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Tree-Level Vacuum Stability in Multi Higgs Models

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In the most general model with two Higgs doublets, if a minimum that preserves the $U(1)_{em}$ symmetry exists, then charge breaking (CB) minima cannot occur. The depth of the potential at a stationary point that breaks CB or CP, relative to the $U(1)_{em}$ preserving minimum, is proportional to the squared mass of the charged or pseudoscalar Higgs, respectively.

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In this talk we will review recent results [1, 2] on symmetry breaking in two Higgs doublet models (2HDM) [3]. Namely, the possibility that minima that break different symmetries can appear simultaneously in the potential, and tunneling between them might occur. In ref. [1] we worked in 2HDM without explicit CP breaking and showed that if there is, at tree level, a minimum that preserves the $U(1)_{em}$ and CP symmetries, that minimum is the global one. Therefore, the stability of this minimum is guaranteed and tunneling to a deeper one that breaks charge conservation or CP becomes impossible. In ref. [2] we extended this analysis to the most general 2HDM and proved that, once a charge-preserving minimum exists, any charge breaking (CB) stationary point that might exist lies above the minimum. Charge conservation is thus assured.

There are many ways of writing the 2HDM tree-level potential, for this talk we will use the one introduced in ref. [4]. With two scalar Higgs doublets in the theory, Φ_1 and Φ_2, both having hypercharges $Y = 1$ [1],

$$\Phi_1 = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_5 + i\varphi_7 \end{pmatrix} \;,\; \Phi_2 = \begin{pmatrix} \varphi_3 + i\varphi_4 \\ \varphi_6 + i\varphi_8 \end{pmatrix} \;, \tag{1}$$

there are four $SU(2)_W \times U(1)_Y$ invariants one can construct with these fields, namely

$$\begin{aligned} x_1 &\equiv |\Phi_1|^2 = \varphi_1^2 + \varphi_2^2 + \varphi_5^2 + \varphi_7^2 \\ x_2 &\equiv |\Phi_2|^2 = \varphi_3^2 + \varphi_4^2 + \varphi_6^2 + \varphi_8^2 \\ x_3 &\equiv Re(\Phi_1^\dagger \Phi_2) = \varphi_1\varphi_3 + \varphi_2\varphi_4 + \varphi_5\varphi_6 + \varphi_7\varphi_8 \\ x_4 &\equiv Im(\Phi_1^\dagger \Phi_2) = \varphi_1\varphi_4 - \varphi_2\varphi_3 + \varphi_5\varphi_8 - \varphi_6\varphi_7 \;. \end{aligned} \tag{2}$$

Notice that under a particular CP transformation in this basis ($\Phi_1 \rightarrow \Phi_1^*$, $\Phi_2 \rightarrow \Phi_2^*$) the invariants x_1, x_2 and x_3 remain the same but x_4 changes sign. The most general tree-level potential is thus composed of all the terms linear and quadratic in the x's, i.e.,

$$\begin{aligned} V \;=\; & a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{44} x_4^2 + \\ & b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{34} x_3 x_4 \;. \end{aligned} \tag{3}$$

The a_i parameters have dimensions of mass squared and the b_{ij} parameters are dimensionless. The potential thus written depends on 14 real parameters but, with a particular choice of basis, one can reduce this number to 11 independent parameters (see, for instance, [5]). The linear terms in x_4 are the ones that explicitly break CP, and if we eliminate them we are left with the 10 parameter CP preserving potential that was used in ref. [1] (with a judicious choice of basis [5] the number of independent real parameters of this potential may be reduced to 9).

It is a well known fact [3] that the 2HDM potential can only have three types of minima. One of them is a charge breaking minimum where three of the fields, at least one of them carrying electrical charge, have non-vanishing vacuum expectation values (vevs). For instance, the fields φ_5, φ_6 and φ_3. In the second possible type of minimum only neutral fields have vevs and there are two possibilities. In the first only two fields have vevs (φ_5 and φ_6, for instance), and we call this the first "normal" minimum, N_1. In the second case there are three vevs, for the fields φ_5, φ_6

[1] The numbering of the real scalar φ fields is chosen for convenience of writing the mass matrices for the scalar particles.

and φ_7, for example. We call this case the N_2 minimum. Notice that when the model is reduced to the potential that explicitly preserves CP, the N_2 minimum spontaneously breaks CP. For this more general case, however, there is *a priori* no physical distinction between the two "normal" minima, both of them preserving charge conservation.

In references [1] and [2] we developed a method to compute the value of the tree-level potential at each of these stationary points, and compare their value. We refer the readers to those publications for details of the calculations, and proceed to present the results and their consequences. For the N_1 minimum, the vevs will be $\varphi_5 = v_1$ and $\varphi_6 = v_2$; for CB, we will have $\varphi_5 = v'_1$, $\varphi_6 = v'_2$ and $\varphi_3 = \alpha$ (this last vev is the charged one that breaks charge conservation); for N_2, $\varphi_5 = v''_1$, $\varphi_6 = v''_2$ and $\varphi_7 = \delta$ (in the case of the CP preserving potential, this last vev is the one that spontaneously breaks CP). The difference between the values of the potential at a CB and an N_1 stationary points is given by

$$V_{CB} - V_{N_1} = \frac{1}{2} Y^T V' = \frac{M^2_{H^\pm}}{2v^2} \left[(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2 \right] \quad , \tag{4}$$

where $M^2_{H^\pm}$ is the value of the squared charged scalar mass at N_1. Then, if N_1 is a minimum, we will necessarily have $M^2_{H^\pm} > 0$ and, given that the quantity in square brackets above is always positive, we conclude that $V_{CB} - V_{N_1} > 0$. Then, the CB stationary point is clearly above the N_1 minimum. Furthermore, it is possible to show that under these circumstances the matrix of CB squared scalar masses is neither positive nor negative definite. As a result, we reach the conclusion the the CB stationary point is a saddle point, and lies above the N_1 minimum.

Results altogether identical are obtained if one compares the N_2 and CB potentials. From [2] we see that

$$V_{CB} - V_{N_2} = \left(\frac{M^2_{H^\pm}}{2v^2} \right)_{N_2} \left[(v'_1 v''_2 - v'_2 v''_1)^2 + \alpha^2 (v''^2_1 + \delta^2) + \delta^2 v'^2_2 \right] \quad , \tag{5}$$

where now $\left(M^2_{H^\pm}\right)_{N_2}$ is the squared charged scalar mass of the N_2 stationary point, and $(v^2)_{N_2} = v''^2_1 + v''^2_2 + \delta^2$. Again, we reach the conclusion that, if N_2 is a minimum, then $V_{CB} - V_{N_2} > 0$, and the CB stationary point lies above the normal minimum, Again, it is possible to demonstrate that the CB stationary point is a saddle point. The conclusion to take from this analysis is that, if a minimum that preserves electric charge conservation exists, it is necessarily deeper than any CB stationary point that might exist in the model. Further, that stationary point is necessarily a saddle point. There is therefore no possibility whatsoever of tunneling from a charge-preserving minimum to a deeper one where charge is broken, and the masslessness of the photon is thus assured.

What about a comparison between the values of the potential at N_1 and N_2 stationary points? Unfortunately we cannot reach any definite conclusion about which of these possible minima is deeper. Following a chain of thought altogether identical to the previous cases, one obtains

$$V_{N_2} - V_{N_1} = \frac{1}{2} \left[\left(\frac{M^2_{H^\pm}}{v^2} \right)_{N_1} - \left(\frac{M^2_{H^\pm}}{v^2} \right)_{N_2} \right] \left[(v''_1 v_2 - v''_2 v_1)^2 + \delta^2 v_2^2 \right] \quad . \tag{6}$$

Depending on which stationary point has a larger value for the squared charged mass, then, either N_1 or N_2 might be deeper. This seems to depend on the particular values of the parameters of the model, both cases *a priori* possible. A very interesting thing happens, though, when we restrict ourselves to the case of the CP preserving potential. In that case the N_1 minimum preserves both electric charge conservation *and* CP, and the N_2 stationary point spontaneously breaks CP. Calling $V_{N_1} = V_N$ and $V_{N_2} = V_{CP}$, and investigating the mass matrices of the 2HDM model (for instance, [1]) we obtain a remarkable result,

$$V_{CP} \, - \, V_N \;=\; \frac{M_A^2}{2\,v^2}\,\left[(v_1^{\prime\prime} v_2 \, - \, v_2^{\prime\prime} v_1)^2 \, + \, \delta^2\, v_2^2\right] \quad , \tag{7}$$

where M_A^2 is, as usual, the squared pseudoscalar mass at the normal (i.e., charge and CP preserving) minimum. The right-hand side of this equation is thus positive and we have, just as in the CB case, $V_{CP} \, - \, V_N \, > \, 0$. The CP stationary point is therefore *above* the normal minimum, but in this case it is not obvious whether it is also a saddle point. Thus no tunneling to a deeper minimum may occur once the potential is at a vacuum that respects both CP and charge conservation. The tree level vacuum, we have therefore shown, is perfectly stable.

An intriguing aspect of these results is the following: if one observes equations (4) and (5), one sees that the difference in the depth of the potential between the normal minimum and the CB stationary point is “controlled" by the charged Higgs squared mass. On the other hand, equation (7) shows that the difference in the value of the potential between the CP and the normal stationary points is proportional to the pseudoscalar squared mass. That is, the depth of the potential at a stationary point that breaks a given symmetry, relative to the normal minimum, depends, in a very straightforward manner, on the mass of the scalar particle directly linked with that symmetry. The absence of charge breaking when normal minima exist seems to be related to the non-existence, in the potential, of cubic terms in the fields. In fact, analysing the Zee model [6] scalar potential - this model consists of the 2HDM plus a charged $SU(2)$ singlet scalar -, where such terms are present, CB minima deeper than the normal ones are discovered [7]. This is not surprising, since charge - and colour - breaking is known to occur in supersymmetric theories [8], for which the scalar potential has, once again, cubic terms in the fields.

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