hand, the predicted θ_{12} covers whole experimental allowed region. The texture G-II is a typical one, which gives also a specific mass hierarchy and the clear lower bound of $|U_{e3}|$. The texture I-V is a typical one, which leads to the inverted mass hierarchy of neutrino masses. The prediction excludes the specific region on the $\sin^2 2\theta_{23} - |U_{e3}|$ plane.

It may be helpful to see which future data might rule out these textures. If the inverted mass hierarchy is shown to be realized by Nature, the textures of C-IV and G-II are ruled out. On the contrary, if mass spectrum is the normal hierarchy, the textures of C-III and I-V are ruled out. Finding $\sin^2 2\theta_{23} < 0.98$ and $|U_{e3}| < 0.04$ rule out the texture C-IV and G-II, respectively.

It is important to discuss the neutrinoless double beta decay rate, which is controlled by the effective Majorana mass:

$$\langle m \rangle_{ee} = \left| \, m_1 c_{12}^2 c_{13}^2 e^{i\rho} + m_2 s_{12}^2 c_{13}^2 e^{i\sigma} + m_3 s_{13}^2 e^{-2i\delta} \, \right| \, , \tag{7}$$

where $\rho = \arg(\lambda_1/\lambda_3)$ and $\sigma = \arg(\lambda_2/\lambda_3)$. This effective mass is just the absolute value of $(M_\nu)_{ee}$ component of the neutrino mass matrix. It is remarked that the neutrinoless double beta decay is forbidden in the textures of type IV, because of $(M_\nu)_{ee} = 0$. Many hybrid textures (thirty-eight ones) predict the lower bound $10 \sim 30$ meV although there are differences of factor in the lower bound predictions for each texture.

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