PROCEEDINGS OF SCIENCE

Polyakov loop at low and high temperatures

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We describe how the coupling of the gluonic Polyakov loop to quarks solves different inconsistencies in the standard treatment of chiral quark models at finite temperature at the one quark loop level. Large gauge invariance is incorporated and an effective theory of quarks and Polyakov loops as basic degrees of freedom is generated. From this analysis we find a strong suppression of finite temperature effects in hadronic observables below the deconfinement phase transition triggered by approximate triality conservation in a phase where chiral symmetry is spontaneously broken (Polyakov cooling). We also propose a simple phenomenological model to describe the available lattice data for the renormalized Polyakov loop in the deconfinement phase. Our analysis shows that non perturbative contributions driven by dimension-2 gluon condensates dominate the behaviour of the Polyakov loop in the regime $T_c < T < 6T_c$.

29th Johns Hopkins Workshop on current problems in particle theory: strong matter in the heavens
1-3 August
Budapest

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PoS(JHW2005)025

1. Introduction

Pure gluodynamics formulated using the Imaginary Time Formalism of Finite Temperature Field Theory has an extra discrete glogal symmetry $\mathbb{Z}(N_c)$, which is the center of the usual gauge group SU(N_c). A natural order parameter for the transition from the confining phase, where $\mathbb{Z}(N_c)$ is preserved, to the deconfining phase, where this symmetry is spontaneously broken, is the traced Polyakov loop (for a comprehensive review see e.g. [1]), defined by

$$L(T) = \langle \mathrm{tr}_c\, \Omega(x) \rangle = \left\langle \frac{1}{N_c} \mathrm{tr}_c\, \mathbf{P} \left(e^{ig \int_0^{1/T} dx_0 A_0(\mathbf{x},x_0)} \right) \right\rangle , \tag{1.1}$$

where $\langle\ \rangle$ denotes vacuum expectation value, tr_c is the (fundamental) color trace, and $\mathbf{P}$ denotes path ordering. A_0 is the gluon field in the (Euclidean) time direction. There have been many efforts in studying the Polyakov loop. A perturbative evaluation of the Polyakov loop was carried out long ago [2] at high temperatures. An update of these calculations was presented by us in [3]. Different renormalization procedures have been proposed on the lattice simulations more recently [4, 5].

In full QCD, i.e. with dynamical fermions, the Polyakov loop appears to be an approximate order parameter, as lattice simulations suggest [6]. This may look a bit puzzling since the center symmetry is largely broken for current quarks. However, as discussed in [7] in the context of chiral quark models, the relevant scale stemming from the fermion determinant is in fact the constituent quark mass, generated by spontaneous chiral symmetry breaking. Thus, one expects large violations of the center symmetry to correlate with chiral symmetry restoration.

We have analyzed the role of large gauge symmetry in a similar framework [8] yielding a unique way of coupling the polyakov loop to effective constituent quarks. In Ref. [9], we have also proposed a model to describe the available lattice data for the renormalized Polyakov loop in terms of the dimension-2 gluon condensate [10, 11]. This model also describes consistently the lattice results for the free energy [12].

2. Large gauge transformations

In the Matsubara formalism of Quantum Field Theory at Finite Temperature the space-time manifold becomes a topological cylinder. In principle, only periodic gauge transformacions are acceptable since the quark and gluon fields are stable under these transformations:

$$g(\vec{x},x_0) = g(\vec{x},x_0+\beta)\,, \tag{2.1}$$

where $\beta = 1/T$. In the Polyakov gauge, where $\partial_0 A_0 = 0$, A_0 is a diagonal and traceless $N_c \times N_c$ matrix. Let us consider for instance the following periodic gauge transformation

$$g(x_0) = e^{i2\pi x_0 \Lambda/\beta}\,, \tag{2.2}$$

where Λ is a color traceless diagonal matrix of integers. Note that it cannot be considered to be close to the identity, and in that sense we call it a large gauge transformation. The gauge transformation on the A_0 component of the gluon field is

$$A_0 \to A_0 + \frac{2\pi}{\beta}\Lambda\,, \tag{2.3}$$

and so gauge invariance manifests as the periodicity of the diagonal amplitudes of A_0 of period $2\pi/\beta$. This invariance is manifestly broken in perturbation theory, since a periodic function is approximated by a polynomial. Nevertheless, we can implement this large gauge symmetry by considering the Polyakov loop or untraced Wilson line as an independent degree of freedom, $\Omega(x)$, which transforms covariantly at x

$$\Omega(x) \to g^{-1}(x)\Omega(x)g(x)\,, \tag{2.4}$$

and, in the Polyakov gauge, $\Omega(x) = e^{i\beta A_0(\vec{x})}$, it becomes gauge invariant.

Fermions break the center symmetry of the gauge group, which is present in all the pure gauge theories. That means that we can only consider periodic gauge transformations (see Eq. (2.1)). In pure gluodynamics at finite temperature one can smooth this condition, and consider aperiodic gauge transformations:

$$g(\vec{x}, x_0 + \beta) = zg(\vec{x}, x_0)\,, \qquad z^{N_c} = 1\,. \tag{2.5}$$

Note that z is not an arbitrary phase but an element of $\mathbb{Z}(N_c)$. An example of such a transformation in the Polyakov gauge es given by

$$g(x_0) = e^{i2\pi x_0 \Lambda / N_c \beta}\,, \tag{2.6}$$

for which $z = e^{i2\pi/N_c}$. The corresponding gauge transformation of the A_0 field and the Polyakov loop Ω is

$$A_0 \to A_0 + \frac{2\pi}{N_c \beta}\Lambda\,, \qquad \Omega \to z\Omega\,. \tag{2.7}$$

We observe that Ω transforms as the fundamental representation of the $\mathbb{Z}(N_c)$ group. From Eq. (2.7) we deduce that $\langle\Omega\rangle = z\langle\Omega\rangle$ and hence $\langle\Omega\rangle = 0$ in the center symmetric or confining phase. More generally, in this phase

$$\langle\Omega^n\rangle = 0 \qquad \text{for} \qquad n \neq mN_c\,, \tag{2.8}$$

with m an arbitrary integer. Obviously, in full QCD the fermion determinant changes the selection rule Eq. (2.8). We will see in section 4 that this violation is large for massless quarks and the usefulness of the center symmetry becomes doubtful.

3. Problems with Chiral Quark Models at finite temperature

The standard treatment of Chiral Quark Models at Finite Temperature presents some inconsistencies. To illustrate this point we will use the Nambu–Jona-Lasinio model. In the Matsubara formalism we have the standard rule to pass from $T = 0$ formulas to $T \neq 0$,

$$\int \frac{dk_0}{2\pi} F(\vec{k}, k_0) \to iT \sum_{n=-\infty}^{\infty} F(\vec{k}, i\omega_n)\,, \tag{3.1}$$

where $\omega_n = 2\pi T(n + 1/2)$ are the fermionic Matsubara frecuencies. Using this rule the chiral condensate at finite temperature at the one loop level is given by

$$\langle\overline{q}q\rangle = 4MT\,\mathrm{tr}_c \sum_{\omega_n} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \vec{k}^2 + M^2}\,, \tag{3.2}$$

where M is the constituent quark mass. Doing the integral and after a Poisson resummation, we have the low temperature behaviour

$$\begin{aligned} \langle \overline{q}q \rangle_T &= \langle \overline{q}q \rangle_{T=0} - 2\frac{N_c M^2 T}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} K_1(nM/T) \\ &\stackrel{\text{Low T}}{\sim} \langle \overline{q}q \rangle_{T=0} - \frac{N_c}{2} \sum_{n=1}^{\infty} (-1)^n \left(\frac{2MT}{n\pi}\right)^{3/2} e^{-nM/T} , \end{aligned} \tag{3.3}$$

where we have used the asymptotic form of the Bessel function $K_n(z)$. This formula can be interpreted in terms of the quark propagator in coordinate space

$$S(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{\not k - M} = (i \not\partial + M) \frac{M^2}{4\pi^2 i} \frac{K_1(\sqrt{-M^2x^2})}{\sqrt{-M^2x^2}} , \tag{3.4}$$

so that at low temperature we get

$$S(\vec{x}, i\beta) \stackrel{\text{Low T}}{\sim} e^{-M/T} , \tag{3.5}$$

which represents the exponential suppression for a single quark at low T. This means that we can write the quark condensate in terms of statistical Boltzmann factors with mass $M_n = nM$. This is a problem since it means that the heat bath is made out of free constituent quarks without any color clustering. The calculation can be extended to any observables which are color singlets in the zero temperature limit, and the general result is that quark models calculations at finite temperature in the one loop aproximation generate all possible quark states, i.e.

$$\mathscr{O}^T = \mathscr{O}^{T=0} + \mathscr{O}_q e^{-M/T} + \mathscr{O}_{qq} e^{-2M/T} + \dots . \tag{3.6}$$

Note that while the term $\mathscr{O}_q$ corresponds to a single quark state, the next term $\mathscr{O}_{qq}$ must be a qq diquark state, corresponding to a single quark line looping twice around the thermal cylinder. It cannot be a $\overline{q}q$ meson state because at one loop this state comes from the quark like going upwards and then downwards in imaginary time, so that the path does not wind around the thermal cylinder and then it is already counted in the zero temperature term $\mathscr{O}^{T=0}$. From Eq. (3.3) we obtain

$$\langle \overline{q}q \rangle_T = \sum_{n=-\infty}^{\infty} (-1)^n \langle \overline{q}(x_0) q(0) \rangle |_{x_0 = in\beta} . \tag{3.7}$$

Note that the zero temperature contribution corresponds to the term $n = 0$ in the sum. Under a gauge transformation of the central type we have $\overline{q}(n\beta)q(0) \to z^{-n}\overline{q}(n\beta)q(0)$. This means that Eq. (3.7) is not gauge invariant, and the quark condensate can be decomposed as a sum of irreducible representations of a given triality n.

Another problem comes from comparison with Chiral Perturbation Theory (ChPT) at Finite Temperature. In the chiral limit the leading thermal corrections to the quark condensate for $N_f = 2$, for instance, are given by

$$\langle \overline{q}q \rangle_T |_{\text{ChPT}} = \langle \overline{q}q \rangle_{T=0} \left(1 - \frac{T^2}{8 f_\pi^2} - \frac{T^4}{384 f_\pi^4} + \cdots \right) . \tag{3.8}$$

Thus, the finite temperature correction is N_c-suppressed as compared to the zero temperature value, since f_π^2 scales as N_c. This feature contradicts our result of Eq. (3.7) obtained by using the standard finite temperature treatment of chiral quark models, from which we deduce that in the large N_c limit there is a finite temperature correction $\sim N_c e^{-M/T}$. To obtain the ChPT result of Eq. (3.8) pion loops have to be considered [13] and dominate for $T \ll M$. The problem is that already without pion loops chiral quark models predict a chiral phase transition at about $T_c \sim 170$ MeV, in remarkable but perhaps unjustified agreement with lattice calculations.

4. The Polyakov loop Chiral Quark Model

We can formally keep track of large gauge invariance by coupling gluons to the model in a minimal way. In the Nambu–Jona-Lasinio model the effective action takes the form

$$\Gamma_{\rm NJL}[S,P] = -iN_c \mathrm{Tr}\log(i\mathbf{D}) - \frac{1}{4G}\int d^4x\, \mathrm{tr}_f\,(S^2+P^2)\,, \tag{4.1}$$

where the Dirac operator is given by

$$i\mathbf{D} = i\,\partial\!\!\!/ - \hat{M}_0 + (S + i\gamma^5 P)\,. \tag{4.2}$$

It is obtained after using the standard bosonization procedure [14] with the introduction of auxiliary bosonic fields (S,P) and after formally integrating out the quarks. The chiral quark model coupled to the Polyakov loop corresponds to simply make the replacement

$$\partial_0 \to \partial_0 - iA_0 \tag{4.3}$$

in the Dirac operator (4.2). As we said in section 2, a perturbative treatment of the A_0 component of gluon field manifestly breaks gauge invariance at finite temperature, and we need to consider the Polyakov loop as an independent degree of freedom. It appears naturally in any finite temperature calculation in the presence of minimally coupled vector fields within a derivative or heat kernel expansion [3, 15]. Our approach is similar to that of [7], except that there a global Polyakov loop is suggested in analogy with the chemical potential. Instead we consider a local Polyakov loop $\Omega(\vec{x})$ coupled to the quarks [8]. From those calculations we deduce the rule of Eq. (3.1), but with the modified fermionic Matsubara frequencies

$$\hat{\omega}_n = 2\pi T(n+1/2+\hat{\nu})\,, \qquad \hat{\nu} = (2\pi i)^{-1}\log\Omega\,, \tag{4.4}$$

which are shifted by the logarithm of the Polyakov loop $\Omega = e^{i2\pi\hat{\nu}}$, i.e. $\hat{\nu}(\vec{x}) = A_0(\vec{x})/(2\pi T)$. The effect of such a shift over a finite temperature fermionic propagator starting and ending at the same point is

$$\tilde{F}(x;x) \to \sum_{n=-\infty}^{\infty} (-\Omega(\vec{x}))^n \tilde{F}(\vec{x},x_0+in\beta;\vec{x},x_0)\,, \tag{4.5}$$

instead of the $(-1)^n$ factor obtained from the standard rule Eq. (3.1) after using Poisson's summation formula and Fourier transformation. [1]

[1] This formula can be interpreted saying that in a quark loop at finite temperature, the quarks pick up a phase (-1) due to Fermi-Dirac statistics, and a non Abelian Aharonov-Bohm factor Ω each time the quarks wind once around the compactified thermal cylinder.

If we integrate over the A_0 gluon field of Eq. (4.3) in a gauge invariant manner [16], this yields a partition function for the quiral quark model of the form

$$Z = \int dU d\Omega\, e^{i\Gamma_G[\Omega]} e^{i\Gamma_Q[U,\Omega]}\,, \tag{4.6}$$

where U is the nonlinearly transforming pion field, dU is the Haar measure of the chiral flavour group $SU(N_f)_R \times SU(N_f)_L$ and $d\Omega$ the Haar measure of the colour group SU(N_c), Γ_G is the effective gluon action whereas Γ_Q stands for the quark effective action. If the gluonic measure is left out $A_0 = 0$ and $\Omega = 1$ we recover the original form of the corresponding chiral quark model, where there exists a one-to-one mapping between the loop expansion and the large N_c expansion both at zero and finite temperature. Equivalently one can make a saddle point approximation and corrections thereof. In the presence of the Polyakov loop such a correspondence does not hold, and we proceed by a quark loop expansion, i.e. a saddle point approximation in the bosonic field U, keeping the integration over the Polyakov loop Ω. This integration must be done according to the QCD dynamics and will restore gauge invariance. Effectively this implies an average over the local Polyakov loop with some normalized weight $\sigma(\Omega;\vec{x})d\Omega$. Here $\sigma(\Omega;\vec{x})$ is the (temperature dependent) probability distribution of $\Omega(\vec{x})$ in the gauge group. For a general function $f(\Omega)$, meaning a ordinary funcion $f(z)$ evaluated at $z = \Omega$, we have

$$\left\langle \frac{1}{N_c} \mathrm{tr}_c f(\Omega) \right\rangle = \int_{\mathrm{SU}(N_c)} d\Omega\, \sigma(\Omega) \frac{1}{N_c} \sum_{j=1}^{N_c} f(e^{i\phi_j}) = \int_0^{2\pi} \frac{d\phi}{2\pi} \hat{\sigma}(\phi) f(e^{i\phi})\,, \tag{4.7}$$

where $e^{i\phi_j}$, $j = 1,\ldots,N_c$ are the eigenvalues of Ω and

$$\hat{\sigma}(\phi) := \int_{\mathrm{SU}(N_c)} d\Omega\, \sigma(\Omega) \frac{1}{N_c} \sum_{j=1}^{N_c} 2\pi\delta(\phi - \phi_j)\,. \tag{4.8}$$

By applying this formalism to the quark condensate, we deduce

$$\langle \overline{q}q \rangle_T = \sum_{n=-\infty}^{\infty} \frac{1}{N_c} \langle \mathrm{tr}_c(-\Omega)^n \rangle \langle \overline{q}(x_0) q(0) \rangle|_{x_0 = in\beta}\,. \tag{4.9}$$

From Eq. (2.8) we observe that in the confining phase and in quenched approximation triality is preserved, so that after gluon average Eq. (4.5) becomes

$$\tilde{F}(x;x) \to \sum_{n=-\infty}^{\infty} \langle (-\Omega(\vec{x}))^{nN_c} \rangle \tilde{F}(\vec{x}, x_0 + inN_c\beta; \vec{x}, x_0)\,. \tag{4.10}$$

At sufficiently low temperature the distribution of the Polyakov loop becomes just the Haar measure, and one can easily deduce the following result

$$\langle \mathrm{tr}_c(-\Omega)^n \rangle_{\mathrm{SU}(N_c)} = \begin{cases} N_c\,, & n = 0 \qquad (4.11) \\ -1\,, & n = \pm N_c \qquad (4.12) \\ 0\,, & \text{otherwise} \qquad (4.13) \end{cases}\,.$$

Taking into account this formula in Eq. (4.9), we observe that the inclusion of the Polyakov loop not only removes the triality breaking terms, but also the thermal contributions are N_c suppressed as compared to the zero temperature value, as is expected from ChPT (see Eq. (3.8)). The quark condensate at finite temperature at one loop level and in quenched approximation is

$$\langle \overline{q}q \rangle_T = \langle \overline{q}q \rangle_{T=0} + \frac{2M^2T}{\pi^2 N_c} K_1(N_c M/T) + \cdots \overset{\text{Low T}}{\sim} \langle \overline{q}q \rangle_{T=0} + 4\left(\frac{MT}{2\pi N_c}\right)^{3/2} e^{-N_c M/T} . \tag{4.14}$$

The dots indicate higher gluonic or sea quark effects. Due to the exponential suppression, the leading thermal corrections at one quark loop level start only at temperatures near the deconfinement phase transition. We have named this effect *Polyakov cooling* [8], because it is triggered by a group averaging of Polyakov loops. This means that in the quenched approximation we do not expect any important finite temperature effect on quark observables below the deconfinement transition, and the biggest change should come from pseudoscalar loops at low temperatures. This is precisely what one expects from ChPT.

We have also studied in Ref. [8] the low energy effective chiral Lagrangian deduced from different constituent quark models at one loop level. This is a local object, and after a derivative expansion it takes the form

$$\mathscr{L}(x) = \sum_n \mathrm{tr}\left[f_n(\Omega(x)) \mathscr{O}_n(x) \right] , \tag{4.15}$$

where tr acs on all internal degrees of freedom, n labels all possible local gauge invariant operators $\mathscr{O}_n(x)$ (i.e. containing covariant derivatives), and $f_n(\Omega(x))$ are temperature dependent functions of the Polyakov loop which replace the numerical coefficients present in the zero temperature case. We focus only on the Polyakov loop with color degrees of freedom, and forget the requirement of a *chiral flavor Polyakov loop* to maintain large flavour symmetry at finite temperature. The full calculation of the low energy constants up tu order $\mathscr{O}(p^4)$ in quenched approximation shows that they become functions of temperature and the Polyakov loop, and after integration of gluons by using Eq. (4.7) they have the same strong suppression at low temperatures observed in Eq. (4.14) $L_i^T - L_i^{T=0} \overset{\text{Low T}}{\sim} e^{-N_c M/T}$.

In order to go beyond the quenched approximation, we will consider the computation of the fermion determinant in the presence of a slowly varying Polyakov loop following the techniques developed in [15]. Such an approximation makes sense in a confining region where there are very strong correlations between Polyakov loops. The fermion determinant can be written as

$$\mathrm{Det}(i\,\not{D} - M) = e^{-\int d^4x \mathscr{L}(x,\Omega)} , \tag{4.16}$$

where $\mathscr{L}$ is the chiral Lagrangian as a function of the Polyakov loop which has been computed at finite temperature in Ref. [8] in chiral quark models. Using this we can estimate the Polyakov loop [2]

$$L = \frac{1}{N_c} \frac{\langle \mathrm{tr}_c \Omega(x)\, \mathrm{Det}(i\,\not{D} - M) \rangle}{\langle \mathrm{Det}(i\,\not{D} - M) \rangle} \overset{\text{Low T}}{\sim} c \frac{8\pi T^2 B}{N_c^2 \sigma^3} e^{-M/T} , \tag{4.17}$$

where B is the vacuum energy density, σ is the string tension and c is a numerical factor which depends on the model. Note that triality is not preserved due to the presence of dynamical quarks, but the relevant scale is the constituent quark mass. So the Polyakov loop can be effectively used as an order parameter. In Fig. 1 we confront such an exponential suppression with unquenched lattice

[2] The integration can be easily computed by using the formula $\langle \mathrm{tr}_c \Omega(x)\; \mathrm{tr}_c \Omega^{-1}(y) \rangle = e^{-\sigma|x-y|/T}$.

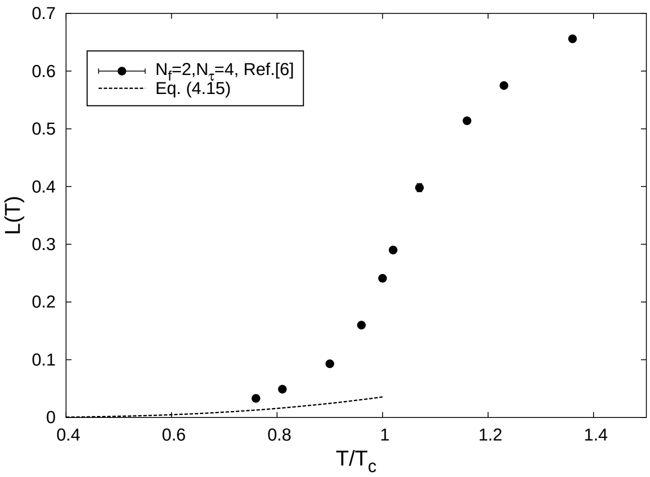

Figure 1: Temperature dependence of the renormalized Polyakov loop in units of the critical temperature. Lattice data correspond to 2-flavor QCD, and has been taken from [6]. The line represents our estimation of the Polyakov loop in the low temperature regime, using $c = 3$ as a suitable value for this model-dependent parameter.

calculations below the phase transition. We observe that Eq. (4.17) could be a good approximation below $0.6T_c$. In any case, lattice data for lower temperatures are desirable to do a more precise analysis.

For the quark condensate we take into account the result of Eq. (4.9), so that

$$\langle \overline{q}q \rangle_T = \frac{\langle \overline{q}q \,\mathrm{Det}(i\,\not{D} - M)\rangle}{\langle \mathrm{Det}(i\,\not{D} - M)\rangle} \overset{\text{Low T}}{\sim} \langle \overline{q}q \rangle_{T=0} \times \left(1 + c' \frac{8\pi T^2 B}{N_c^2 \sigma^3} e^{-2M/T} \right) , \tag{4.18}$$

where again c' depends on the particular model. In other chiral quark models, similar results are obtained by replacing $2M \to M_V$ (the ρ meson mass) [8]. The Polyakov cooling persists although is a bit less effective, and for instance the temperature dependence of the low energy constants of the effective chiral Lagrangian becomes $L_i^T - L_i^{T=0} \overset{\text{Low T}}{\sim} e^{-M_V/T}$.

Finally, on top of this one must include higher quark loops, or equivalently mesonic excitations. They yield exactly the results of ChPT [13] and for massless pions dominate at low temperatures. Thus, we see that when suitably coupled to chiral quark models the Polyakov loop provides a quite natural explanation of results found long ago on purely hadronic grounds.

5. Polyakov loop above T_c

In this section we focus on the behaviour of the Polyakov loop in the deconfining phase. In that phase chiral symmetry is restored and the degrees of freedom are quarks and gluons. A perturbative evaluation of the Polyakov loop was carried out in [2] in pure gluodynamics to NLO, which corresponds to $\mathscr{O}(g^4)$ in the Landau gauge. In the Polyakov gauge Eq. (1.1) becomes

$$L(T) = \frac{1}{N_c} \left\langle \mathrm{tr}_c e^{igA_0(\vec{x})/T} \right\rangle = 1 - \frac{g^2}{2T^2} \frac{1}{N_c} \langle \mathrm{tr}_c(A_0^2) \rangle + \frac{g^4}{24T^4} \frac{1}{N_c} \langle \mathrm{tr}_c(A_0^4) \rangle + \cdots , \tag{5.1}$$

where we have considered a series expansion in the gluon field.[3] To describe the dynamics of the $A_0(\mathbf{x})$ field we use the 3-dimensional reduced effective theory of QCD, obtained from the Euclidean QCD action by integrating the non stationary Matsubara gluon modes and the quarks [3, 17]. Let $D_{00}(\mathbf{k})\delta_{ab}$ denote the 3-dimensional propagator for the gluon field, then[4]

$$\langle A_{0,a}^2 \rangle = (N_c^2-1)T \int \frac{d^3k}{(2\pi)^3} D_{00}(\vec{k})\,. \tag{5.2}$$

To lowest order in perturbation theory the propagator becomes $D_{00}^P(\vec{k}) = 1/(\vec{k}^2+m_D^2)$, where m_D is the Debye mass, which to one loop [18] writes $m_D = gT(N_c/3+N_f/6)^{1/2}$. We can compute $\langle A_0^2\rangle$ and $\langle A_0^4\rangle$ by taking derivatives of the vacuum energy density of the 3-dimensional theory, already computed to four loops in [19]. The contribution from $g^2\langle A_0^4\rangle$ starts at $\mathscr{O}(g^6)$, while that of $g^2\langle A_0^2\rangle$ starts at $\mathscr{O}(g^3)$. So, the replacement of Eq. (5.1) with

$$L(T) = \exp\left[-\frac{g^2\langle A_{0,a}^2\rangle}{4N_cT^2}\right] \tag{5.3}$$

becomes correct up to $\mathscr{O}(g^5)$. This gaussian ansatz is exact in the large N_c limit. We obtain

$$\langle A_{0,a}^2\rangle^{\mathrm{P}} = -\frac{N_c^2-1}{4\pi} m_D T - \frac{N_c(N_c^2-1)}{8\pi^2} g^2T^2 \left(\log\frac{m_D}{2T} + \frac{3}{4}\right) + \mathscr{O}(g^3)\,. \tag{5.4}$$

This result can be deduced in two forms: by derivating the vacuum energy density [19], or by identifying Eq. (5.3) with the perturbative result of Ref. [2]. Note that this formula holds also in the unquenched theory, since to this order, N_f only appears through the Debye mass. The perturbative contributions to the Polyakov loop at $\mathscr{O}(g^3)$ and $\mathscr{O}(g^4)$ have been displayed in Fig. 2 and compared to the lattice data of the renormalized Polyakov loop of Ref. [4]. It has a rather flat logarithmic dependence with temperature and only seems to reproduce these data for the highest temperature value $6T_c$. The results of Ref. [19] would provide the $\mathscr{O}(g^5)$ and $\mathscr{O}(g^6)$ terms. Unfortunately, this perturbative result is obtained in covariant gauges, generating a spurious gauge dependence beyond $\mathscr{O}(g^4)$. In any case, numerically these terms do not make a substantial contribution as they are qualitatively and quantitatively similar to those in Ref. [2].

It is clear that perturbation theory cannot explain by itself lattice data, and we propose to account for non perturbative contributions coming from condensates. We consider adding to the propagator new phenomenological pieces driven by positive mass dimension parameters:

$$D_{00}^{\mathrm{NP}}(\vec{k}) = \frac{m_G^2}{(\vec{k}^2+m_D^2)^2}\,. \tag{5.5}$$

This piece produces a non perturbative contribution to the gluon condensate, namely, $\langle A_{0,a}^2\rangle^{NP} = (N_c^2-1)Tm_G^2/(8\pi m_D)$. If we assume that m_G is temperature independent, the condensate will

[3] Note that the odd order terms vanish due to the conjugation symmetry of QCD, $A_\mu(x) \to -A_\mu^T(x)$.

[4] We consider $A_0 = \sum_a T_a A_{0,a}$, with the standard normalization for the Hermitian generators of the SU(N_c) Lie algebra in the fundamental representation, $\mathrm{tr}(T_aT_b) = \delta_{ab}/2$.

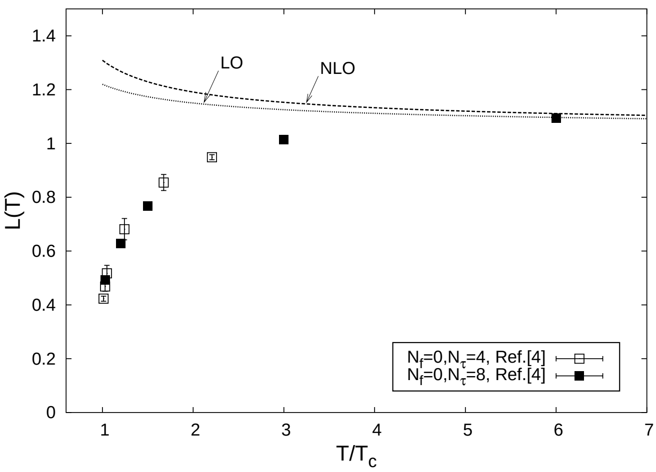

Figure 2: Temperature dependence of the renormalized Polyakov loop in units of the critical temperature. Lattice data correspond to quenched QCD, and have been taken from [4]. We plot the prediction of perturbation theory at LO $\mathscr{O}(g^3)$ and up to NLO $\mathscr{O}(g^4)$ in pure gluodynamics [2].

also be temperature independent, modulo radiative corrections. Adding the perturbative and non perturbative contributions, we get for the Polyakov loop

$$-2\log L = \frac{g^2\langle A_{0,a}^2\rangle^{\mathrm{P}}}{2N_cT^2} + \frac{g^2\langle A_{0,a}^2\rangle^{\mathrm{NP}}}{2N_cT^2}\,, \tag{5.6}$$

where $\langle A_{0,a}^2\rangle^{\mathrm{P}}$ is given by Eq. (5.4). Note that, module radiative corrections, $\langle A_{0,a}^2\rangle^{\mathrm{P}}$ scales as T^2 while $\langle A_{0,a}^2\rangle^{\mathrm{NP}}$ is temperature independent. We will rewrite this formula as

$$-2\log L = a + b\left(\frac{T_c}{T}\right)^2\,, \tag{5.7}$$

where the parameters a and b are expected to have only a weak temperature dependence.

The lattice data for $-2\log L$ versus $(T_c/T)^2$ are displayed in Fig. 3. The nearly straight line behaviour is clear, which means the unequivocal existence of a temperature power correction driven by a dimension 2 gluon condensate. If we fi t the lattice data by using the perturbative value of a up to NLO, i.e. $\mathscr{O}(g^4)$, one obtains:

$$b = \begin{cases} 2.16(4)\,, \\ 2.99(12)\,, \end{cases} \qquad g^2\langle A_{0,a}^2\rangle^{\mathrm{NP}} = \begin{cases} (0.97(1)\ \mathrm{GeV})^2\,, & N_f = 0 \\ (0.86(2)\ \mathrm{GeV})^2\,, & N_f = 2 \end{cases}\,, \tag{5.8}$$

with $\chi^2/\mathrm{DOF} = 1.05, 1.87$ respectively. A fi t of the lattice data with a treated as a free parameter gives

$$a = \begin{cases} -0.23(1)\,, \\ -0.31(6)\,, \end{cases} \quad b = \begin{cases} 1.72(5)\,, \\ 2.19(13)\,, \end{cases} \quad g^2\langle A_{0,a}^2\rangle^{\mathrm{NP}} = \begin{cases} (0.87(2)\ \mathrm{GeV})^2\,, & N_f = 0 \\ (0.73(3)\ \mathrm{GeV})^2\,, & N_f = 2 \end{cases}\,, \tag{5.9}$$

with $\chi^2/\mathrm{DOF} = 0.80, 0.25$ respectively. In our fi ts we include lattice data for temperatures above $1.03T_c$ in the $N_f = 0$ case and above $1.15T_c$ in the $N_f = 2$ case.

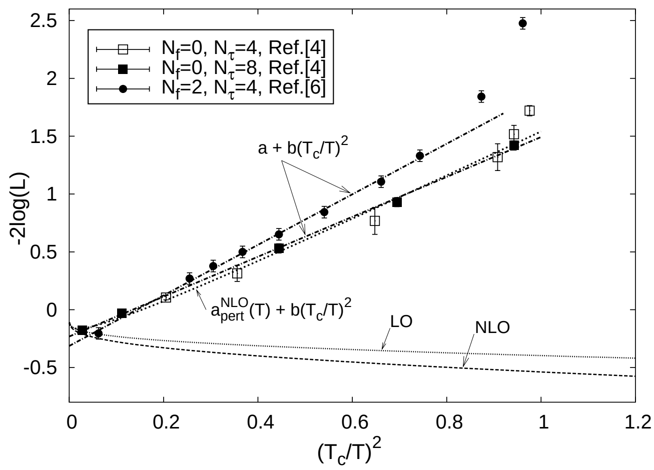

Figure 3: The logarithmic dependence of the renormalized Polyakov loop versus the inverse temperature squared. Lattice data from [4, 6]. Fits with *a* adjustable constant and predicted by NLO perturbation theory are displayed. Purely perturbative LO and NLO results for $N_f = 0$ are shown for comparison.

Recent analyses of the heavy quark free energy with the model proposed in Eq. (5.5) (see [12]) suggest the possibility that α_s at finite temperature has a smoother behaviour than the predicted by perturbation theory in the interval $T_c < T < 6T_c$. This is in contrast with existing analyses [6, 20], where the authors find a very large value for α_s in this regime.

We can compare our result for the gluon condensate with finite temperature determinations based on the study of non perturbative contributions to the pressure in pure gluodynamics [21]. These results yield for the gluon condensate $(0.93 \pm 0.07\ \text{GeV})^2$ in the temperature region used in our fit and in Landau gauge, which is in good agreement with Eqs. (5.8) and (5.9). We also can compare with zero temperature determinations of the gluon condensate $g^2\langle A^2_{\mu,a}\rangle$ in the Landau gauge and in quenched QCD. From the gluon propagator $(2.4 \pm 0.6\ \text{GeV})^2$ [22] and from the quark propagator $(2.1 \pm 0.1\ \text{GeV})^2$ [11]. [5] We observe a remarkable agreement, taking into account that these results refer to different temperatures and gauges.

Acknowledgments

This work is supported in part by funds provided by the Spanish DGI and FEDER funds with Grant No. BFM2002-03218, Junta de Andalucía Grant No. FQM-225 and EU RTN Contract No. CT-2002-0311(EURIDICE).

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