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Relativistic description of heavy baryons*

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The masses of the ground state heavy baryons consisting of two light (u,d,s) and one heavy (c,b) quarks are calculated in the heavy-quark–light-diquark approximation within the constituent quark model. The light quarks, forming the diquark, and the light diquark in the baryon are treated completely relativistically. The expansion in v/c up to the second order is used only for the heavy (b and c) quarks. The diquark-gluon interaction is taken modified by the form factor describing the light diquark structure in terms of the diquark wave functions. An overall reasonable agreement of the obtained predictions with available experimental data and previous theoretical results is found.

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The description of baryons within the constituent quark model is a very important problem in quantum chromodynamics. Since the baryon is a three-body system, its theory is much more complicated compared to the two-body meson system. The quark-diquark picture of a baryon is the popular approximation widely used to describe the baryon properties [1]. Here we apply this picture for studying the ground states of baryons with one heavy quark. It is assumed that the heavy-quark–light-diquark configuration dominates. This approximation cannot be rigorously deduced since the heavy quark influences the interaction of light quarks. We assume that the influence on the diquark dynamics is small. This assumption could be justified by the presumed universal nature of the diquark [1]. Otherwise the diquark properties would be very different in such hadronic systems as baryons, tetraquarks, pentaquarks, etc.

We use the relativistic quark model (RQM) based on the quasipotential approach for the calculations of heavy baryon masses in the quark-diquark picture (for details see [2]). The quasipotentials of the interaction of two light quarks in a diquark and the heavy quark interaction with a light diquark in a baryon were constructed following the similar procedure for mesons and doubly heavy baryons [3]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the qq and $q\bar{q}$ interaction. An important role in this construction is played by the Lorentz-structure of the confining interaction. In our previous analysis of mesons [3], we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. We also keep fixed all parameters of the quark interaction potential and quark masses which were found from studying meson spectra and decays.

At a first step, we calculate the masses and form factors of the light diquark. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment is required. To achieve this goal in describing light diquarks, we closely follow our recent consideration of the spectra of light mesons [4] and adopt the same procedure to make the relativistic quark potential local. The quasipotential equation is solved numerically for the complete relativistic potential which depends on the diquark mass in a complicated highly nonlinear way. The obtained ground state masses of scalar and axial vector light diquarks are presented in Table 1. These masses are in good agreement with values found within the Nambu–Jona-Lasinio (NJL) model [5], Bethe-Salpeter equation (BSE) with different approximations for the kernel [6, 7] and lattice calculations [8]. We also calculate the matrix element of the quark current between diquark states using the obtained diquark wave functions. As a result we find the corresponding form factor which determines the diquark interaction with the gluon field and thus takes into account the internal diquark structure.

At a second step, we calculate the masses of heavy baryons as the bound state of a heavy quark and light diquark. For the potential of the heavy-quark–light diquark interaction we use the expansion in p/m_Q. Since the light diquark is not heavy enough for the applicability of a p/M_d expansion, it should be treated fully relativistically. To achieve this goal and simplify the potential we follow the same procedure, which was used for light quarks in a diquark, and replace the diquark energies $E_d(p) \equiv \sqrt{\mathbf{p}^2 + M_d^2}$ by $E_d \equiv (M^2 - m_Q^2 + M_d^2)/(2M)$. This substitution makes the Fourier

Quark content	Diquark type	Mass				
		our [2] RQM	[5] NJL	[6] BSE	[7] BSE	[8] Lattice
$[u,d]$	S	710	705	737	820	694(22)
$\{u,d\}$	A	909	875	949	1020	806(50)
$[u,s]$	S	948	895	882	1100	
$\{u,s\}$	A	1069	1050	1050	1300	
$\{s,s\}$	A	1203	1215	1130	1440	

Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q,q']$ and symmetric $\{q,q'\}$ in flavour, respectively.

Baryon	$I(J^P)$	Theory						Experiment
		our [2]	[9]	[10]	[11]	[12]	[13]	PDG [14]
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2284.9(6)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2451.3(7)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2515.9(2.4)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2466.3(1.4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2574.1(3.3)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2647.4(2.0)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5624(9)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5820		5824.2(9.0)	5847	
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

Table 2: Masses of the ground state heavy baryons (in MeV).

transform of the potential local. At leading order in p/m_Q the resulting potential for the S-wave states is the same for scalar and axial vector diquarks. The masses of baryons with spin 1/2 and 3/2, containing the axial vector diquark, are degenerate in this approximation since the spin-spin interaction arises only at first order in p/m_Q. The p/m_Q corrections break down this leading order degeneracy of heavy baryon states. We calculate the masses of the ground state heavy baryons with the account of all corrections of order p^2/m_Q^2 by solving numerically the corresponding quasipotential equation. The obtained values of the baryon masses are given in Table 2 in comparison with some theoretical predictions [9, 10, 11, 12, 13] and experimental data [14].

The heavy-quark symmetry ($1/m_Q$ expansion) and the $SU(3)$ flavour symmetry are applied in Refs. [11, 15, 12] to evaluate the masses of baryons with a single heavy quark. At lowest order in

$SU(3)$ breaking these masses obey the equal-spacing rule:

$$J=\frac{1}{2},\quad M_{\Sigma_Q}+M_{\Omega_Q}=2M_{\Xi'_Q};\qquad J=\frac{3}{2},\quad M_{\Sigma^*_Q}+M_{\Omega^*_Q}=2M_{\Xi^*_Q},\quad Q=b,c. \tag{1}$$

The corrections to this rule, estimated on the basis of chiral perturbation theory (light meson loops) combined with heavy-quark symmetry, are found to be small [11]. The equal-spacing rule holds also for the hyperfine mass splittings [11]:

$$\delta_{\Sigma_Q}+\delta_{\Omega_Q}=2\delta_{\Xi_Q};\qquad \delta_{\Sigma_Q}=M_{\Sigma^*_Q}-M_{\Sigma_Q};\quad \delta_{\Xi_Q}=M_{\Xi^*_Q}-M_{\Xi'_Q};\quad \delta_{\Omega_Q}=M_{\Omega^*_Q}-M_{\Omega_Q}. \tag{2}$$

The accuracy of the relation (2) is estimated [12] to be of order 1 MeV for $Q=c$ and 0.3 MeV for $Q=b$. Our values of baryon masses satisfy rather well both the mass relations (1) and (2). This supplies a strong additional support to our model, since it means that the model incorporates the important features of broken $SU(3)$ flavour symmetry and heavy quark expansion of QCD in a reasonable way.

It is important to emphasize that, in calculating the heavy baryon masses, we do not use any free adjustable parameters. The light diquark in our approach is not considered as a point-like object. Instead we use its wave functions to calculate diquark-gluon form factors and, thus, take into account the finite (and relatively large) size of the light diquark. The light quarks in the diquark and the light diquark in the heavy baryon are treated completely relativistically. We use the v/c expansion only for heavy (b and c) quarks. The overall reasonable agreement of our model predictions given in Table 2 with both available experimental data and the results of significantly distinct theoretical approaches gives further grounds for the heavy-quark–light-diquark picture of heavy baryons.

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