

## "Seesawing" away the hierarchy problem

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We describe a model for the scalar sector where all interactions occur either at an ultra-high scale  $\Lambda_U \sim 10^{16} - 10^{19}$  GeV or at an intermediate scale  $\Lambda_I = 10^9 - 10^{11}$  GeV. The interaction of physics on these two scales results in an SU(2) Higgs condensate at the electroweak (EW) scale,  $\Lambda_{EW}$ , through a seesaw-like Higgs mechanism,  $\Lambda_{EW} \sim \Lambda_I^2 / \Lambda_U$ , while the breaking of the SM SU(2)×U(1) gauge symmetry occurs at the intermediate scale  $\Lambda_I$ . The EW scale is, therefore, not fundamental but is naturally generated in terms of ultra-high energy phenomena and so the hierarchy problem is alleviated. We show that our “seesaw-Higgs” model predicts the existence of sub-eV neutrino masses which are generated through a “two-step” seesaw mechanism in terms of the same two ultra-high scales:  $m_\nu \sim \Lambda_I^4 / \Lambda_U^3 \sim \Lambda_{EW}^2 / \Lambda_U$ . We also show that our seesaw Higgs model can be naturally embedded in theories with *tiny* extra dimensions of size  $R \sim \Lambda_U^{-1} \sim 10^{-16}$  fm, where the seesaw induced EW scale arises from a violation of a symmetry at a distant brane if there are 7 *tiny* extra dimensions.

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## 1. Introduction

A long standing problem in modern particle physics is the apparent enormous hierarchies of energy/mass scales observed in nature. Disregarding the “small” hierarchies in the masses of the known charged matter particles, there seems to be two much larger hierarchies: the first is the hierarchy between the fundamental grand unified scale  $\Lambda_U \sim \mathcal{O}(10^{16})$  GeV [or Planck scale  $\Lambda_U \sim \mathcal{O}(10^{19})$  GeV], and the EW scale,  $\Lambda_{EW} \sim \mathcal{O}(100)$  GeV, and the second is the hierarchy between the EW scale and the neutrino mass scale  $m_\nu \sim \mathcal{O}(10^{-2})$  eV. This apparent hierarchical structure of scales have fueled a lot of activity in the past 30 years in the search for new physics beyond the Standard Model (SM).

The  $\Lambda_U - \Lambda_{EW}$  hierarchy, when viewed within the SM framework, is usually referred to as the gauge Hierarchy Problem (HP) of the SM, which is intimately related to the SM Higgs sector responsible for the generation of the EWSB scale,  $v_{EW} \sim \Lambda_{EW}$ , through the SM Higgs mechanism. The HP of the SM raises a technical difficulty known as the naturalness (or fine tuning) problem, i.e., there is a problem of stabilizing the  $\mathcal{O}(\Lambda_{EW})$  mass scale of the Higgs against radiative corrections without an extreme fine tuning (to one part in  $\Lambda_{EW}^2/\Lambda_U^2$ ). It should, however, be emphasized that this fine-tuning problem of the SM may be just a technical difficulty which reflects our ignorance in explaining the simultaneous presence of the two disparate scales  $\Lambda_U$  and  $\Lambda_{EW}$ , and *may have nothing to do* with the more fundamental question of the origin of scales which we will address in this work: *why do we observe in nature such large hierarchies between the fundamental GUT or Planck scale  $\Lambda_U$ , the EW scale  $\Lambda_{EW}$  and the neutrino mass-scale  $m_\nu$ ?*

In this letter we propose a simple model [1], where the only fundamental scale is the GUT or Planck scale  $\Lambda_U$ , while the EW and neutrino mass scales both arise due to interactions between this fundamental scale  $\Lambda_U$  and a new intermediate ultra-high scale  $\Lambda_I \sim 10^9 - 10^{11}$  GeV, i.e.,  $\Lambda_{EW} \ll \Lambda_I \ll \Lambda_U$ . The intermediate scale is viewed as the scale of breaking of the unification group which underlies the physics at the scale  $\Lambda_U$  (see e.g., [2]). Our model then naturally accounts for the existence of both the EW and sub-eV neutrino mass scales by means of a “two-step” seesaw between the two ultra-high mass scales  $\Lambda_U$  and  $\Lambda_I$ : the first  $\Lambda_U - \Lambda_I$  seesaw generates the EW scale  $\Lambda_{EW} \sim \Lambda_I^2/\Lambda_U$  and then a second  $\Lambda_U - \Lambda_{EW}$  seesaw gives rise to the sub-eV neutrino mass scale  $m_\nu \sim \Lambda_{EW}^2/\Lambda_U \sim \Lambda_I^4/\Lambda_U^3$ . Our model does not address the fine tuning problem - we assume that some higher symmetry at the fundamental scale  $\Lambda_U$  is responsible for protecting the EW Higgs mass scale.

## 2. The seesaw-Higgs model

Let us schematically define the Lagrangian of our seesaw-Higgs model as follows:

$$\mathcal{L} = \mathcal{L}_{SM}(f, G) + \mathcal{L}_Y(\Phi, f) + \mathcal{L}_S(\Phi, \varphi, \chi) + \mathcal{L}_\nu(\Phi, \varphi, \chi, \nu_L, \nu_R), \quad (2.1)$$

where  $\Phi$  is an SU(2) scalar doublet and  $\varphi, \chi$  are “sterile” SU(2)-singlets that do not interact with the SM particles. Also,  $\mathcal{L}_{SM}(f, G)$  contains the usual SM’s fermions and gauge-bosons kinetic terms,  $\mathcal{L}_Y(\Phi, f)$  contains the SM-like Yukawa interactions and

$$\mathcal{L}_\nu(\Phi, \varphi, \chi, \nu_L, \nu_R) = -Y_D \ell_L \Phi \nu_R + Y_M \varphi \bar{\nu}_R^c \nu_R + Y'_M \chi \bar{\nu}_R^c \nu_R + h.c., \quad (2.2)$$

$$\mathcal{L}_S(\Phi, \varphi, \chi) = |D_\mu \Phi|^2 + |\partial_\mu \varphi|^2 + |\partial_\mu \chi|^2 - V, \quad (2.3)$$

with

$$V = \lambda_1 (|\Phi|^2 - |\chi|^2)^2 + \lambda_2 (|\varphi|^2 - \Lambda_U^2)^2 + \lambda_3 (\text{Re}(\varphi^\dagger \chi) - \Lambda_I^2)^2 + \lambda_4 (\text{Im}(\varphi^\dagger \chi) - \Lambda_I^2)^2, \quad (2.4)$$

where all  $\lambda_i$  are positive real constants, naturally of  $\mathcal{O}(1)$ . Note that the above total Lagrangian conserves lepton number  $L$  if we assign lepton number 2 to both singlets  $\varphi$  and  $\chi$ , i.e., if  $L_\varphi = L_\chi = 2$ .

### 3. The seesaw-Higgs mechanism and the Electroweak scale

The seesaw-Higgs potential in Eq. 2.4 gives rise to the desired seesaw-condensate of  $\Phi$ . In particular, the minimization of  $V$  which only contains terms at energy scales  $\Lambda_U$  and  $\Lambda_I$  leads to:

$$\begin{aligned} \langle \varphi \rangle &= \Lambda_U, \\ \langle \Phi \rangle &= \langle \chi \rangle = \frac{\Lambda_I^2}{\Lambda_U} \equiv v_{EW} \sim \Lambda_{EW}, \end{aligned} \quad (3.1)$$

where  $\langle \Phi \rangle = v_{EW} = \Lambda_I^2/\Lambda_U$  is the condensate required for EWSB, when the fundamental scale  $\Lambda_U$  is taken to be around the GUT scale,  $\Lambda_U \sim \mathcal{O}(10^{16})$  GeV, and the intermediate scale is  $\Lambda_U \sim \mathcal{O}(10^9)$  GeV, or when  $\Lambda_U \sim \mathcal{O}(10^{19})$  GeV (the Planck scale) and  $\Lambda_U \sim \mathcal{O}(10^{10.5})$  GeV.

After EWSB we are left with 5 physical neutral scalars:  $H$  which is a SM-like light Higgs with a mass  $M_H \sim 2\sqrt{\lambda_1}v_{EW}$ , 3 superheavy neutral states  $S_1, S_2, A_1$  with masses  $M_{S_1} \sim \sqrt{\lambda_3}\Lambda_U, M_{S_2} \sim 2\sqrt{\lambda_2}\Lambda_U, M_{A_1} \sim \sqrt{\lambda_4}\Lambda_U$  and 1 massless axial state  $A_M$  which is the ‘Majoron’[4] associated with the spontaneous breakdown of Lepton number (by the condensate of the two singlets, see next section).

### 4. A two-step seesaw and the neutrino mass scale

When the singlet  $\varphi$  forms its condensate,  $\langle \varphi \rangle = \Lambda_U$ , the second term in  $\mathcal{L}_v(\Phi, \varphi, \chi, \nu_L, \nu_R)$  will lead to a right-handed Majorana mass which will naturally be of that order:  $m_V^M = Y_M \Lambda_U$ .<sup>1</sup> The SU(2) condensate  $\langle \Phi \rangle = \Lambda_I^2/\Lambda_U \sim \Lambda_{EW}$  will generate a Dirac mass for the neutrinos of size  $m_V^D \sim Y_D \Lambda_{EW}$  through the first term in  $\mathcal{L}_v(\Phi, \varphi, \chi, \nu_L, \nu_R)$ . Then, the neutrino mass matrix acquires the classic seesaw structure which, upon diagonalization, yields two physical Majorana neutrino states: a superheavy state  $\nu_h$  with a mass  $m_{\nu_h} \sim Y_M \Lambda_U$  and a superlight state  $\nu_\ell$  with a mass:

$$m_{\nu_\ell} = \frac{(m_V^D)^2}{m_V^M} = \frac{Y_D^2 \Lambda_I^4}{Y_M \Lambda_U^3} \sim \frac{Y_D^2 \Lambda_{EW}^2}{Y_M \Lambda_U}. \quad (4.1)$$

The neutrino mass scale is, therefore, subject to a two-step seesaw mechanism, the first (in the scalar sector) generates the Dirac neutrino mass  $m_V^D \sim Y_D \Lambda_{EW}$ , which then enters in the off diagonal neutrino mass matrix to give the classic ‘seesawed’ Majorana mass in (4.1) by a second  $m_V^M - m_V^D$  seesaw in the neutrino mass matrix. The presence of this extremely small scale,  $m_{\nu_\ell} \sim \mathcal{O}(\Lambda_{EW}^2/\Lambda_U)$ , well below the EW scale, is therefore naturally explained in terms of the two

<sup>1</sup>Note that, since  $\chi$  forms a condensate of  $\mathcal{O}(\Lambda_{EW})$ , its contribution to the Majorana neutrino mass term will be negligible compared to that of  $\varphi$  which forms the condensate of  $\mathcal{O}(\Lambda_U)$ .

ultra-high scales  $\Lambda_U$  and  $\Lambda_I$ . For example, if  $\Lambda_U \sim \mathcal{O}(10^{16})$  GeV and  $\Lambda_I \sim \mathcal{O}(10^9)$  (which gives  $\Lambda_{EW} \sim \mathcal{O}(100)$  GeV) we obtain for  $Y_D \sim Y_M \sim \mathcal{O}(1)$ :  $m_{\nu_\ell} \sim \mathcal{O}(10^{-3})$  eV, roughly in accord with current mixing data. A value of  $\Lambda_U$  at the Planck scale could still be consistent with the double-seesaw sub-eV neutrino masses, if  $\Lambda_I = \mathcal{O}(10^{10.5})$  GeV (again giving  $\Lambda_{EW} \sim \mathcal{O}(100)$  GeV) when  $Y_D^2/Y_M \sim \mathcal{O}(10^3)$  GeV. This may happen if e.g., the heavy Majorana mass term is of the order of the intermediate scale  $\Lambda_I$ , and the Dirac mass term is of  $\mathcal{O}(100)$  MeV (consistent with most light leptons and down quark masses).

## 5. The seesaw-Higgs model from tiny extra dimensions

If there are extra compact spatial dimensions (ECSD) which are populated with multiple 3-branes, then, as was shown in [3], the violation of flavor symmetries on these distant branes can be carried out to our brane by "messenger" scalar fields that can propagate freely in the bulk between the branes. In particular, the profile of these messenger fields at all points on our wall (i.e., on the interference between the bulk and our brane) "shines" the flavor violation which appears as a boundary condition on our 3-brane.

In our case, this "shining" mechanism can be utilized to generate the seesaw-Higgs potential through the "shined" value of the condensate of a messenger field  $\eta$  on our wall [1]:

$$\langle \eta \rangle \sim \frac{\Gamma(\frac{n-2}{2})}{4\pi^{\frac{n}{2}}} \frac{M_\star}{(M_\star R)^{n-2}}, \quad (5.1)$$

where  $M_\star \sim \Lambda_U$  is the fundamental  $4+n$  mass scale and  $R$  is the size of the ECSD. In particular, an interaction term on our wall of the form:

$$S_{us} = \int_{us} d^4x \eta(x, y^i = 0) \eta(x, y^i = 0) \phi^\dagger(x) \chi(x) + h.c. . \quad (5.2)$$

will yield the term  $\Lambda_I^2 \phi^\dagger \chi$  of the seesaw-Higgs potential in (2.4), if  $\langle \eta \rangle = \Lambda_I$ . Thus, using (5.1) with  $M_\star \sim 10^{16}$  GeV, we find that the desired intermediate scale (i.e.,  $\langle \eta \rangle = \Lambda_I \sim 10^9$  GeV in order to get the seesaw-induced EW scale) is obtained if there are  $n = 7$  tiny extra transverse dimensions of size  $R \sim M_\star^{-1} \sim 10^{-16}$  fm.

## References

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