

Hadronic mass and q^2 moments in $B \rightarrow X_u l \nu$

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We present OPE predictions for the hadronic mass and q^2 moments in inclusive semileptonic charmless B decays, with a lower cut on the charged lepton energy and an upper cut on the hadronic invariant mass. We include non-perturbative corrections through $O(1/m_b^3)$ and perturbative contributions through $O(\alpha_s^2 \beta_0)$. We also investigate the range of the cut on the hadronic mass for which the local OPE can be considered valid and give estimates of the residual theoretical uncertainty.

The study of these moments is important to constrain the effect of the Weak Annihilation (WA) contributions and the shape of the distribution function, providing a more precise inclusive determination of $|V_{ub}|$.

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The determination of the V_{ub} element of the CKM matrix, together with a sensible estimate of the theoretical uncertainties involved in its calculation, is an important but challenging task. The magnitude of V_{ub} , combined with V_{cb} , determines the length of the left side of the Unitarity Triangle, allowing for a stringent test of the Standard Model [1] and a providing a powerful tool to detect hints of new physics. While V_{cb} , thanks to recent developments, has reached a theoretical uncertainty below 2%, the situation of V_{ub} is much worse, with uncertainties about 10% [2, 3]. Therefore improvements are needed here.

The difficulties related with the determination of the V_{ub} from inclusive semileptonic decays are well known. Inclusive decay rates can be calculated using a double expansion in α_s (parton model) and Λ_{QCD}/m_b (Heavy Quark Expansion). The measurement of $B \rightarrow X_u \ell \bar{\nu}$ requires experimental cuts to suppress the large $B \rightarrow X_c \ell \bar{\nu}$ background. Unfortunately, the introduction of kinematic cuts restricts the phase space available for the $B \rightarrow X_u \ell \bar{\nu}$ decay, spoiling the global properties of the OPE.

Several possibilities have been explored in the literature [4], involving cuts in the hadronic invariant mass (M_X), lepton energy (E_l), leptonic invariant mass (q^2), light-cone component of the hadronic four-momentum (P_+), and combinations of them. Each of these cases requires different techniques and a careful estimates of the uncertainties involved.

In this talk, I will present a calculation of the moments in the hadronic invariant mass of the $B \rightarrow X_u \ell \bar{\nu}$ distribution [5], and try to suggest the reason for its usefulness. Of course, as soon as the measurements of such moments will be available, one should verify their consistency with moments of $B \rightarrow X_c \ell \bar{\nu}$ and $B \rightarrow X_s \gamma$ in the OPE framework. The present combined fit [6] of all moments of inclusive B decays (hadronic mass, lepton energy) shows very good consistency of the available data and allows for the determination of the b quark mass and expectation values of the dominant power suppressed operators. However the main motivations for our analysis comes from recent experimental developments. The high statistics accumulated at the B-factories by BaBar and Belle allows for the measurement, based on fully reconstructed events, of hadronic invariant mass distributions in inclusive $B \rightarrow X_u \ell \bar{\nu}$. This new generation of analysis can discriminate between charmless events and charmed background, even for relatively high values of hadronic invariant mass M_X . For instance, BaBar has been able to measure invariant mass distribution and first two moments with promising accuracy [7]. However, the experimental error is much smaller if one applies a cut on the hadronic mass M_X^{cut} close to the kinematic boundary for charm production. On the other hand, theoretical calculations of inclusive B decays rely on the OPE, whose convergence is spoiled by severe cuts. A possible solution could be to raise M_X^{cut} just enough to suppress non-perturbative effects that cannot be accounted for by the OPE (B meson distribution function effects). I will show a simple example later on, but before let me briefly describe the content of our calculation, and succinctly discuss the theoretical uncertainties involved.

We determine the normalized integer moments of the squared invariant mass $\langle M_X^{2n} \rangle$, and in particular the central moments defined as $U_1 = \langle M_X^2 \rangle$, $U_{2,3} = \langle (M_X^2 - \langle M_X^2 \rangle)^{2,3} \rangle$. The physical hadronic invariant mass is related to parton level quantities by $M_X^2 = \bar{\Lambda}^2 + 2m_b \bar{\Lambda} E_0 + m_b^2 s_0$; therefore we can express the moments of M_X^2 in term of combinations of moments of parton energy E_0 and invariant mass of the parton s_0 . Using the same approach, we can easily obtain also the moments

of the distribution in q^2 . The building blocks that allow to construct both M_X^2 and q^2 moments are:

$$\begin{aligned} \mathcal{M}_{(i,j)} &= \frac{1}{\Gamma_0} \int dE_0 ds_0 dE_\ell s_0^i E_0^j \frac{d^3\Gamma}{dE_0 ds_0 dE_\ell} \\ &= T_{(i,j)} + \frac{\mu_\pi^2}{m_b^2} B_{(i,j)} + \frac{\mu_G^2}{m_b^2} C_{(i,j)} + \frac{\rho_D^3}{m_b^3} D_{(i,j)} + \frac{\rho_{LS}^3}{m_b^3} E_{(i,j)} + \frac{\alpha_s}{\pi} A_{(i,j)}^{(1)} + \frac{\alpha_s^2 \beta_0}{\pi^2} A_{(i,j)}^{(2)} + \dots \end{aligned} \quad (1)$$

The moments of E_0 and s_0 and of their product are obtained in the local OPE and are expressed in terms of the heavy quark parameters. We employ the “kinetic” scheme [8] (Wilsonian scheme with a hard factorization scale $\mu \simeq 1$ GeV). We start from on-shell expressions, and express the on-shell parameters in terms of the μ -dependent “kinetic” parameters: kinetic mass of the b quark $m_b(\mu)$, kinetic expectation value $\mu_\pi^2(\mu)$, and Darwin expectation value $\rho_D^3(\mu)$. At $\mu \rightarrow 0$ one recovers the results of the on-shell scheme.

In the case of charmless decay, we introduce an additional parameter $X_\mu \equiv 8 \ln m_b^2 / \mu_{4q}^2$. It is well known [9, 10] that the coefficient function $D_{(0,0)}$ (see Eq. 1) has a logarithmic divergence as $m_q \rightarrow 0$, i.e. contains a term proportional to $\ln(m_b^2/m_q^2)$. This problem is solved when we include the contribution of the Weak Annihilation (WA) operator. In fact, a one-loop penguin diagram that mixes the four-quark operator into ρ_D^3 replaces $\ln m_b^2/m_q^2$ by $\ln \mu_{4q}^2/m_b^2$, where μ_{4q} is the normalization point of the WA operator. Varying the parameter X_μ allows us to estimate the effects of WA. The associated variation in the results for the moments accounts for the non-valence piece in the expectation value of the WA operator (flavor singlet WA). This contribution does not distinguish between B^+ and B^0 (non-singlet WA effect).

The calculation is implemented in a FORTRAN code¹ and includes non-perturbative corrections $\mathcal{O}(1/m_b^2)$ [11] and $\mathcal{O}(1/m_b^3)$ [9], and perturbative corrections $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2 \beta_0)$ [12]. Partial checks have been performed to reproduce previous results [13].

There are several sources of theoretical uncertainty in our predictions for the moments. Let me briefly list them and describe the methods used to estimate their size. We should consider: (i) Uncalculated $\mathcal{O}(\alpha_s^2)$ and perturbative corrections to the Wilson coefficients; (ii) Missing $\mathcal{O}(1/m_b^4)$ non-perturbative effects; (iii) Error from the scale in X_μ ; (iv) WA contributions; (v) Effects of Fermi Motion.

We estimate the effect of missing higher order corrections (i) and (ii), by varying the parameters μ_π^2 , μ_G^2 , α_s , ρ_D^3 , ρ_{LS}^3 , and m_b in an uncorrelated way. Alternatively, we can estimate the size of missing higher orders comparing the results obtained for the moments using different rearrangements of non-perturbative corrections. As I mentioned previously, the effects of (iii) and (iv) are closely related [5]: varying μ_{4q} over a reasonable interval, we estimate the effect of the flavor singlet WA. Those contributions are concentrated at maximal q^2 (small hadronic invariant mass) and are therefore suppressed in the moments of M_X^2 . Finally, we estimate the effect of Fermi motion (v), introduced by the M_X cut, by smearing the tree-level differential rate with an exponential distribution function. This estimate depends on the functional form adopted, for instance an exponential form leads to more conservative estimates than a Gaussian ansatz. While find relatively small effects for the first moment, the second and the third one are more sensitive. A more complete and satisfactory treatment of these effects is required, and it is currently under investigation.

¹ Available on request from the authors of Ref. [5].

As an example of the possibilities offered by the analysis based on the moments, we present an estimate of the error on the determination of m_b based on U_1 only. We consider a mild $M_X^2 < 5.6 \text{ GeV}^2$ cut, that has the advantage of removing the experimentally most poorly known region of phase-space for charmless decays, reducing the experimental error by almost a factor two, respect to the case without the cut [14]. The experimental uncertainty, considering the statistics that can be accumulated in the near future, can be estimated as $\delta U_1^{\text{exp}} = 11\%$ [14]. On the theoretical side, considering the sources of uncertainty listed above, we obtain $\delta U_1^{\text{th}} = 9\%$ at $M_X^{\text{cut}} = 2.5 \text{ GeV}$ [5]. The combined effect of theoretical and experimental uncertainties, together with the use of the linearized formulas of Ref. [5], roughly leads to $\delta m_b \simeq 80 \text{ MeV}$.

To summarize, we presented predictions for the first three moments of the hadronic invariant mass distribution in $B \rightarrow X_u \ell \bar{\nu}$, that includes corrections through $O(\alpha_s^2 \beta_0)$ and $O(1/m_b^3)$ and cuts on the lepton energy and on the invariant hadronic mass. The calculation is implemented in a FORTRAN code, available on request. The work is in progress: we are currently improving our code, allowing all possible combined cuts (including q^2 and light-cone variables). A lot is still to be done, in order to constrain WA (singlet/non-singlet) and better understand Fermi motion effects. An analysis based on moments with high M_X^{cut} is challenging for experiments, but reduces the theoretical uncertainties related with OPE predictions. The preliminary results are promising.

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