The Unitarity Triangle through $B_d \to \rho^{\pm}\pi^{\mp}$ decays

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We analyze the impact of the CP-violating observables in the $B_d \to \rho^{\pm}\pi^{\mp}$ system, combined with the precise measurement of $\sin 2\beta$, in the extraction of the CKM matrix. We explore two strategies for determining the Unitarity Triangle in these modes. Computing the penguin parameters $(r_{\pm}, \phi_{\pm})$ and the ratio of two trees (r_t, ϕ_t) within QCD factorization yields a precise determination of $(\bar{\rho}, \bar{\eta})$, reflected by a weak dependence on $\phi_{\pm}$, which is shown to be a second order effect, as in the $B_d \to \pi^+\pi^-$ system. Moreover, we find that the dependence on penguin amplitudes $r_{\pm}$ in $B_d \to \rho^{\pm}\pi^{\mp}$ is less pronounced than in the $B_d \to \pi^+\pi^-$ case, since penguin contributions $r_{\pm} \approx r_{\pi\pi}/3$, implying an important simplification in our analysis. Independent experimental tests of the factorization framework are proposed and discussed, using $B_d \to \pi^+\pi^-$ and $B_d \to \rho^{\pm}\rho^{\mp}$ modes.

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One of the most relevant challenges for the *B*-factories is the determination of the three angles of the Unitarity Triangle (UT) of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. To date, besides the precisely measured angle β, from the "gold-plated" mode $B_d \to J/\psi K_S$, the extraction of the two remaining angles, namely α and γ, is obscured by our lack of knowledge about the hadronic dynamics inside mesons. Although their extraction is mainly limited theoretically by the so-called penguin pollution, CP violation in the charmless *B* decays, such as $B_d \to \pi\pi, \pi\rho$ and similar modes, could be of great help in their extractions.

In this work, we propose a transparent analysis of exploring the UT through the CP violation in $B_d \to \rho^{\mp}\pi^{\pm}$, combined with the "gold-plated" mode $B_d \to J/\psi K_S$. Contrary to the $B_d \to \pi^-\pi^+$ mode $B_d \to \rho^{\mp}\pi^{\pm}$ exhibits two transition amplitudes, namely $A_{\pm} \equiv A(B^0 \to \pi^{\pm}\rho^{\mp}) = T_{\pm}\left(e^{i\gamma} + \frac{P_{\pm}}{T_{\pm}}\right)$, where $T_{\pm}$ and $P_{\pm}$ are respectively the corresponding tree and penguin amplitude. Since the determination of the penguin-to-tree amplitude is relevant for our analysis, we use QCD factorization (QCDF) [2] for their estimate.

The time-dependent decay rates for $B_d \to \rho^{\pm}\pi^{\mp}$ decays are defined by 6 Observables C, ΔC, S, ΔS, $\Gamma^{\rho\pi}$ and $\mathscr{A}^{\rho\pi}$, insufficient input to predict model independently the 8 theoretical parameters, namely 7 hadronic parameters $|T_{\pm}|, |P_{\pm}|, \phi_{\pm}, \phi_t$ and one weak phase γ related to these modes. To disentangle the CKM dependence from the hadronic parameters, we write the penguin-to-tree ratio $(P/T)_{\pm} = \pm r_{\pm}e^{i\phi_{\pm}}/\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$, where $(r_{\pm}, \phi_{\pm})$ are pure strong interaction quantities and $(\bar{\rho}, \bar{\eta})$ are the perturbatively improved Wolfenstein parameters [3]. Neglecting the very small effects from electroweak penguin contributions in our processes, one can express the penguin parameters $r_+ e^{i\phi_+}$ and $r_- e^{i\phi_-}$, respectively, in the form [2,4,5]

$$\frac{-(a_{V,4}^c + r_A[b_{P,3} + b_{P,4} + b_{V,4}])}{a_{V,1} + a_{V,4}^u + r_A[b_{V,1} + b_{P,3} + b_{P,4} + b_{V,4}]}, \quad \frac{-(a_{P,4}^c - r_{\chi}^{\pi}a_{P,6}^c + k_A[b_{V,3} + b_{V,4} + b_{P,4}])}{a_{P,1} + a_{P,4}^u - r_{\chi}^{\pi}a_{P,6}^u + k_A[b_{P,1} + b_{V,3} + b_{V,4} + b_{P,4}]}. \quad (1)$$

A recent analysis gives [6]:

$$\begin{aligned}
r_+ &= 0.04 \pm 0.01, & \phi_+ &= 0.18 \pm 0.27, \\
r_- &= 0.03 \pm 0.02, & \phi_- &= -0.02 \pm 0.42, \\
r_t &= 0.89 \pm 0.22, & \phi_t &= 0.02 \pm 0.03,
\end{aligned} \quad (2)$$

where the error includes an estimate of potentially important power corrections. In order to obtain additional insight into the structure of hadronic *B*-decay amplitudes, it will be also interesting to extract these quantities from other *B*-channels, via $SU(3)$-symmetry [7], or using other methods.

Since the parameters $r_{\pm}$ and ϕ_t are small quantities, ("*CP − violating*" S, "*CP − conserving*" ΔS) and their corresponding rescaled quantities ("*CP − violating*" $\bar{S}$, "*CP − conserving*" $\Delta\bar{S}$) [7] are respectively well approximated, at the lowest order in $r_{\pm}$, by [4,7]:

$$\begin{aligned}
S &\doteq \text{fct}_1(r_{\pm}, \phi_{\pm}, r_t, \tau, \bar{\eta}), & \Delta S &\doteq \text{fct}_2(r_{\pm}, \phi_{\pm}, r_t, \phi_t, \tau, \bar{\eta}), \\
\bar{S} &\doteq \bar{\text{fct}}_3(r_{\pm}, \phi_{\pm}, \tau, \bar{\eta}), & \Delta\bar{S} &\doteq \bar{\text{fct}}_4(r_{\pm}, \phi_{\pm}, \phi_t, \tau, \bar{\eta}),
\end{aligned} \quad (3)$$

where the observable $\tau \equiv \cot\beta$ has been introduced to relate the parameter $\bar{\rho}$ to $\bar{\eta}$, namely $\bar{\rho} = 1 - \tau\bar{\eta}$, to simplify our analysis in terms of only one CKM parameter. Thus, assuming that the parameter τ (or $\sin 2\beta$) is known one could extract precisely $\bar{\eta}$, as we will see below. Taking
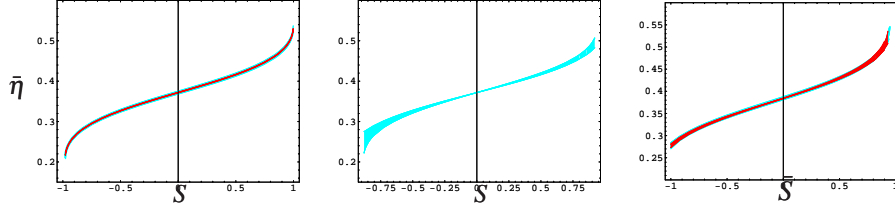
Figure 1: *CKM phase $\bar{\eta}$ as a function of the mixing-induced CP asymmetry S or $\bar{S}$ in $B_d \to \rho^{\pm}\pi^{\mp}$ within the SM for $\sin 2\beta = 0.739$. The dark (light) band reflects the theoretical uncertainty in the penguin phases $\phi_{\pm}$ (penguin amplitude $r_{\pm}$) in the left and right plots, however in the middle one the band reflects the theoretical uncertainty in the tree ratio r_t.*

$\tau = 2.26 \pm 0.22$, $\bar{\eta} = 0.35 \pm 0.04$ [8] and our penguin parameters results in (2), we find from (3) that

$$S = -0.26 \; {}^{+0.28}_{-0.19}(\tau) \; {}^{+0.47}_{-0.39}(\bar{\eta}) \; \underbrace{{}^{-0.02[+0.01]}_{+0.02[-0.001]}}_{(r_+[\phi_+])} \; \underbrace{{}^{+0.04[-0.01]}_{-0.04[-0.005]}}_{(r_-[\phi_-])} \; {}^{-0.005}_{+0.02}(r_t), \tag{4}$$

$$\bar{S} = -0.42 \; {}^{+0.27}_{-0.18}(\tau) \; {}^{+0.49}_{-0.39}(\bar{\eta}) \; \underbrace{{}^{-0.04[+0.02]}_{+0.04[-0.01]}}_{(r_+[\phi_+])} \; \underbrace{{}^{-0.01[-0.005]}_{+0.01[+0.01]}}_{(r_-[\phi_-])} . \tag{5}$$

We note that the sensitivity of τ or $\sin 2\beta$ in extracting S is significant, however the dominant uncertainty is related to the $\bar{\eta}$ which for the purpose of predicting S has been borrowed from a standard CKM fit [8]. Distinctly, the large sensitivity of S to $\bar{\eta}$ is analogous to the fact that in turn $\bar{\eta}$ depends weakly on S. Concerning $\bar{S}$, which is free from r_t per definition, the impact of $\bar{\eta}$ and τ (or $\sin 2\beta$) are significant, however less than in the case of S. In Fig. 1, we have plotted the CKM parameter $\bar{\eta}$ as function of S, showing the $(r_{\pm}, \phi_{\pm})$ (left-plot) and the r_t (middle-plot) uncertainties. We find that the sensitivity of $\bar{\eta}$ on the strong phase $\phi_{\pm}$ is rather mild compared to the penguin amplitude $r_{\pm}$. This is not surprising, since the dependence on $\phi_{\pm}$ enters in $\bar{\eta}_S = G(r_{\pm}, \phi_{\pm}, r_t, \tau, S)$ only at second order. As a cross check of our strategy one can extract $\bar{\eta}$, independently of r_t, as function of $\bar{S}$, $\bar{\eta}_{\bar{S}} = K(r_{\pm}, \phi_{\pm}, \tau, \bar{S})$. In this case, however the sensitivity of $\bar{\eta}_{\bar{S}}$ on the strong phase $\phi_{\pm}$ is more pronounced than in $\bar{\eta}_S$, as shown in Fig. 1 (right-plot). This is traced back to the fact that the strong phase $\phi_{\pm}$ in $\bar{\eta}_{\bar{S}}$, contrary to $\bar{\eta}_S$, enters at first order.

Moreover, the $B_d \to \rho^{\pm}\pi^{\mp}$ decays offer the possibility to explore the two individual direct CP asymmetries between $B^0(\overline{B}^0) \to \rho^+\pi^-$ and $\overline{B}^0(B^0) \to \rho^-\pi^+$ decay rates, namely $C_{\pm}$, For any given values of $r_{\pm}$ and $\phi_{\pm}$ a measurement of $C_{\pm}$ defines a curve in the $(\bar{\rho}, \bar{\eta})$-plane, as sketched in Fig. 2 where a model-independent correlation, within the SM, between the penguin parameters $r_{\pm}$ and $\phi_{\pm}$ for different values for $C_{\pm}$ is shown. In the determination of $\bar{\eta}$ and $\bar{\rho}$ described here discrete ambiguities do in principle arise. However, they can be excluded by other information on the UT (for further details on these ambiguities, see [10]).

In [9,10], it has been shown that the sensitivity of $\bar{\eta}$, in $B_d \to \pi^+\pi^-$ modes, on the strong phase $\phi_{\pi\pi}$ is rather mild, since its dependence enters in $\bar{\eta}$ only at second order. Hence, using the lowest order result in $\phi_{\pi\pi}$ is most likely a very good approximation to the exact result (see [9,10]). In an analogous manner, the same argument holds for $B_d \to \rho^+\rho^-$ channels. The corresponding equations are similar to those defined for $B_d \to \pi^+\pi^-$ $\big($see eqs. (36) and (37) in [10]$\big)$. Therefore,
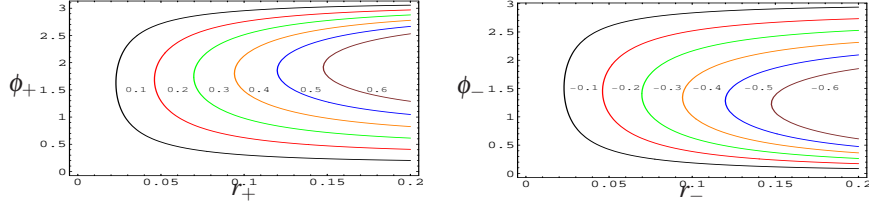
Figure 2: *Contours of constant $C_{\pm}$ in the ($r_{\pm}$, $\phi_{\pm}$)-plane for fixed $\bar{\rho} = 0.20$ and $\bar{\eta} = 0.35$.*

it is interesting to define the ratio [6]:

$$\underbrace{\frac{S_{\rho\rho}(1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2})}{S_{\pi\pi}(1 + \tau S_{\rho\rho} - \sqrt{1 - S_{\rho\rho}^2})}}_{\mathscr{R}^{\mathrm{exp}}} = \underbrace{\frac{1 + r_{\rho\rho}\cos\phi_{\rho\rho}}{1 + r_{\pi\pi}\cos\phi_{\pi\pi}}}_{\mathscr{R}^{\mathrm{the}}}. \tag{6}$$

Note that the lhs of (6), namely $\mathscr{R}^{\mathrm{exp}}$, depends only on experimental observables, and is therefore a measurable quantity, to be confronted with the theoretical informations encoded in the rhs of (6) $\mathscr{R}^{\mathrm{the}}$. Since $r_{\pi\pi} \sim r_{\rho\rho} \sim 0.1$ are small, we expand the ratio $\mathscr{R}^{\mathrm{the}}$ in these two parameters to get to lowest order $\mathscr{R}^{\mathrm{the}} \doteq 1 + \mathrm{Re}[r_{\rho\rho}\, e^{i\phi_{\rho\rho}} - r_{\pi\pi}\, e^{i\phi_{\pi\pi}}]$, where in QCDF $r_{\rho\rho}\, e^{i\phi_{\rho\rho}} - r_{\pi\pi}\, e^{i\phi_{\pi\pi}} \approx \frac{r_{\chi}^{\pi} a_6^c}{a_1}$ is very sensitive to the chirally enhanced terms r_{χ}^{π}. Using QCDF, we find $\frac{r_{\chi}^{\pi} a_6^c}{a_1} = -0.06 \pm 0.014$, leading to $\mathscr{R}^{\mathrm{the}}_{\mathrm{QCDF}} = 0.94 \pm 0.014$ [6]. On the other hand, using the most recent experimental data reported either by BaBar or Belle concerning $(S_{\pi\pi}, S_{\rho\rho}, \tau)$ [11], we find $\mathscr{R}^{\mathrm{exp}}_{\mathrm{BaBar}} = 1.01 \pm 0.11$ and $\mathscr{R}^{\mathrm{exp}}_{\mathrm{Belle}} = 0.81 \pm 0.11$. Although these two values suffer from large uncertainties, their central values agree quite remarkably with the QCDF one given above.

We proposed strategies to extract information on weak phases from CP violation observables in $B_d \to \rho^{\mp}\pi^{\pm}$ decays even in the presence of hadronic contributions related to penguin amplitudes. Assuming knowledge of the penguin pollution, an efficient use of mixing-induced CP violation in $B_d \to \rho^{\mp}\pi^{\pm}$ decays, can be made by combining it with the corresponding observable from $B_d \to J/\psi K_S$, $\sin 2\beta$, to obtain the UT parameters $(\bar{\rho}, \bar{\eta})$. The sensitivity on the hadronic quantities is discussed. In particular, there are no first-order corrections in $\phi_{\pm}$. Moreover, we found that the dependence on $r_{\pm}$ in $B_d \to \rho^{\pm}\pi^{\mp}$ is less pronounced than in the $B_d \to \pi^+\pi^-$ case, since $r_{\pm} \approx r_{\pi\pi}/3$, leading to an important simplification in our analysis. Finally, a new experimental test of the factorization ansatz is presented using $B_d \to \pi^+\pi^-$ and $B_d \to \rho^{\pm}\rho^{\mp}$ modes.

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