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String Theory

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We review basic aspects as well as recent developments in string theory. We discuss the correspondence between superstring theory on curved spaces and gauge theories in flat space-time, the computation of gluon scattering amplitudes using a topological string and the relation between four-dimensional supersymmetric gauge theories and bosonic matrix models. We consider non-critical strings and the pure spinor covariant quantization of superstrings, and review the studies of black holes in string theory, including the entropy calculation, higher curvature corrections and black hole topology in space-time dimension higher than four. Finally, we outline direct and indirect possible experimental signatures of string theory.

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1. Introduction

The current description of nature is based on two amazingly successful theories: the Standard Model theory of particle interactions and General Relativity theory of gravity. The Standard Model is a quantum theory, while general relativity is a classical theory. Performing an expansion in $\hbar$ in order to quantize General Relativity meets infinities that cannot be controlled: the theory is not renormalizable. What is the energy scale where quantum gravity is relevant?

The four-dimensional gravitational Newton coupling is $G_N = l_p^2$, where $l_p = 1.6 \times 10^{-33} cm$ is the Planck length. The corresponding energy scale is the Planck mass $M_p = l_p^{-1} = 1.2 \times 10^{19} GeV$. The dimensionless effective gravity coupling is $g_{eff} = G_N E^2$, where E is the energy scale. Therefore, the effects of gravity grow as the energy increases. At the Planck scale energy, gravity will have the strength of the order of the Standard Model interactions. Therefore, the traditional quantum gravity scale has been taken to be the Planck energy $M_{Quantum\ Gravity} \sim M_p$. We will see later how this discussion can be modified to allow lower quantum gravity scales.

This extremely high Planck scale energy raises the question, whether a theory of quantum gravity can ever be tested experimentally. One can say that a most important experiment has already been done, namely the Big-Bang creation of the universe. Classical gravity is not an adequate framework to discuss the physics of that event.

As a particle physics analogy of the above discussion, consider the physics of the weak interactions. In the Fermi theory of weak interactions, where four leptons interact at a point, the dimension two Fermi coupling G_F is analogous to the dimension two gravitational Newton coupling G_N. The dimensionless effective coupling is $g_{eff} = G_F E^2$, and as in gravity the interaction strength grows as energy increases. At the energy scale of the W-boson mass M_W, the effective coupling is strong and, as for gravity, we meet divergences in perturbation theory. These divergences have a physical origin: they signal the breakdown of the effective theory that we use in order to describe the weak interactions, and point to new physics. The divergences are resolved by introducing additional degrees of freedom, the weak interaction gauge bosons, leading to the Standard Model description. The lesson we would like to draw from this analogy is that the difficulty in quantizing gravity is not a technical one. As for the weak interactions, the divergences in the perturbative quantum computations signal new physics with additional degrees of freedom. Our task is to identify these new degrees of freedom.

What are the challenges that we face when trying to construct a complete theory of the forces of nature? First, we wish to have a consistent theory of quantum gravity: a theory that reduces to General Relativity at low energies, and where quantum computations can be made to any required order of accuracy. The theory should explain fundamental issues of quantum gravity, such as the microscopic origin of black holes entropy. Second, the theory has to incorporate the Standard Model and should therefore contain at low energies gauge fields and chiral matter. Third, the theory should explain the big bang singularity and its resolution, and the theory should explain the Standard Model structure, the gauge groups and couplings, the three generations pattern and the values of the Standard Model parameters.

The candidate theory that will be discussed in this lecture is string (M) theory [8]. We will start the discussion with a brief string theory status report, with respect to the challenges posed above. First, string theory is a consistent perturbative theory of quantum gravity, and as far as we know also non-perturbatively. It does explain the microscopic origin of black holes entropy for certain charged black holes that allow a controlled computation. Second, string theory incorporates the Standard Model. A large number of string models that include the standard model in addition to other degrees of freedom have been constructed. As to the third challenge, string theory has not yet explained neither the big bang singularity nor the particular structure of the Standard Model, as we will discuss in more detail in the next section.

The lecture is organized as follows. In the section 2 we review the basics of string theory. We will consider both the perturbative and non-perturbative aspects of the theory, and discuss the lowering of the string, quantum gravity and compactification scales in brane-world scenarios. In section 3 we will present the correspondence between gauge theories and string theory on curved spaces. We will consider in some detail holography and its realization via the AdS/CFT correspondence. We will discuss the computation of gluon scattering amplitudes using a topological string and the relation between four-dimensional supersymmetric gauge theories and bosonic matrix models. We will present the non-critical strings and the pure spinor covariant quantization of superstrings. In section 4 we will consider black holes in string theory. We will discuss the black hole entropy, the higher curvature corrections to the black hole free energy and the rich structure of classical black holes in space-time dimensions higher than four. In section 5 we will discuss possible direct and indirect experimental signatures of string theory. At the end of the lecture we list several reviews for further reading on the topics discussed in the lecture.

2. Basics of String Theory

In string theory the point particle is replaced by a vibrating string. The string oscillates and this gives particles with masses being the energy of the oscillations. The string interaction allows splitting and joining of strings. There are two important comments to be made on the string interaction vertex. First, unlike ordinary quantum field theory, where we need to specify the interactions when constructing the field theory action, in string theory this is dictated by the theory. Second, unlike point particle interactions which occur at a point in space-time, string theory interactions are not point-like. This smearing of the interaction point is the reason behind the resolution of the high-energy divergences of the quantum computations.

We should note also that the mathematics of string theory is very different from the mathematics of point particles. While in the latter we use the mathematics based on the concept of a point, in the former we need a mathematics based on the concept of a loop. In physical terms, a string probes the world in a very different way from the way that a point-like particle probes it [4].

The string has a tension $T_s = \frac{1}{l_s^2}$, where l_s is called the string length. A very important open problem is to determine the value of the string scale l_s. The traditional viewpoint, based on perturbative string analysis, has been that $l_s \sim l_p \sim 10^{-33} cm$.

The spectrum of oscillations of the string include massless and massive modes. The massless oscillations include the spin two graviton (closed string), the photon (open string) and other particles. The massive oscillations correspond to massive particles with mass of the order of the string scale $m \sim M_s = l_s^{-1}$.

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At low energy $E \ll M_s$ we see only the massless excitations of the string. Their interaction low-energy effective action can be computed from string theory and reads schematically

$$S_{Low\ \ energy} \sim \int \sqrt{g}(\mathscr{R} + F_{\mu\nu}F^{\mu\nu} + ...) \ , \tag{2.1}$$

where $\mathscr{R}$ is the Ricci scalar curvature and $F_{\mu\nu}$ is the gauge field strength. The first term in (2.1) gives General Relativity, while the second term gives the electromagnetic and Yang-Mills interactions. Thus, string theory includes correctly both the gravitational and gauge interactions. Here it is important to stress that these interactions are not put in by hand, but are derived from string theory.

In addition, string theory includes at tree level higher curvature corrections $(l_s^2\mathscr{R})^n$. These additional higher curvature corrections, allow the resolution of certain (time-like) singularities of space-time. Thus, while classical general relativity and point particle physics are singular on certain backgrounds, the physics of string theory is regular. However, whether the physics of string theory on big-bang type singularities, such as FRW universe, is smooth when these higher curvature corrections are included is not yet known.

The consistency of string theory (the absence of the conformal anomaly) implies extra dimensions, in addition to the $3+1$ space-time dimensions. Such extra dimensions have not been observed yet, and this raises the question what is their scale R_c?

The traditional viewpoint based on perturbative string theory, which will be reexamined later in the talk, has been that $R_c \sim l_p \sim 10^{-33} cm$. Thus, when the universe expanded from a point, while $3+1$ dimensions opened up, there have been other dimensions that remained curled-up and compact. In this scenario, the properties of the compact internal space determine the low energy data in the $(3+1)$-dimensional universe, such as the spectrum of particles and their interactions. We should note, however, that in string theory the notion of dimension is a derived one, determined by the string, and moreover is not a good one at the quantum level of gravity. What this means, as we shall see later, is that we can write descriptions of the same quantum system, using different number of dimensions.

Let us introduce now the notion of supersymmetry. Supersymmetry is a symmetry relating bosons to fermions: every fermion has a bosonic superpartner and vice versa. Thus, the quarks have squarks partners, the gluons have gluinos partners and the Higgs has the Higgsino. Why is this mathematical structure relevant?[1]

The standard arguments in favor of supersymmetric nature are the gauge hierarchy and the unification of gauge couplings. The former issue is raised by asking why the characteristic energy scale of the Standard Model $M_{SM} \sim M_W$ is much smaller than the characteristic energy scale of gravity M_p. This question is deeply rooted in the structure of quantum field theories, which we invoke in order to describe the forces of nature. In the quantum field theory framework it is practically impossible for us to keep such a hierarchy of scales when we include radiative corrections. This can be done, however, in a supersymmetric quantum field theory, due to the delicate cancellation between the radiative corrections of bosons and fermions. As to the unification of couplings, one finds a remarkable accuracy with which supersymmetry unifies the couplings of the electro-

[1] See R. Rattazzi's talk in this conference.

magnetic, the weak and the strong forces at around $10^{16}GeV$. What does string theory have to say about supersymmetry?

The way we understand string theory, it requires supersymmetry for consistency, at least at some high energy scale. As discussed above, gravity and gauge interactions are built in string theory. We could have even said that gravity and gauge interactions are predicted by string theory, if historically string theory had been invented before General Relativity and gauge theory. In string theory supersymmetry is built in, and we can view this theoretically on equal footing as the existence of gravity and gauge interactions. Since supersymmetry has not been observed yet, we can consider this as a prediction of string theory. With supersymmetry strings become superstrings.

In ten dimensions superstring theory has one parameter: the string scale M_s. The string coupling is a modulus: the vacuum expectation value of a scalar field, the dilaton ϕ, $g_s = \langle e^{\phi} \rangle$. Compactification to four dimensions introduces other parameters (moduli) describing the volume and shape of the internal space. The moduli appear in four dimensions as massless scalars. Massless scalars have not been observed in nature, and much work is done on building mechanisms to lift the moduli, i.e. giving mass to the scalars. The introduction of fluxes in the internal space is one such working scenario.

At weak string coupling $g_s \ll 1$, the string scale M_s and the compactification scale R_c lie just below the Planck scale energy at $10^{18}GeV$, far beyond experiment. Model building at $g_s \ll 1$ is done, for instance, by a compactification on six dimensional Calabi-Yau spaces, which provides a rich class of supersymmetric models in four dimensions. One gets naturally GUT groups such as $SU(5)$, $SO(10)$ and E_6 and three generations of chiral fermions. However, one can build a very large number of models, with different gauge groups and matter content. There is no principle yet that selects the gauge groups and matter that we observe. A basic feature of these models is the existence of a large number of scalar fields, the moduli.

A natural question is whether superstring theory is unique, i.e. how many different consistent superstring theories can be constructed. Here there is an amazing result: there is just one theory. The different consistent superstring theories are related to each other by dualities [12]. An important duality is the S-duality which inverts the string coupling $g_s \to \frac{1}{g_s}$, namely when one string theory is weakly coupled, its S-dual theory is strongly coupled. It exchanges elementary string states (particles) with string solitonic objects (monopoles). Moreover, if we consider the type IIA superstring theory in ten dimensions at strong coupling, we find that its weakly coupled S-dual theory is eleven-dimensional. Thus, when going to strong coupling a new dimension is opened up. We can describe the same physics using two theories, one in ten dimensions and the other in eleven dimensions. The eleven dimensional theory is called M-theory. At low energies it is described by eleven-dimensional supergravity. This tells us that the concept of dimension may not be a good one when considering quantum gravity.

Compactification of the eleven-dimensional theory on seven-dimensional spaces, such as G_2 holonomy manifolds, allows model building at $g_s \gg 1$. Issues such as getting low-energy gauge groups and chiral matter as well as proton decay, doublet-triplet splitting and axions have been considered in these strong string coupling scenarios.

D-branes are objects in string theory, on which open strings can end [9]. Open string massless excitations include gauge fields. Closed string massless excitations include the graviton. The brane-world scenarios are constructed by assuming that we live on D-branes. In such scenarios, the

Standard Model particles (open strings) are confined to the branes, and the extra dimensions can be probed only by the closed string modes such as the graviton. The brane-world scenarios allow sub-millimeter compact dimensions, and predict deviations from the Newton gravitational force law, $F \sim \frac{1}{r^2}$. With D compact extra dimensions, the gravitational force law changes to $F \sim \frac{1}{r^{D+2}}$. Such deviations are currently being looked for experimentally. The brane-world scenarios allow also low string scales $TeV < M_s < 10^{18} Gev$, which can be potentially probed by the LHC.

The main obstacle to quantitative realistic scenarios is the supersymmetry breaking mechanism. The world that we see is not supersymmetric, so how is supersymmetry broken in nature? In brane-world scenarios, generic choice of compactification with fluxes break supersymmetry on the worldvolume of the D-branes, and the soft supersymmetry breaking terms are calculable. Another mechanism to break supersymmetry is to use branes-antibranes systems, where by antibranes we mean D-branes that are charged oppositely to D-branes. Generic branes-antibranes systems break supersymmetry. However, there is a tachyon in the open string spectrum, and we typically lack a controlled computational scheme.

Extra dimensions add a beautiful ingredient to the unification of forces. Since the gravitational coupling runs as a power with the energy, gravity can unify with the other forces by introducing extra dimensions which change this power.

Finally, a comment on the landscape of string vacua. The string equations have a large number of meta-stable de-Sitter type solutions describing different worlds. This raises the question how our world is picked? Recently, anthropic (environmental) arguments have been invoked, where some parameters, such as the cosmological constant, are assumed to be anthropically determined, allowing the prediction of others.

3. Gauge Fields and Strings

In this section we discuss some aspects of the relation between gauge theories and string theory.

3.1 The AdS/CFT Correspondence

One of the remarkable theoretical results of the recent years has been the discovery that superstring theory on certain curved space-times is equivalent to gauge theories in one dimension lower, living on the boundary of the curved space-time. This equivalence seems to provide an example of the Holographic Principle, according to which the number of degrees of freedom of a quantum theory of gravity in a region of space increases with the area enclosing it. Here we take the theory of quantum gravity to be superstring theory. This drastic reduction in the number of degrees of freedom of quantum gravity is in sharp contrast to local quantum field theories where the number of degrees of freedom is proportional to the volume of space-time. In the examples that we will discuss, the degrees of freedom of quantum gravity are the superstring degrees of freedom, and the holographic degrees of freedom on the boundary of the curved space-time are gauge theory degrees of freedom in flat space-time.

The AdS/CFT correspondence provides a large class of examples of the duality between superstring theory and gauge theories [1]. The curved background is asymptotically five-dimensional

Anti-de-Sitter (AdS) space. The four-dimensional boundary theory is a superconformal gauge theory.

The AdS metric can be written as

$$ds^2 = \frac{R_{AdS}^2}{r^2} dr^2 + \frac{r^2}{R_{AdS}^2}(-dt^2 + d\vec{x}^2) \ . \quad (3.1)$$

$(t, \vec{x})$ are the four-dimensional space-times coordinates, where the gauge field theory lives. The radial coordinate r corresponds to the gauge theory energy scale. Large r is the UV region, while small r is the IR. R_{AdS} is the radius of the anti-de-Sitter space.

Consider first the comparison of the superstring theory parameters and the gauge field theory parameters. On the string side there are two dimensionless expansion parameters: the string coupling g_s and the curvature expansion parameter $\frac{l_s}{R_{AdS}}$. There are two dimensionless expansion parameters for the gauge field theory: the Yang-Mills coupling g_{YM} and $\frac{1}{N_c}$, where by N_c we denote the number of colors. They are related by:

$$g_s = g_{YM}, \qquad \frac{l_s}{R_{AdS}} \sim (g_{YM}^2 N_c)^{-\frac{1}{4}} \ . \quad (3.2)$$

Consider the large N_c limit

$$N_c \to \infty, \qquad \lambda_{t\ Hooft} \equiv g_{YM}^2 N_c = Fixed \ . \quad (3.3)$$

On the string theory side, it means considering the tree level theory since we set the string coupling g_s to zero.

When $\lambda_{t\ Hooft} \gg 1$, we have $\frac{R_{AdS}}{l_s} \gg 1$, that is the radius of the AdS space is large in string units. This means that the curvature of the AdS space is small and the classical gravity approximation provides a good description. On the other hand, when $\lambda_{t\ Hooft} \ll 1$, we have $\frac{R_{AdS}}{l_s} \ll 1$, which means that the curvature of the AdS space is large in string units. Therefore, the classical gravity approximation breaks down. However, now the perturbative gauge theory description is a good one. We see that the duality is of the strong-weak duality type: when one set of degrees of freedom is strongly coupled, the dual set is weakly coupled and vice-versa. In particular, we can study the strong coupling regime of the gauge theory using classical gravity in five dimensions, while we can study strongly coupled gravity using perturbative Yang-Mills theory.

The AdS/CFT correspondence provides workable calculational tools. It associates to each gauge invariant local operator of the gauge theory a superstring excitation. For instance, the dilaton field ϕ in the string spectrum corresponds to the dimension four gauge invariant operator TrF^2. The generating functional of the correlation functions of the field theory is the string partition function with appropriate boundary conditions at infinity in the radial direction.

How do we distinguish on the superstring side of the duality, between conformal gauge theories and confining ones? The answer is that we use the same superstring theory and only change the curved background. For instance, if the AdS space is cut at a scale $r_0 \sim \Lambda_{QCD} R_{AdS}^2$, then while at energy scales higher than r_0, the gauge theory is scale invariant, the IR physics at energy scales lower than r_0 is drastically modified. Various backgrounds describing QCD-like theories have been constructed. They exhibit the confinement of quarks via a linear potential

$$V_{q\bar{q}} \sim \sigma L \ , \quad (3.4)$$

and a mass gap

$$\langle TrF^2(x)TrF^2(0)\rangle \sim exp(-m|x|) \ . \tag{3.5}$$

One should note, that although superstring scattering amplitudes at high energy are soft due to the string extended nature, in the warped space geometry one gets the hard scattering (power law) behavior of glueballs at high energy. Also, one can see on the superstring side the confinement-deconfinement phase transition of the gauge theory. It is simply the fact that there are two five-dimensional curved geometries, where one dominates the superstring (gravity) partition function at low temperature and the other dominates at high temperature.

While a large number of supersymmetric gauge theories have been studied at strong coupling using classical gravity, we still lack a good control on the analysis of non-supersymmetric gauge theories. In particular, dual string descriptions of QCD or pure Yang-Mills have not been constructed yet.

3.2 Perturbative QCD and Topological Strings

Another type of relation between superstring theory and gauge theories involves a truncated version of superstrings called topological strings. On the gauge theory side, gluons scattering amplitudes have remarkable properties when written in terms of spinor variables [2]. For example, the tree level scattering amplitudes that are maximally helicity violating (MHV) can be expressed in terms of a simple holomorphic or antiholomorphic function. To interpret these results, it has been suggested to Fourier transform the scattering amplitudes from momentum space to twistor space. The transformed amplitudes are supported on certain holomorphic curves, which suggests a topological string description. It has been argued that the perturbative expansion of $\mathcal{N}=4$ super Yang-Mills theory is equivalent to the D-instanton expansion of a topological string whose target space is a Calabi-Yau supermanifold.

At tree level, this analysis holds for n gluons scattering in non-supersymmetric pure Yang-Mills theory, which are of phenomenological importance. For instance, for $n=4$, the nonzero tree level amplitude is the MHV amplitude with helicities $++--$ (or its permutations). For $n=5$, the nonzero amplitudes are MHV amplitudes such as $++---$ or $+++--$. These amplitudes dominate two-jet and three-jet production in hadron colliders at very high energies. Perturbative gluon amplitudes can be computed via a dual description of topological string theory on twistor space. This has provided simplified computational schemes and tree level recursion relations [3]. In addition it inspires the computation and search for recursion relations of loop amplitudes. Of much importance it the search for a possible relation between the AdS/CFT correspondence and the type of duality discussed here. There are two fundamental differences: while the AdS/CFT duality is of the strong-weak type, here both theories are weakly coupled. In addition, while in AdS/CFT the computation is of off-shell correlation functions, here the computations involve on-shell amplitudes. Still, one expects a contact between the two dualities, since on-shell gluon loop amplitudes provide information on anomalous dimensions of twist operators, which are computed using the AdS/CFT correspondence. An important underlying property is expected to be an integrable structure at the planar large N_C limit of the gauge theory.

[2] See Lance's Dixon talk in this conference.

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3.3 Supersymmetric Gauge Theories and Matrix Models

Another relation between gauge theories and string theory is based on topological strings, and field theoretic arguments which imply that the (holomorphic) F-terms of a large class of strongly coupled confining $\mathcal{N}=1$ supersymmetric gauge theories can be computed exactly by a large N computation in a bosonic matrix model [7]. The assumption is that the relevant fields in the IR are the glueball superfields S_i and the framework provides a means of computing their exact effective superpotential. This is done by evaluating the planar diagrams of the matrix model. The generic glueball superpotential is a sum of logarithmic superpotential terms and an infinite perturbative sum in the S_i. Even if the matrix model is not solvable, one can still compute the superpotential to arbitrary power of S_i by evaluating matrix model diagrams. Further development has been the observation that the loop equations for the matrix model associated with the $\mathcal{N}=1$ gauge theory are equivalent to the generalized Konishi anomaly equations. This allows the study of $\mathcal{N}=1$ gauge theories with chiral matter. When considering the supersymmetric gauge theory in a gravitational background one has corrections to the effective superpotential ,which can be computed either via generalized Konishi anomaly equations, or summing up non-planar diagrams in the matrix model. An essential ingredient in the analysis is the existence of a supersymmetric vacuum. An important question is whether there is a way to generalize the discussion when supersymmetry is broken. The analysis can be generalized, in the case when there is a holomorphic parameter whose tuning can move the theory from a non-supersymmetric vacuum to a supersymmetric one. It is still not clear whether this is the only possible generalization.

3.4 Non-critical Superstrings

The critical dimension for superstrings in flat space-time is determined by the vanishing of the conformal anomaly to be $d=10$. In dimensions $d<10$, the conformal mode of the worldsheet metric (Liouville mode) φ is dynamical and needs to be quantized as well [11]. This is analogous to the longitudinal mode of the gauge boson, which becomes dynamical when radiative corrections break the gauge symmetry. Such strings are called non-critical, and their quantum consistency is an important issue. The total conformal anomaly vanishes for the non-critical strings due to the Liouville background charge. Non-critical strings may provide an alternative to string compactifications and may provide a dual description of four-dimensional gauge theories such as QCD. Examples of backgrounds one wishes to study are

$$ds^2 = d\varphi^2 + a^2(\varphi) d\vec{x}^2 \ , \tag{3.6}$$

where $\vec{x}=(x_1,...,x_{d-1})$, and with other background fields turned on. String theory on such warped backgrounds is expected to provide a dual description of gauge theories. Depending on the form of the warp factor $a^2(\varphi)$, the gauge theory can be confining, or at a conformal fixed point.

A complication in the study of non-critical superstrings is that unlike the critical case, there is no consistent approximation where supergravity provides a valid effective description. The reason being that the d-dimensional supergravity low-energy effective action contains a cosmological constant type term of the form

$$S \sim \int d^d x \sqrt{G} e^{-2\Phi} \left(\frac{d-10}{l_s^2} \right) \ , \tag{3.7}$$

which vanishes only for $d = 10$. This implies that the low energy approximation $E \ll l_s^{-1}$ is not valid when $d \neq 10$, and the higher order curvature terms of the form $\left(l_s^2 \mathscr{R}\right)^n$ cannot be discarded. A manifestation of this is that solutions of the d-dimensional supergravity equations have typically curvatures of the order of the string scale $l_s^2 \mathscr{R} \sim O(1)$ when $d \neq 10$.

Another complication is that interesting target space curved geometries include Ramond-Ramond (RR) field fluxes, and we face the need to quantize the strings in such backgrounds. A simple non-critical string background that can be studied is the linear dilaton. This is a flat space (in the string frame), with a dilaton scalar field that is linear in a radial coordinate. Such a background can be studied in the conventional Ramond-Neveu-Schwarz (RNS) formalism and the resulting theory is consistent. A framework to study curved geometries that include RR fields fluxes, is a target space covariant formulation of non-critical superstrings, as we will discuss next. Its applications are a subject of current investigations.

3.5 Pure Spinor Superstrings

The standard RNS description of the superstring has various shortcomings. First, it does not exhibit manifestly the super-Poincaré space-time symmetry. Second, the calculation of loop scattering amplitudes is difficult due to the need to sum over spin structures and the required integration over the supermoduli. Indeed, so far computations have been done only up to the two-loop order. In addition, vanishing theorems are difficult to prove. Another shortcoming of the RNS formalism, as we noted above, is that it does not treat the RR fields and the NSNS fields on an equal footing, due to the non-polynomial coupling of the Ramond- Ramond (RR) fields to the spin fields of the CFT. Therefore, studying string backgrounds with RR fields turned on is extremely difficult in the RNS formalism.

An alternative scheme to define superstrings is the pure spinor formalism [2]. In this framework the variables are the Grassmann-odd $\theta^\alpha, \alpha = 1, \ldots, 16$ superspace coordinates, their conjugate momenta p_α, a Grassmann-even spinor ghost field λ^α and its conjugate w_α. The requirement for the vanishing of the conformal anomaly, while having the space-time Lorentz symmetry, implies that the ghost field λ^α is constrained by

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0 \ , m = 0, ..., 9 \ . \tag{3.8}$$

This set of equations defines a pure spinor.

Physical states in the pure spinor formalism are defined as super-Poincaré covariant ghost-number $+1$ states in the cohomology of the nilpotent BRST operator

$$Q_B = \oint \lambda^\alpha d_\alpha \ , \tag{3.9}$$

where d_α is the superspace covariant derivative.

The low-energy supergravity action emerges from the pure spinor construction in a manifestly covariant space-time supersymmetric manner. Scattering amplitudes up to the 2-loop order have been computed and agree with the RNS formalism. In addition, some vanishing theorems have been proved in the pure spinor formalism using zero modes counting. In curved spaces, quantum consistency of string theory on backgrounds such as $AdS_5 \times S^5$ has been studied. Detailed analysis of the excitation spectrum in $AdS_5 \times S^5$ has not been performed yet. More work is needed in order for the pure spinor scheme to become a simple workable tool.

4. Black Holes in String Theory

In classical General Relativity black holes have a horizon. It is a surface in space-time, which when crossed does not allow a return.

4.1 Black Hole Entropy

Due to quantum effects, black holes emit thermal radiation. This has raised the questions: what are the internal constituents of the black holes that explain this temperature, and what is the microscopic origin of the Bekenstein-Hawking entropy associated with black holes

$$S_{BH} = \frac{A_H}{4G_N} , \tag{4.1}$$

where A_H is the area of the horizon.

Since superstring theory is proposed as a framework to quantum gravity, one expects that it will provide answers to these questions. Indeed, one finds that for certain charged black holes, D-brane states are the internal constituents. In the limit of large coupling one has the black hole geometry, while in the limit of small coupling one can count the D-brane states, using the D-brane field theory. One of the important results of string theory concerning quantum gravity is that for a large class of extremal and near-extremal black holes, calculations of the macroscopic thermodynamic (Bekenstein-Hawking) entropy and the microscopic statistical entropy have been performed and agree.

4.2 Higher Curvature Corrections

In string theory one adds higher curvature corrections to Einstein's gravity. Thus, one would like to know the implications of these terms to the quantum black hole physics. The question can be dealt in a precise way for a certain class of charged black holes. These are supersymmetric black holes in four dimensions that carry electric and magnetic charges. They are solutions of $\mathcal{N} = 2$ supergravity, which is the effective low energy theory of superstrings compactified on six-dimensional manifolds called Calabi-Yau spaces. These solutions are asymptotically flat black holes, with $AdS_2 \times S^2$ near horizon geometry. An important proposal concerning the free energy of these black holes has been made [7].

According to the proposal, the partition function of the black holes Z_{BH} is related to the partition function of topological strings Z_{top} by $Z_{BH} = |Z_{top}|^2$. Z_{BH} is evaluated as a function of magnetic charges and electric potentials, while Z_{top} is evaluated as a function of Calabi-Yau moduli which are determined at the horizon by the magnetic charges and electric potentials via the attractor equation. Z_{BH} and Z_{top} are defined by a perturbative expansion in $1/Q$, where Q is the charge of the black hole. The proposal agrees with the calculations of Wald's generalized entropy formula, using the Lagrangian of $\mathcal{N} = 2$ supergravity with R^2 terms [6].

It is of importance to ask whether higher curvature corrections to the entropy of non-supersymmetric black holes such as the near extremal $\mathcal{N} = 2$ black holes can be analyzed in both a macroscopic and microscopic descriptions. An important ingredient in the analysis is the attractor equation that determines the moduli on the horizon in terms of the charges, independently of their values at infinity. This is not the case for general non-supersymmetric black holes, but it is correct, at least to leading order in the curvature, when the black hole horizon is $AdS_2 \times S^2$. These issues are currently being studied.

4.3 Black Holes in Higher Dimensions

Black holes in more than four space-time dimensions exhibit a rich structure. For instance, while in four dimensions the topology of the event horizon of an asymptotically flat stationary black hole is uniquely determined to be the two-sphere S^2 [5], this is not the case in higher dimensions. Hawking's theorem requires the integrated Ricci scalar curvature with respect to the induced metric on the event horizon to be positive. This condition applied to two-dimensional manifolds determines uniquely the topology. However, while this condition applies also in higher dimensions it is much less powerful.

The classification of the topology of the event horizons in higher dimensions is more complicated. For instance, for five-dimensional asymptotically flat stationary black holes, in addition to the known S^3 topology of event horizons, stationary black hole solutions with event horizons of $S^2 \times S^1$ topology (Black Rings) have been constructed.

The classification of the topology of event horizons in higher dimensions has been studied. While in five dimensions event horizons have been found to be S^3 and $S^2 \times S^1$, in six and higher dimensions there seems to be an even richer structure. For instance, in six dimensions, the requirement that the horizon is cobordant to a four-sphere (topological censorship), implies that simply connected event horizons are homeomorphic to S^4 or $S^2 \times S^2$. A complete classification is still lacking.

5. Discussion: Experimental Signatures of Strings

As we discussed in the lecture, strings are very different objects than point particles. However, they have not been observed yet. In the following we will consider possible experimental signatures for strings.

- Supersymmetry and the mechanism of supersymmetry breaking: in superstring theory, supersymmetry is an essential ingredient. However, as far as the superstrings are concerned, supersymmetry can be present at a very high scale compared to the LHC's energy regime at the TeV. Also, even if supersymmetry is found in LHC, this will not confirm that superstrings are the correct description of nature. However, since supersymmetry is natural in superstring theory, it will certainly boost the attempts to build the UV completion of the low-energy supersymmetric theory via superstrings.

- Detection of string scalar fields: there are many scalar fields in generic superstring compactifications to four dimensions, representing the shape and size of the internal dimensions. Seeing such scalars experimentally will provide important informations, but will not uniquely determine superstrings as the correct building blocks of nature.

- Magnetic monopoles: magnetic monopoles (of the 't Hooft Polyakov type) are natural objects in string theory and have played an important role in the duality picture. Their mass can vary a lot depending on the scenario. In particular, low string scale scenarios can have

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monopoles at the TeV. So far no magnetic monopoles have been observed. Seeing such magnetic monopoles experimentally will provide important informations, but will not uniquely determine superstrings as the correct degrees of freedom.

- Strings in the sky: the development of strongly coupled string theory and the discovery of D-branes to which open strings are attached, allows the construction of models with strings that may be detected in the sky [10]. This may provide a direct evidence for string theory.

- Extra dimensions: string theory predicts extra dimensions. Their size and shape depends on the scenario. Compact dimensions may have a size in the range from sub-millimeter to the Planck scale. One can also have wrapped non-compact dimensions. Having extra dimensions does not prove that string theory is the right framework. However, strings behave differently than point particles in background of extra dimensions, and have unique signatures.

- Low scale strings: it is not excluded theoretically and experimentally that strings can have a string scale as low as the TeV. However, there is no known evidence that the TeV scale is preferred. If indeed strings will show up at the TeV, we will verify that strings are playing a role in the building blocks of nature. On the other hand, we will face the question whether string theory is in fact just an effective theory, and when we will go to higher energy scales, other fundamental degrees of freedom will reveal themselves.

- Low scale quantum gravity: it is possible that the quantum gravity scale is lower than the traditional Planck scale, for instance a the TeV regime. In such theoretical scenarios, quantum gravity effects such as the production of quantum black holes are important. If realized, we will have a laboratory to examine the quantum nature of gravity and analyze whether string theory is the right theory of quantum gravity, or maybe there exists another one.

We finish the lecture by saying that an obvious theoretical signature for superstrings is the construction of a superstring model that reproduces all the low energy (Standard Model and cosmology) data.

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