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$B \to \pi\pi$ Hadronic Amplitudes from QCD Light-Cone Sum Rules *

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We analyse $B \to \pi\pi$ decays, obtaining the relevant hadronic amplitudes from QCD light-cone sum rules (LCSR). In this approach the $B \to \pi\pi$ hadronic matrix elements with annihilation topology are finite and turn out to be small with respect to the factorizable amplitude. The pattern of $B \to \pi\pi$ amplitudes obtained from LCSR, including emission, penguin and annihilation effects, does not reveal significant deviations from naive factorization and agrees with the predictions of QCD factorization if the latter approach is taken with moderate values of the annihilation parameters. We find a substantial discrepancy between our predictions and the current data for $B \to \pi^+\pi^-$ and $B \to \pi^0\pi^0$ indicating a missing piece of $\Delta I = 1/2$ transition amplitude.

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1. Introduction

The current data on charmless B decays, in particular on $B \to \pi\pi$, represent a challenge for QCD factorization (QCDF) [1], the method used to evaluate the hadronic matrix elements in these decays. The comparison to the data based on the isospin expansion of the decay amplitudes:

$$A(B^- \to \pi^-\pi^0) = \langle \pi^-\pi^0|H_{\rm eff}|B^-\rangle = \frac{3}{\sqrt{2}}A_2\,, \;\; A(\bar{B}^0 \to \pi^+\pi^-) = \langle \pi^+\pi^-|H_{\rm eff}|\bar{B}^0\rangle = A_2 + A_0\,,$$
$$A(\bar{B}^0 \to \pi^0\pi^0) = \langle \pi^0\pi^0|H_{\rm eff}|\bar{B}^0\rangle = 2A_2 - A_0\,, \tag{1.1}$$

reveals that the isospin-two amplitude A_2 determining the $B^- \to \pi^-\pi^0$ decay is in a reasonable agreement with theoretical predictions, (in factorizable approximation, using the $B\to\pi$ form factor calculated from LCSR , e.g. in [2, 3]), whereas one needs additional contributions to the isospin zero amplitude A_0 which are generated by the $\Delta I = 1/2$ pieces of the effective Hamiltonian, that is, presumably, by the penguin and annihilation effects. However, even when QCD corrections and estimates for the penguin contributions are included, QCDF is only marginally consistent with the data for $B \to \pi\pi$, therefore it is important to employ alternative methods.

Here we present the analysis of two-pion B- decay amplitudes based on the method of QCD light-cone sum rules (LCSR). The method was adjusted for $B \to \pi\pi$ in [4], where the soft-gluon emission mechanism in these decays has been studied. Further LCSR results for $B \to \pi\pi$ which we use in our analysis include estimates of hadronic matrix elements with penguin topology [5], and the most recent calculation of annihilation effects in $B \to \pi\pi$ in [6].

2. Outline of the Method

In order to obtain LCSR for a hadronic matrix element of a certain operator O in the effective Hamiltonian, one uses [4] as a starting object the correlation function defined as:

$$F^{(O)}_\alpha(p,q,k) = -\int d^4x\, e^{-i(p-q)x}\int d^4y\, e^{i(p-k)y}\langle 0|T\left\{j^{(\pi)}_{\alpha 5}(y)O^u(0)j^{(B)}_5(x)\right\}|\pi^-(q)\rangle, \tag{2.1}$$

where $j^{(\pi)}_{\alpha 5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j^{(B)}_5 = m_b\bar{b}i\gamma_5 d$ are the quark currents interpolating the pion and the B meson, respectively. This correlation function is calculated in QCD at large spacelike external momenta squared $(p-k)^2, (p-q)^2, P^2 = (p-q-k)^2$, in a form of the operator product expansion (OPE), combining the short-distance amplitudes with long-distance pion distribution amplitudes of various twist. The result of this calculation is then matched to the hadronic dispersion relations, subsequently in the pion ($j^{(\pi)}_{\alpha 5}$) and B meson ($j^{(B)}_5$) channels, constructed in such a way that the final sum rule relation contains the matrix element $\langle \pi\pi|O|B\rangle$ in the ground-state contribution. The details of the procedure and of the calculation for various topologies can be found in [4, 5, 6]. Note that obtaining $B \to \pi\pi$ hadronic matrix elements one uses an additional assumption of the local quark-hadron duality, allowing the transition (analytical continuation) from a large spacelike to the large timelike scale m_B^2.

Turning, for example, to the most recent calculation of the annihilation effect [6], we emphasize that due to sufficient virtuality of the underlying correlation function, the OPE diagrams with annihilation topology are free from end-point divergences. Both contributions of hard and soft gluons are taken into account. A finite result for the hadronic matrix element of the current-current O_1^u operator with annihilation topology is obtained, including an imaginary part which contributes to the strong phase. In addition, an important factorizable contribution from the quark-penguin operator O_6 has been found. For the annihilation with hard gluons considered in [6], we have modified the method suggested in [4], to avoid the problem of calculating two-loop multi-scale diagrams. Instead of performing the QCD calculation based on the vacuum-to-pion correlation function, we start from the pion-pion correlator, thereby reducing the calculation to one-loop diagrams.

3. Results for $B \to \pi\pi$

The input is specified and explained in [6], in particular we use the one-loop pole b quark mass $m_b = 4.7 \pm 0.1$ GeV and $\alpha_s(m_Z) = 0.1187$ [7]. First, we reevaluate the LCSR result of [2] (see also [3]) for $B \to \pi$ form factor $f^+_{B\pi}(0) = 0.26 \pm 0.02_{[a^\pi_{2,4}]} \pm 0.03_{[param]}$, where the uncertainties induced by Gegenbauer moments and by other sum rule parameters are shown separately. This estimate provides the main input for the factorizable $B \to \pi\pi$ amplitude, that is the hadronic matrix element of the operator $O_1^u = (\bar{d}\Gamma_\mu u)(\bar{u}\Gamma^\mu b)$ in the emission topology:

$$\langle \pi^+\pi^-|O_1^u|\bar{B}^0\rangle_E \simeq i f_\pi f^+_{B\pi}(0) m_B^2 \,. \tag{3.1}$$

Using the sum rules for hard and soft gluon contributions obtained in [6], we estimate the ratios of the hadronic matrix elements in the annihilation topology to the factorizable amplitude (3.1):

$$r_A^{(\pi\pi)} = \frac{\langle \pi^+\pi^-|\tilde{O}_2^u|\bar{B}^0\rangle_A}{\langle \pi^+\pi^-|O_1^u|B^0\rangle_E} = \left[-0.67^{+0.47}_{-0.87} + i\left(3.6^{+0.5}_{-1.1}\right)\right] \times 10^{-3}\,, \tag{3.2}$$

where $\widetilde{O}_2^u = (\bar{u}\Gamma_\mu \frac{\lambda^a}{2} u)(\bar{d}\Gamma^\mu \frac{\lambda^a}{2} b)$. The factorizable annihilation via quark-penguin operator $O_6^d = -2(\bar{d}(1+\gamma_5)d)(\bar{d}(1-\gamma_5)b)$ yields a considerably larger hadronic matrix element:

$$R_A^{(\pi\pi,6)} = \frac{\langle \pi^+\pi^-|O_6^d|\bar{B}^0\rangle_A}{\langle \pi^+\pi^-|O_1^u|B^0\rangle_E} = 0.23^{+0.05}_{-0.08}\,. \tag{3.3}$$

However, the small Wilson coefficient reduces the effect in the decay amplitudes to the same level as for the annihilation via the current-current operator. Thus, the contributions of annihilation amplitudes are found very small, at the same level as the other nonfactorizable effects (soft-gluon emission, charming and gluonic penguins) estimated from LCSR in Refs. [4, 5]. Altogether, the smallness of the corrections to the leading-order factorizable amplitude reveals a good convergence of the OPE series for the correlation function and justifies the use of the adopted approximation in LCSR, that is, including only $O(\alpha_s)$ and twists ≤ 4, as well as omitting the small $O(s_0^\pi/m_B^2)$ corrections in each term of OPE.

Having at hand the estimates of $r_A^{(\pi\pi)}$ and $R_A^{(\pi\pi,6)}$, and for the other analogous parameters determining the hadronic matrix elements with penguin topology, we perform the phenomenological analysis of $B \to \pi\pi$ channels, with all nonfactorizable parts of the amplitudes calculated from LCSR, except the emission with hard gluons which is estimated using QCDF [1].

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	$a^{dir}_{CP}(B^+ \to \pi^+\pi^0)$	$a^{dir}_{CP}(B^0 \to \pi^+\pi^-)$	$a^{dir}_{CP}(B^0 \to \pi^0\pi^0)$
BaBar	-0.01 ± 0.10	0.09 ± 0.16	0.12 ± 0.56
Belle	0.02 ± 0.08	0.56 ± 0.14	0.44 ± 0.56
Average	0.01 ± 0.06	0.37 ± 0.10	0.28 ± 0.40
This work	0	$-0.04 \pm 0.01 \pm 0.01$	$0.70^{+0.19+0.08}_{-0.29-0.08}$

Table 1: Direct *CP*-asymmetries: current data [8] compared with the LCSR predictions[6]

In particular, we adopt $|V_{ub}| = (4.22 \pm 0.26) \cdot 10^{-3}$ (the errors added in quadrature) and use a representative interval $\gamma = (58.6 \pm 10)^\circ$ from [8]. The results for the decay rates are:

$$BR(B^+ \to \pi^+\pi^0) = \left(6.7^{+1.8+0.9}_{-1.5-0.8}\right) \times 10^{-6}\,,\; BR(B^0 \to \pi^+\pi^-) = \left(9.7^{+2.3+1.2}_{-1.9-1.2}\right) \times 10^{-6}$$
$$BR(B^0 \to \pi^0\pi^0) = \left(0.29^{+0.24+0.07}_{-0.12-0.07}\right) \times 10^{-6}\,, \tag{3.4}$$

where the errors represent the variation of the LCSR parameters and of the CKM factors, respectively. The direct *CP* asymmetries are presented in Table 3. We find that the general picture does not qualitatively deviate from the naive factorization. Our results are also consistent with QCDF, if the divergent annihilation diagrams there are modelled by moderate logarithmic factors. On the other hand,our predictions disagree with the current data for $BR(B^0 \to \pi^+\pi^-)$ and $BR(B^0 \to \pi^0\pi^0)$ [8] and probably also for the direct CP-asymmetry in $B^0 \to \pi^+\pi^-$, indicating a missing $\Delta I = 1/2$ transition amplitude.

An important direction of future studies is the LCSR analysis of the $B \to \pi K$ and $B \to K\bar{K}$ channels including calculable $SU(3)$ violation effects which are expected [9] to be important. Furthermore, one may also use for charmless B decays the alternative LCSR with B meson distribution amplitude recently suggested in [10] in full QCD and in [11] in the SCET framework.

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