

Hard spectator-scattering in $B \rightarrow \pi\pi$ decays at NNLO*

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We compute the 1-loop (NNLO) corrections to hard spectator-scattering in tree-dominated hadronic B decays. Depending on the values of hadronic input parameters the corrections are shown to have a significant impact on the $B \rightarrow \pi\pi$ branching fractions.

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1. Introduction

Hadronic B -decays into two light mesons are a rich source of information about the flavour structure of the Standard Model (CKM matrix) and of its possible extensions. On the theoretical side the task of relating the fundamental parameters to the large number of branching ratios and CP asymmetries on which experimental data is available is complicated by strong interaction effects. In recent years, these have become more tractable through the development of the QCD factorization (QCDF) formalism [1] and the soft-collinear effective theory (SCET) [2, 3].

At leading power in an expansion in Λ_{QCD}/m_b the matrix elements of operators in the weak effective Hamiltonian obey a factorization formula [1]

$$\langle M_1 M_2 | O_i | \bar{B} \rangle = f_+^{B \rightarrow M_1}(0) \int du T_i^{\text{I}}(u) \phi_{M_2}(u) + \int d\omega du dv T_i^{\text{II}}(\omega, v, u) \phi_{M_1}(v) \phi_{M_2}(u). \quad (1.1)$$

The hard-scattering kernels $T_i^{\text{I}}, T_i^{\text{II}}$ are perturbatively calculable in the strong coupling, while the form factors f_+ and light-cone distribution amplitudes (LCDAs) ϕ encapsulate nonperturbative properties of the initial and final state particles. Both kernels are currently known to $\mathcal{O}(\alpha_s)$.

Within the set of hadronic final states, the $\pi\pi$ system is special. Isospin symmetry allows to extract the strong amplitudes if the CKM angle γ is known. Consequently it can be considered a test case for a theoretical description of QCD dynamics such as QCDF. Indeed, the naive factorization result predicts a $1/N_c^2 \sim 0.1$ colour suppression of $BR(\bar{B}_d \rightarrow \pi^0 \pi^0)$, stronger than what is observed in experiment. At $\mathcal{O}(\alpha_s)$ the kernel T_i^{I} for the colour-suppressed tree amplitude $\alpha_2(\pi\pi)$ receives corrections proportional to the large Wilson coefficient C_1 that nearly cancel the LO contribution, increasing its sensitivity to the spectator-scattering term T_i^{II} . This motivates a computation of the 1-loop current-current contributions (NNLO, $\mathcal{O}(\alpha_s^2)$) to T_i^{II} for the tree operators Q_1 and Q_2 , on which we report below. The 1-loop correction also introduces a new source of strong phases (from spectator-scattering) not present in the NLO (tree) contribution to T_i^{II} .

2. Matching calculation

Within the SCET framework, the spectator-scattering kernel T_i^{II} arises through two consecutive matching steps (e.g., third paper of [2]). The SCET_I operators relevant to this calculation are

$$\begin{aligned} \mathcal{O}^{\text{I}}(t) &= \bar{\chi}(tn_-) \frac{\not{n}_-}{2} (1 - \gamma_5) \chi(0) \left[\tilde{C}_A \bar{\xi}(0) \not{n}_+ (1 - \gamma_5) h_v(0) \right. \\ &\quad \left. - \frac{1}{m_b} \int ds \tilde{C}_B(s) \bar{\xi}(0) \not{n}_+ i \not{D}_{\perp c}(sn_+) (1 + \gamma_5) h_v(0) \right], \\ \mathcal{O}^{\text{II}}(t, s) &= \bar{\chi}(tn_-) \frac{\not{n}_-}{2} (1 - \gamma_5) \chi(0) \frac{1}{m_b} \bar{\xi}(0) \frac{\not{n}_+}{2} i \not{D}_{\perp c}(sn_+) (1 + \gamma_5) h_v(0). \end{aligned} \quad (2.1)$$

The operator \mathcal{O}^{I} is associated with the first term on the right-hand side of (1.1). It includes the matching coefficients \tilde{C}_A, \tilde{C}_B such that its matrix element reproduces the full form factor f_+ . Our new result refers to the 1-loop matching coefficient of $\mathcal{O}^{\text{II}}(t, s)$. The matrix element of this operator factorizes into the LCDA $\phi_{M_2}(u)$ and the matrix element

$$\frac{1}{m_b} \langle M_1 | \bar{\xi}(0) \frac{\not{n}_+}{2} i \not{D}_{\perp c}(sn_+) (1 + \gamma_5) h_v(0) | \bar{B} \rangle. \quad (2.2)$$

Performing a second matching to SCET_{II}, this matrix element factorizes into $J_{\parallel} * \phi_B * \phi_{M_1}$ [4] such that

$$T^{\text{II}}(\omega, u, v) = \int dv' H^{\text{II}}(u, v') J_{\parallel}(1 - v'; \omega, v). \quad (2.3)$$

The jet kernel J_{\parallel} has already been computed in [5]. To obtain the kernel H^{II} , we computed the amplitude for $b \rightarrow q_c A_{\perp c} \bar{q}_{\bar{c}} q_{\bar{c}}$ up to one loop, where the b quark is at rest and the other partons are collinear with the light mesons, both in QCD and in SCET_I. On the SCET_I side the contribution of the operator O^{I} serves to eliminate the “factorizable” QCD diagrams and provides a further subtraction term proportional to the known 1-loop kernel $T^{\text{I}(1)}$. The renormalized 1-loop matrix element of O^{II} is given in terms of the Brodsky-Lepage kernel and the anomalous dimension kernel of the subleading SCET_I current. Here care must be taken to ensure the vanishing of evanescent operator matrix elements. After these subtractions, the remaining difference between the QCD and SCET_I amplitudes is infrared finite and is absorbed into an $\mathcal{O}(\alpha_s)$ correction $H^{\text{II}(1)}$. This together with J_{\parallel} provides a complete result for spectator-scattering at NNLO.

To incorporate our results into the colour-suppressed tree amplitude we write, following the notation of [6],

$$\begin{aligned} \alpha_2(M_1 M_2) = & C_2 + \frac{C_1}{N_c} + \frac{C_1}{N_c} \frac{\alpha_s(\mu_b) C_F}{4\pi} V_2(\pi) + \frac{\pi \alpha_s(\mu_{\text{hc}}) C_F}{N_c} \left(\frac{C_1}{N_c} [H_2^{\text{tw}2}(\pi\pi) I_{\parallel} + H_2^{\text{tw}3}(\pi\pi)] \right. \\ & \left. + \frac{\alpha_s(\mu_{\text{hc}})}{4\pi N_c} \frac{9f_{M_1} \hat{f}_B(\mu_{\text{hc}})}{m_b f_+(0) \lambda_B(\mu_{\text{hc}})} [C_1 R_1 + C_2 R_2] \right). \end{aligned} \quad (2.4)$$

An analogous equation with $C_1 \leftrightarrow C_2$ holds for the colour-allowed tree amplitude α_1 . I_{\parallel} is the jet function correction [second paper of [5], Eq. (96)], while for the new contributions R_1 and R_2 , we find, using asymptotic LCDAs ($x_b = m_b^2/\mu_{\text{hc}}^2$),

$$\begin{aligned} R_1 = & C_F \left(-\frac{1}{2} \ln^2 x_b + \frac{1}{2} \ln x_b + \frac{9}{2} - \frac{3\pi^2}{4} + 2i\pi \right) \\ & + \left(C_F - \frac{C_A}{2} \right) \left(\left[2 + \frac{2\pi^2}{3} \right] \ln x_b - \frac{74}{5} - 2\pi^2 + \frac{32}{5} \zeta(3) - \left(1 + \frac{2\pi^2}{5} \right) i\pi \right), \\ R_2 = & 3 \ln x_b - \frac{163}{20} + \frac{\pi^2}{3} - \frac{14}{5} \zeta(3) + \left(-3 + \frac{2\pi^2}{15} \right) i\pi. \end{aligned} \quad (2.5)$$

The finiteness of these expressions proves factorization of spectator-scattering at $\mathcal{O}(\alpha_s^2)$.

3. Phenomenological implications

Numerically, with input parameters defined in [6], we obtain

$$\begin{aligned} \alpha_2(\pi\pi) = & 0.17 - [0.17 + 0.08i]_{V_2} + \begin{cases} [0.10 \cdot (1.32 + 0.40i)]_{H_2^{\text{tw}2} I_{\parallel} + R_{1,2}} + [0.06]_{H_2^{\text{tw}3}} & \text{(default)} \\ [0.28 \cdot (1.61 + 0.49i)]_{H_2^{\text{tw}2} I_{\parallel} + R_{1,2}} + [0.17]_{H_2^{\text{tw}3}} & \text{(S4)} \end{cases} \\ = & \begin{cases} 0.20(0.18) - 0.04(-0.08)i & \text{(default)} \\ 0.62(0.47) + 0.05(-0.08)i & \text{(S4)} \end{cases} \end{aligned} \quad (3.1)$$

The various terms and factors correspond to those in (2.4) and we show the numbers for the default parameter set and the set S4 that provides a better overall description of hadronic two-body modes.

Mode	default		S4		Expt. [7]
	NLO	NNLO	NLO	NNLO	
$\alpha_1(\pi\pi)$	$0.99 + 0.02i$	$0.99 - 0.01i$	$0.88 + 0.02i$	$0.86 - 0.07i$	–
$\alpha_2(\pi\pi)$	$0.18 - 0.08i$	$0.20 - 0.04i$	$0.47 - 0.08i$	$0.62 + 0.05i$	–
$\text{Br}(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	8.86	8.89	5.17	5.10	5.0 ± 0.4
$\text{Br}(\bar{B}^0 \rightarrow \pi^0 \pi^0)$	0.35	0.37	0.70	1.11	1.5 ± 0.3
$\text{Br}(B^- \rightarrow \pi^- \pi^0)$	6.03	6.30	5.07	6.10	5.5 ± 0.6

Table 1: Tree amplitude coefficients α_1 and α_2 , and the CP-averaged $\pi\pi$ branching ratios in units of 10^{-6} in the default and S4 scenarios of [6], showing the effect of the new NNLO correction.

The numbers in parentheses in the second line give the result from [6] for comparison. Depending on parameters the 1-loop correction to spectator-scattering can result in a significant enhancement of the real part of the amplitude, predominantly from the jet function correction [cf. second paper of [5], Eq. (132)], and a substantial correction to the imaginary part (strong phase) from the new hard-matching correction. We confront the old NLO and our new (partial) NNLO results with the experimental data on the three branching ratios (Table 1), and observe that that the agreement is rather good with set S4. More details on the numerical analysis and the (substantial) theoretical uncertainties can be found in [8]. The numbers for the branching fractions in the Table should be considered as preliminary, since the NNLO correction to spectator-scattering is still missing for the penguin amplitudes (see [9] for a related calculation). For this reason we do not discuss CP asymmetries, which we expect to be affected by the spectator-scattering phase.

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