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The $B \to \pi$ form factor from light-cone sum rules in soft-collinear effective theory

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Recently, we have derived light-cone sum rules for exclusive B-meson decays into light energetic hadrons from correlation functions within soft-collinear effective theory [1]. In these sum rules the short-distance scale refers to "hard-collinear" interactions with virtualities of order $\Lambda_{\rm QCD} m_b$. Hard scales (related to virtualities of order m_b^2) are integrated out and enter via external coefficient functions in the sum rule. Soft dynamics is encoded in light-cone distribution amplitudes for the B-meson, which describe both the factorizable and non-factorizable contributions to exclusive B-meson decay amplitudes. Factorization of the correlation function has been verified to one-loop accuracy. Thus, a systematic separation of hard, hard-collinear, and soft dynamics in the heavy-quark limit is possible.

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The QCD factorization theorems for exclusive, energetic B-decays, first proposed in [2], identify short-distance effects that can be systematically calculated in perturbation theory. Non-perturbative effects are parametrized in terms of a few universal functions such as form factors and light-cone distribution amplitudes, on which our information is rather restricted. Moreover, phenomenological applications are limited by the insufficient information on (power-suppressed) non-factorizable terms. These restrictions should be tackled by the large data sets available by current and future B- physics experiments on the experimental side, and by the development of suitable non-perturbative methods on the theory side.

The quantum field theoretical framework corresponding to the QCD factorization theorems is the soft collinear effective theory (SCET) [3, 4]. In contrast to the well-known heavy-quark effective theory (HQET), the recently proposed SCET does not correspond to a local operator expansion. While HQET is only applicable to B decays, when the energy transfer to light hadrons is small, for example to $B \to D$ transitions at small recoil to the D meson, it is not applicable, when some of the outgoing, light particles have momenta of order m_b; then one faces a multiscale problem: a) $\Lambda = \text{few} \times \Lambda_{\text{QCD}}$, the *soft* scale set by the typical energies and momenta of the light degrees of freedom in the hadronic bound states; b) m_b, the *hard* scale set by the heavy-b-quark mass; c) the hard-collinear scale $\mu_{\text{hc}} = \sqrt{m_b \Lambda}$ appears via interactions between soft and energetic modes in the initial and final states. The dynamics of hard and hard-collinear modes can be described perturbatively in the heavy-quark limit $m_b \to \infty$. The separation of the two perturbative scales from the non-perturbative hadronic dynamics is determined by the small expansion parameter $\lambda = \sqrt{\Lambda/m_b}$. On a technical level the implementation of power counting in λ for fields and operators in SCET corresponds directly to the well-known method of regions for Feynman diagrams [5].

A simple example where the above considerations apply is given by the $B \to \pi$ transitions form factor. In the large recoil-energy limit the heavy-to-light form factors obey relations [6] that are broken by radiative and power corrections. Each form factor can be decomposed into two basic contributions, one piece that factorizes into a perturbatively calculable coefficient function T_i and light-cone distribution amplitudes ϕ_B and ϕ_π for heavy and light mesons, respectively, and a second contribution, where the hard-collinear interactions are *not* factorizable, leaving one universal “soft” form factor ξ_π [7]:

$$\langle \pi | \bar{\psi} \Gamma_i \, b | B \rangle = C_i(E, \mu_I)\, \xi_\pi(\mu_I, E) + T_i(E, u, \omega, \mu_{\text{II}}) \otimes \phi_+^B(\omega, \mu_{\text{II}}) \otimes \phi_\pi(u, \mu_{\text{II}}) + \text{subleading terms}\,.$$

Here C_i is a short-distance function arising from integrating out hard modes, and T_i contains hard and hard-collinear dynamics related to spectator scattering. Consequently μ_I is a factorization scale below m_b, while μ_{II} is a factorization scale below μ_{hc}. Both functions can be computed as perturbative series in α_s (the effective theories for the two short-distance regimes are known as SCET_{I} and SCET_{II}). We have shown in [1] that light-cone sum rules can be formulated for the *soft* (i.e. non-factorizable) part of the form factor *within* SCET_{I}. In the following we briefly discuss the basic idea using the tree-level construction in an examplary mode. For the derivation of all further non-trivial results, the phenomenological application, and also for the general notation used in this short letter, we guide the reader to the original publication [1].

Within SCET_{I} the non-factorizable (i.e. end-point-sensitive) part of the $B \to \pi$ form factor in the heavy-quark limit is described by the current operator

$$J_0(0) = \bar{\xi}_{\rm hc}(0) W_{\rm hc}(0) Y_s^\dagger(0) h_v(0)\,, \qquad \langle \pi(p')|\, J_0(0)\,|B(m_B v)\rangle = (n_+ p')\, \xi_\pi(n_+ p', \mu_{\rm I})\,, \tag{1}$$

where $\xi_{\rm hc}$ is the "good" light-cone component of the light-quark spinor with $\not{n}_- \xi_{\rm hc} = 0$, and $W_{\rm hc}$ and $Y_s^\dagger$ are hard-collinear and soft Wilson lines. Finally, h_v is the usual HQET field. The heavy quark is nearly on-shell in the end-point region. In SCET$_{\rm I}$, this is reflected by the fact that hard sub-processes (virtualities of order m_b^2) are already integrated out and appear in coefficient functions multiplying J_0, which can be determined from the matching of the corresponding QCD current on SCET$_{\rm I}$. Therefore we will *not* introduce an interpolating current for the B meson as in the usual light-cone sum-rule approach. Instead, the short-distance (off-shell) modes in SCET$_{\rm I}$ are the hard-collinear quark and gluon fields, and therefore the sum rules should be derived from a dispersive analysis of the correlation function

$$\Pi(p') = i \int d^4x\, e^{ip'x}\, \langle 0|T[J_\pi(x) J_0(0)]|B(p_B)\rangle\,, \tag{2}$$

where $p_B^\mu = m_B v^\mu$, and the interpolating current J_π for a pion in the effective theory is chosen as

$$J_\pi(x) = -i\,\bar{\xi}_{\rm hc}(x)\,\not{n}_+ \gamma_5\, \xi_{\rm hc}(x) - i\,\left(\bar{\xi}_{\rm hc} W_{\rm hc}(x) \not{n}_+ \gamma_5 Y_s^\dagger q_s(x) + h.c.\right)\,. \tag{3}$$

with $\langle 0|J_\pi|\pi(p')\rangle = (n_+ p')\, f_\pi$. Here we denoted soft and hard-collinear quark fields in SCET as q_s and $\xi_{\rm hc}$, respectively. Notice that soft-collinear interactions require a multipole expansion of soft fields, which is always understood implicitly. We also point out that the effective theory SCET$_{\rm I}$ contains SCET$_{\rm II}$ as its infrared limit (i.e. when the virtuality of the hard-collinear modes is lowered to order Λ^2). For this reason, the hard-collinear fields that define the interpolating current J_π also contain the collinear configurations that show up as hadronic bound states (see also the discussion in [8]).

In the following we will consider a reference frame where $p'_\perp = v_\perp = 0$ and $n_+ v = n_- v = 1$. In this frame the two independent kinematic variables are $(n_+ p') \simeq 2E_\pi = \mathcal{O}(m_b)$ and $0 > (n_- p') = \mathcal{O}(\Lambda)$ with $|n_- p'| \gg m_\pi^2/(n_+ p')$. The dispersive analysis will be performed with respect to $(n_- p')$ for fixed values of $(n_+ p')$. As with all QCD sum-rule calculations, the procedure consists in writing the correlator in two different ways. On the hadronic side, one can write

$$\Pi^{\rm HAD}(n_- p') = \Pi(n_- p')\Big|_{\rm res.} + \Pi(n_- p')\Big|_{\rm cont.}\,, \tag{4}$$

where the first term represents the contribution of the pion, while the second takes into account the role of higher states and continuum above an effective threshold $\omega_s = \mathcal{O}(\Lambda^2/n_+ p')$. The former can be rewritten as

$$\Pi(n_- p')\Big|_{\rm res.} = \frac{\langle 0|J_\pi|\pi(p')\rangle\langle \pi(p')|J_0|B(p_B)\rangle}{m_\pi^2 - p'^2} = \frac{(n_+ p')^2\, \xi_\pi(n_+ p')\, f_\pi}{m_\pi^2 - p'^2}\,. \tag{5}$$

On the SCET side of the sum rule, the tree-level result (see Fig. 1(a)), for the correlation function involves one hard-collinear quark propagator, which reads

$$S_F^{\rm hc} = \frac{i}{n_- p' - \omega + i\eta}\, \frac{\not{n}_-}{2}\,, \tag{6}$$

where $\omega = n_- k$, and k^μ is the momentum of the soft light quark that will end up as the spectator quark in the B meson. The propagator is always off shell and always induces light-like separations as long as $|n_- p'| \sim \Lambda$. This leads to matrix elements of operators that are formulated only in terms of soft fields, which are separated along the light cone and thus define light-cone distribution amplitudes for the B meson in HQET [9]. The final result already has the form of a dispersion integral in the variable $n_- p'$:

$$\Pi(n_- p') = f_B m_B \int_0^\infty d\omega \frac{\phi_-^B(\omega)}{\omega - n_- p' - i\eta}, \tag{7}$$

where the B light-cone distribution amplitude enters through

$$\langle 0|\bar{q}(x_-)\, Y_s \gamma_5 \frac{\not n_+ \not n_-}{2} Y_s^+ h_v(0)|B(p_B)\rangle = i f_B m_B \int d\omega'\, e^{-i\omega' \frac{n_+ x}{2}} \phi_-^B(\omega'). \tag{8}$$

The result for the SCET side of the sum rule shows that the considered correlation function in the (unphysical) Euclidean region factorizes into a perturbatively calculable hard-collinear kernel and a soft light-cone wave function for the B meson, where the convolution variable ω represents the light-cone momentum of the spectator quark in the B meson.

Finally, $\Pi(n_- p')\Big|_{\text{cont.}}$ on the hadronic side can be written again according to a dispersion relation. Moreover, assuming global quark–hadron duality, we identify the spectral density with its perturbative expression above some threshold ω_s. The Borel transform with respect to the variable $n_- p'$ introduces the Borel parameter ω_M and reads in this case $\hat{B}(\omega_M)[1/(\omega - n_- p')] = 1/\omega_M\, e^{-\omega/\omega_M}$. As usual, the physical role of the Borel parameter is to enhance the contribution of the hadronic-resonance region, where the virtualities of internal propagators have to be smaller than the hard-collinear scale. Equating the two representations of the correlator, using global quark–hadron duality (and neglecting the pion mass), we obtain the final sum rule for the soft form factor at tree level (see also [10]):

$$\xi_\pi(n_+ p') = \frac{f_B m_B}{f_\pi (n_+ p')} \int_0^{\omega_s} d\omega\, e^{-\omega/\omega_M} \phi_-^B(\omega). \tag{9}$$

The result for ξ_π has the correct scaling with Λ/m_B as obtained from the conventional sum rules or from the power counting in SCET. The resulting estimate for the soft form factor at maximum recoil, $0.27^{+0.09}_{-0.11}$, is compatible with other determinations. The uncertainty is dominated by the variation of the sum-rule parameters and by the product $f_B \phi_-^B$.

In [1], we have also reproduced the result for the factorizable form-factor contribution from hard-collinear spectator scattering [7]. For this purpose we have to consider the correlation function

$$\Pi_1(p') = i \int d^4x\, e^{ip'x} \langle 0|T[J_\pi(x) J_1(0)]|B(p_B)\rangle, \quad J_1 \equiv \bar{\xi}_{\text{hc}}\, g \not A_{\text{hc}}^\perp h_v, \tag{10}$$

where we used the light-cone gauge. The leading contribution is given by the diagram in Fig. 1(b) which involves the insertion of one interaction vertex from the order-λ soft-collinear Lagrangian. We derived the remarkable feature of the SCET-sum-rule approach to the $B \to \pi$ form factor that the ratio of factorizable and non-factorizable contributions is independent of the B-meson wave function to first approximation, and about 6%, which is in line with the power counting used in

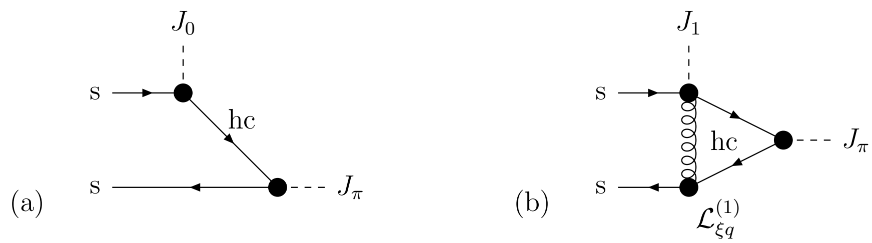

Figure 1: (a) Leading contribution to the correlation function for the non-factorizable SCET current J_0. (b) Leading contribution to the sum rule for the factorizable SCET current J_1.

QCD factorization [2], but contradicts the assumptions of the so-called pQCD approach [11]. The non-trivial issues related to the factorization of the correlation function arise beyond tree level. We have shown that the $O(\alpha_s)$ short-distance radiative corrections from hard-collinear loops preserve factorization, i.e. the encountered IR divergences correspond to the renormalization of the B-meson distribution amplitude(s) in HQET (three-particle Fock states have been neglected so far). At this point our approach differs from the conventional sum rules formulated in QCD, where the separation of three different scales (m_b, $\mu_{\rm hc}$, Λ) is usually not attempted. However, in a recent article [14] Lee proposes to formulate sum rules within the effective field theory framework of SCET for the conventional set-up where the pion is represented by a collinear light-cone distribution amplitude and the B meson is interpolated by a current in HQET. We stress that in that case the radiative corrections would involve soft *and* collinear momentum regions; according to the general discussion in [8, 12, 13], these are not expected to factorize. Indeed the occurrence of end-point singularities that spoil factorization has been observed in [14]. Our choice of interpolating the pion instead avoids the problem of collinear end-point divergences. In this way the SCET-based sum rules generalize the ideas of QCD factorization to the non-factorizable parts of exclusive decay amplitudes, and allow for a systematic and consistent expansion in terms of $1/m_b$ and α_s, where the hadronic information is encoded in light-cone distribution amplitudes for the B meson and non-perturbative sum-rule parameters. At present, because of the limited information on the B-meson wave function, the theoretical uncertainties are larger than those found in QCD light-cone sum rules.

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