

## Final state interactions and long-distance effects in $B \rightarrow \pi\pi K$ and $B \rightarrow K\bar{K}K$ decays \*

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Charged and neutral  $B$  decays into the  $\pi^+\pi^-K$ ,  $K^+K^-K$  and  $K_S^0K_S^0K_S^0$  systems have been studied. The  $\pi^+\pi^-$  and  $K^+K^-$  final state interactions in the  $S$ -wave isoscalar state are treated in an unitary way. The long-distance contributions called charming penguins are introduced in the model. Effective mass distributions, branching ratios and some asymmetries are successfully compared with the recent BaBar and Belle data. A particularly large negative direct CP-violating asymmetry in the charged  $B$  decays into  $f_0(980)K$  is evaluated for one set of the charming penguin amplitudes.

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## 1. Introduction

A good description of final state interactions is crucial to obtain a precise determination of the Cabibbo-Kobayashi-Maskawa matrix elements. Recent experimental results on the  $B$  decays into three pseudoscalar mesons are a rich source of information about weak decay amplitudes and two body mesonic strong interactions. Direct CP-violation effects can appear in these reactions. Here we study the  $B$  decays into  $\pi\pi K$  and  $K\bar{K}K$  channels. An important part of these processes is a production of the  $\pi\pi$  or  $K\bar{K}$  pairs in the isospin zero  $S$ -wave. A very prominent maximum of the  $f_0(980)$  resonance has been seen in the  $\pi\pi$  spectra obtained by the BaBar and Belle collaborations (see, for example refs.[1-4]). One can expect that the other scalar resonance  $f_0(600)$  plays some role at lower  $\pi\pi$  effective masses. Both resonances are incorporated in a natural way in the unitary three-channel model of pion-pion, kaon-kaon and four-pion interactions [5]. We use this model to calculate the final state interactions in the different  $B$ -decay channels.

The penguin diagrams are essential in studies of  $B$  decays into  $\pi\pi K$  and  $K\bar{K}K$ . However, different penguin amplitudes interfere destructively in the decay  $B \rightarrow f_0(980)K$ . This leads to much too small branching ratios calculated in the factorization approach. Thus we consider the long-distance contributions originating from enhanced charm quark loops. These so-called charming penguin terms correspond, for example, to weak decays of  $B$  to intermediate  $D_s^{(*)}D^{(*)}$  states followed by transitions to the  $f_0(980)K$  final states via  $c\bar{c}$  annihilations. The charming penguins have been used in fits of experimental branching fractions for two-body charmless  $B$  decays [6-7]. Below we show that these long-distance contributions are also needed in analyses of  $B$  decays into three pseudoscalar mesons  $\pi\pi K$  or  $K\bar{K}K$ .

## 2. Model for the $B \rightarrow \pi\pi K$ and $B \rightarrow K\bar{K}K$ decays

We have constructed a model for the following charged and neutral  $B$  decays:  $B^\pm \rightarrow (\pi\pi)_S K^\pm$ ,  $B^\pm \rightarrow (K\bar{K})_S K^\pm$ ,  $B^0 \rightarrow (\pi\pi)_S K^0$ ,  $B^0 \rightarrow (K\bar{K})_S K^0$ ,  $\bar{B}^0 \rightarrow (\pi\pi)_S \bar{K}^0$  and  $\bar{B}^0 \rightarrow (K\bar{K})_S \bar{K}^0$ . Here by  $(\pi\pi)_S$  and  $(K\bar{K})_S$  we mean  $\pi^+\pi^-$  or  $\pi^0\pi^0$  and  $K^+K^-$  or  $K^0\bar{K}^0$  pairs in isospin zero  $S$ -wave.

Below we outline only the main features of the amplitudes described in more detail in [8]. The decay amplitudes consist of two parts both related to weak transitions  $b \rightarrow s\bar{n}n$  and  $b \rightarrow s\bar{s}s$ , where  $n$  denotes  $u$  or  $d$  quarks. The first part corresponds to the factorization approximation with some QCD corrections. The second one is the long-distance amplitude  $A_{LD}$  with  $c$ - or  $u$ -quark loops. Its form for the  $B^- \rightarrow (\pi^+\pi^-)_S K^-$  decay reads:

$$A_{LD} = \frac{G_F}{\sqrt{3}} \chi [C(m_{\pi\pi}) \Gamma_1^{n*}(m_{\pi\pi}) + C(m_K) \Gamma_1^{s*}(m_{\pi\pi})], \quad (2.1)$$

where  $G_F$  is the Fermi coupling constant,  $\chi$  is a constant which value is close to  $30 \text{ GeV}^{-1}$ ,  $m_K$  is the kaon mass,  $\Gamma_1^n(m_{\pi\pi})$  and  $\Gamma_1^s(m_{\pi\pi})$  are the non-strange and strange pion scalar form factors depending on the effective pion-pion mass  $m_{\pi\pi}$ . The charming penguin contribution is written similarly as in ref. [6]:

$$C(m) = -(M_B^2 - m^2) f_\pi F_\pi (V_{ub} V_{us}^* P_u + V_{tb} V_{ts}^* P_t), \quad (2.2)$$

where  $f_\pi F_\pi = 0.042 \text{ GeV}$ ,  $P_t$  and  $P_u$  are complex parameters multiplied by the products of the Cabibbo-Kobayashi-Maskawa matrix elements  $V$ . We have performed numerical calculations using

two sets of parameters fitted to data on two-body charmless  $B$ -decays. We consider the model I in which [6]  $P_t = (0.068 \pm 0.007) \exp[i(1.32 \pm 0.10)]$  and  $P_u = (0.32 \pm 0.14) \exp[i(1.0 \pm 0.27)]$ , and the model II [7] with  $P_t = (0.08 \pm 0.02) \exp[-i(0.6 \pm 0.5)]$  and  $P_u = 0$ .

In our treatment of final state interactions we use not only the pion scalar form factors  $\Gamma_1^n(m_{\pi\pi})$  and  $\Gamma_1^s(m_{\pi\pi})$  but also the kaon scalar form factors  $\Gamma_2^n(m_{K\bar{K}})$  and  $\Gamma_2^s(m_{K\bar{K}})$ . The first and the second form factors are responsible for the transitions from the  $\bar{n}n$  quark pair and from the  $\bar{s}s$  quark pair to the  $\pi\pi$  pair, respectively. Similarly the third and the fourth form factors correspond to the  $\bar{n}n \rightarrow K\bar{K}$  and to the  $\bar{s}s \rightarrow K\bar{K}$  transitions. In the  $B$ -decay processes transitions from the  $\pi\pi$  channel to the  $K\bar{K}$  channel and vice versa are possible. They are incorporated in the two-body scattering matrix  $T$ . The interchannel couplings are included in the following formulae:

$$\Gamma_i^{n,s}(m) = R_i^{n,s}(m) + \sum_{j=1}^2 \langle k_i | R_j^{n,s}(m) G_j(m) T_{ij}(m) | k_j \rangle, \quad (2.3)$$

where  $|k_i\rangle$  and  $|k_j\rangle$  represent the wave functions of the two mesons in the momentum space and the indices  $i, j = 1, 2$  refer to the  $\pi\pi$  and  $K\bar{K}$  channels, respectively. The center of mass channel momenta are given by  $k_1 = \sqrt{m^2/4 - m_\pi^2}$  and  $k_2 = \sqrt{m^2/4 - m_K^2}$ .

In the numerical calculations we use the solution  $A$  of the coupled channel model [5]. The functions  $G_i(m)$  are the free particle Green's functions defined in [5] and  $R_i^{n,s}(m)$  are the production functions responsible for the initial formation of the meson pairs prior to rescattering. The production functions have been derived by Meißner and Oller in the one-loop approximation of the chiral perturbation theory [9].

Let us remark that in this approach a sum of many Breit-Wigner terms typical for the so-called isobar model, usually used in phenomenological fits to Dalitz plots, is replaced by a set of the coupled meson-meson amplitudes  $T$ . These amplitudes can be expressed in terms of the phase shifts  $\delta_{\pi\pi}$ ,  $\delta_{KK}$  and the inelasticity  $\eta$  known from other experiments. Thus no arbitrary phases nor relative intensity free parameters for different resonances are needed. In addition, both the  $T$ - and the  $\Gamma$ -matrices satisfy unitarity constraints. A generalization of Watson's theorem to multi-channel  $B$ -decays is thus realized.

### 3. Results

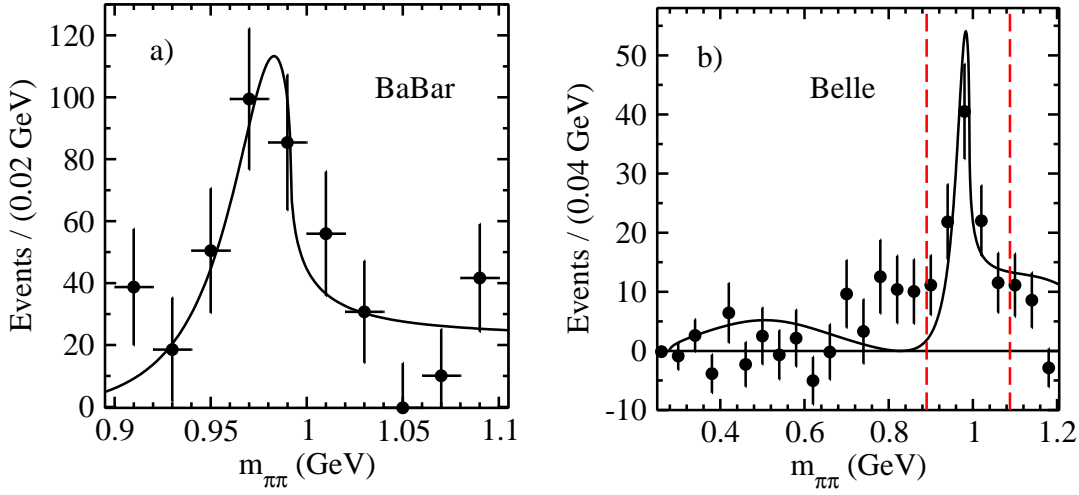
Table 1 shows that the results of our calculations compare reasonably well with the results of the experimental analyses averaged by HFAG in spring 2005.

In Fig.1a we compare the model I with the BaBar data [1] for the  $B^\pm \rightarrow \pi^+ \pi^- K^\pm$  decays. One can see that the theoretical solid line describes well the shape of the  $f_0(980)$  resonance. Fig. 1b shows the Belle data [4] in comparison with our model for the neutral  $B^0$  decays to  $\pi^+ \pi^- K^0$  in a wide range of effective masses. The model provides us with an absolute prediction for the branching ratio of the  $B^0$  decay into  $f_0(980)K^0$  if the effective mass distributions in the range of the  $f_0(980)$  resonance for the charged  $B$  decays are reproduced.

For the model I one obtains a particularly large negative direct CP-violation asymmetry.

**Table 1:** Branching ratios  $\mathcal{B}$  in units of  $10^{-6}$ , direct CP asymmetry  $\mathcal{A}_{CP}$ , time dependent asymmetry parameters  $\mathcal{A}$  and  $\mathcal{S}$  for the  $B \rightarrow f_0(980)K, f_0(980) \rightarrow \pi^+\pi^-$  decay. The model errors come from uncertainties of the charming penguin amplitudes.

decay mode	observable	experiment HFAG	Model I $\chi = 33.5 \text{ GeV}^{-1}$	Model II $\chi = 23.5 \text{ GeV}^{-1}$
$B^\pm$	$\mathcal{B}$	$8.49^{+1.35}_{-1.26}$	8.49 (fit)	8.46 (fit)
	$\mathcal{A}_{CP}$	$-0.13^{+0.19}_{-0.12}$	$-0.52 \pm 0.12$	$0.20 \pm 0.20$
$B^0$	$\mathcal{B}$	$6.0 \pm 1.6$	$5.9 \pm 1.6$	$5.8 \pm 2.8$
	$\mathcal{A}$	$-0.14 \pm 0.22$	$0.01 \pm 0.10$	$0.0004 \pm 0.0010$
	$\mathcal{S}$	$-0.39 \pm 0.26$	$-0.63 \pm 0.09$	$-0.77 \pm 0.0004$



**Figure 1:**  $\pi^+\pi^-$  mass distributions in the  $B^\pm \rightarrow \pi^+\pi^-K^\pm$  (a) and in the  $B^0 \rightarrow \pi^+\pi^-K^0$  decays (b). Vertical lines delimit a band of the  $f_0(980)$  events.

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