

Magnetic properties of quark matter and compact stars

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A microscopic origin of magnetic field in quark matter is discussed. Manifestation of ferromagnetism in quark matter is suggested with the one-gluon-exchange interaction or an effective interaction. Magnetic field due to ferromagnetic quark matter amounts to $O(10^{15-17}G)$, which is enough to magnetars.

29th Johns Hopkins Worhshop on current problems in particle theory: strong matter in the heavens 1-3 August Budapest

*Speaker.

[†]This work is supported by the Japanese Grant-in-Aid for Scientifi c Research Fund of the Ministry of Education, Culture, Sports, Science and Technology (13640282,16540246). It is also partially supported by the Grant-in-Aid for the 21th Century COE "Center for the Diversity and Universality in Physics" from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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1. Introduction

Nowadays the phase diagram of QCD in the temperature (T)-density (ρ) plane has been explored by many people. Here we are concentrated on the magnetic aspect of quark matter [1]. In Fig. 1 we depict some magnetic phases in QCD: it is well-known that pion condensation (PIC) exhibits a specific magnetic property in hadron matter [2], and we may expect ferromagnetism (FM) [3] and spin density wave (SDW) [4] in quark matter, while their properties have not been well understood.



Figure 1: Phase diagram in the temperature-density plane.

Our interest on the magnetic properties of quark matter has been stimulated by recent discovery of magnetars, which have the huge magnetic fi eld of $O(10^{15})$ G [5]. They have been fi rst discovered by the $P - \dot{P}$ diagram (Fig. 2); assuming the dipole radiation for pulsars we can estimate their ages and magnetic fi eld strength by observing their periods (*P*) and its time derivatives (\dot{P}). Then we can see three groups of pulsars in the $P - \dot{P}$ plane with different ages and magnetic fi eld strength *B*. Besides the usual radio pulsars with $B = O(10^{12})$ G and the millisecond pulsars with $B = 10^9$ G, we can see a new class of pulsars with $B = 10^{15}$ G. Recently, some cyclotron absorption lines have been observed in some magneters, which may also suggest the huge magnetic fi eld of 10^{15} G. Unfortunately there is still ambiguity about identifi cation of particles responsible to the absorption lines, but if they are confirmed they give a direct evidence of the huge magnetic fi eld.

The origin of such strong magnetic fields has been a long standing problem since the first discovery of a pulsar in early seventies. A naive working hypothesis tells that the strong magnetic field in pulsars are generated by squeezing the magnetic flux during the stellar evolution from the main sequence stars. This hypothesis looks to work well for usual radio pulsars with the radius R = 10km, by taking the sun with $R = 10^{10}$ cm and $B = 10^3$ G as a typical main-sequence star. However, it may break down once we try to explain the magnetic field of 10^{15} G for magnetars: the radius of a magnetar has to be O(100m), which is much shorter than the Schwartzschild radius of O(1km) for a canonical mass of $O(1M_{\odot})$. Thus the discovery of magnetars looks to give a chance to reconsider the origin of magnetic field in compact stars.

It would be interesting to consider a microscopic origin, since there are widely developed hadron matter inside compact stars and the strong-interaction energy scale of O(1MeV) is rather large in comparison with the magnetic interaction energy $E_{\text{int}} = \mu B$ with the magnetic moment μ even for $B = 10^{15}$ G.¹ The spontaneous spin polarization or ferromagnetism is a candidate in

¹In a recent paper, Makishima also suggested a hadronic origin of the magnetic field for binary X-ray pulsars [7].





Figure 2: $P - \dot{P}$ diagram for pulsars, taken from a review article[6].

hadron matter. There have been repeatedly done many calculations of the polarized nuclear matter with realistic nuclear forces, but they have always given negative results (see recent results [8]). These results look rather robust and we may not expect spontaneous spin polarization in nuclear matter. In the following we shall consider a possibility of ferromagnetism in quark matter.

2. Ferromagnetism in quark matter

We discuss a possibility of ferromagnetism in quark matter in analogy with the itinerant electron gas [3]. For ferromagnetism of itinerant electrons, Bloch suggested a mechanism where the Fock exchange interaction induces spontaneous spin polarization [9]. Consider the spin polarized electron gas interacting by the Coulomb interaction in the background of positively charged ions. Since the direct interaction gives no contribution due to the charge neutrality, the Fock exchange interaction gives the leading contribution as the interaction energy. It favors spin alignment to effectively avoid the Coulomb repulsion due to the Pauli principle, while the kinetic energy is increased due consequently. Hence, if the interaction effect exceeds the kinetic energy increase, we can expect spontaneous ferromagnetism. Recently his idea has been positively proved experimentally[10]. Then one may ask whether the similar mechanism also works for quark matter.

2.1 One-gluon exchange (OGE) interaction

We begin with an OGE action:

$$I_{int} = -g^2 \frac{1}{2} \int d^4x \int d^4y \left[\bar{\psi}(x) \gamma^{\mu} \frac{\lambda_a}{2} \psi(x) \right] D_{\mu\nu}(x,y) \left[\bar{\psi}(y) \gamma^{\nu} \frac{\lambda_a}{2} \psi(y) \right],$$
(2.1)

where $D^{\mu\nu}$ denotes the gluon propagator.

Consider a free quark with momentum k. Since the spin operator Σ is not commutable with the Dirac operator $H_D = \not{k} - m$, we first have to define spin polarization in a covariant way. We can do this by using the Pauli-Lubanski vector W^{μ} , $W^{\mu} = -1/4\varepsilon_{\mu\nu\rho\sigma}k^{\nu}\sigma^{\rho\sigma}$, for a particle with momentum k and the spin vector a^{μ} ; the former is a generalization of the Pauli matrix σ and the latter is a generalization of the quantization axis ζ in non-relativistic theories. For any space-like vector a^{μ} satisfying two constraints,

$$a \cdot k = 0, a^2 = -1, \tag{2.2}$$

we can define the projection operator P(a) on each spin polarization $\zeta = \pm 1$,

$$P(a) = (1 + \gamma^5 \phi)/2. \tag{2.3}$$

We can easily see that $[P(a), H_D] = 0$ and $P^2 = P$. Thus we can construct the eigen-spinor $u^{(\zeta)}(k)$ s.t. $P(a)u^{(\zeta)}(k) = \zeta u^{(\zeta)}(k)$ with $\zeta = \pm 1$, which is simultaneously the energy eigenstate, $H_{\text{free}}u^{(\zeta)}(k) = E_k u^{(\zeta)}(k)$ with $H_{\text{free}} = \alpha \cdot \mathbf{k} + \beta m$ and $E_k = \sqrt{m^2 + \mathbf{k}^2}$. As a consequence of this definition, we are aware of many choices of the spin vector to describe spin polarization in the relativistic way. Here we restrict ourselves to a class of the spin vectors, which is reduced to ζ in the rest frame of each particle. Thus we specify two degrees of freedom of spin polarization by using a vector ζ . There is no way to determine the relevant one a priori. The most favorable form of the spin vector over all the particles should be determined by the energy principle, but it would be very difficult to solve it in a general way. Instead, we can assume some forms by physical considerations. A general form of a^{μ} can be written as

$$a^{\mu}(k) = c_1(k^2, (k \cdot \zeta)^2, n \cdot k)(k \cdot \zeta)k^{\mu} + c_2(k^2, (k \cdot \zeta)^2, n \cdot k)\zeta^{\mu} + c_3(k^2, (k \cdot \zeta)^2, n \cdot k)(k \cdot \zeta)n^{\mu}$$
(2.4)

where $\zeta^{\mu} = (0, \zeta)$ and $n^{\mu} = (1, 0)$. The coefficient functions q should be determined under the constraints (2.2). We, hereafter, consider two special forms: one is

$$a^{0} = \frac{\mathbf{k} \cdot \boldsymbol{\zeta}}{m}, \mathbf{a} = \boldsymbol{\zeta} + \frac{\mathbf{k}(\boldsymbol{\zeta} \cdot \mathbf{k})}{m(E_{k} + m)},$$
(2.5)

and the other is

$$a^{0} = \frac{E_{k}(\zeta \cdot \mathbf{k})}{m\sqrt{(\zeta \cdot \mathbf{k})^{2} + m^{2}}}, \mathbf{a} = \frac{m^{2}\zeta + (\zeta \cdot \mathbf{k})\mathbf{k}}{m\sqrt{(\zeta \cdot \mathbf{k})^{2} + m^{2}}}.$$
(2.6)

The first one is given by the Lorentz transformation from the rest frame, and the second one maximizes the mean-value of the spin operator [11]. We can see that the maximal-spin choice (2.6) plays an important role for the effective zero-range interaction [11].

With the spin vector thus defined, we can write down the OGE interaction. Since quark matter is color neutral, only the Fock exchange interaction gives a leading order contribution in the Feynman gauge,

$$f_{\mathbf{k},\zeta;\mathbf{q},\zeta'}^{\text{Fock}} \equiv \frac{1}{N_c^2} \frac{1}{N_f} \sum_{c,d} f_{\mathbf{k},\zeta,c;\mathbf{q},\zeta',d} = g^2 \frac{N_c^2 - 1}{4N_c^2 N_f E_k E_q} \left[2m^2 - k \cdot q - m^2 a \cdot b \right] \frac{1}{(k-q)^2},$$
(2.7)

where N_c and N_f denotes the number of colors and flavors, respectively, and $f_{\mathbf{k},\zeta,c,i;\mathbf{q},\zeta',d,j} = \delta_{ij}f_{\mathbf{k},\zeta,c;\mathbf{q},\zeta',d}$ indicates the interaction between two quarks with momenta **k** and **q**, spins ζ and ζ' , colors *c* and *d* and flavors *i* and *j*. Note that the spin dependent term described by $a \cdot b$ in Eq. (2.7) implies that the interaction in the axial-vector channel is responsible to the spin polarization (c.f. §2.2). It implicitly depends on the vectors ζ, ζ' , and its form is given by the definite choice of the spin vector. On the other hand, we can easily see that the Fock exchange interaction is reduced to the simple form,

$$f_{\mathbf{k},\boldsymbol{\zeta};\mathbf{q},\boldsymbol{\zeta}'}^{\text{Fock}} = -g^2 \frac{N_c^2 - 1}{4N_c^2 N_f} \frac{1 + \boldsymbol{\zeta} \cdot \boldsymbol{\zeta}'}{|\mathbf{k} - \mathbf{q}|^2}$$
(2.8)

m_a=300 MeV, α_c=2.2

in the non-relativistic limit for any choice of the spin vector.

 m_q =300 MeV, α_c =2.2

244 0.025 δε 242 0.02 χŋ 240 0.015 $n_{a}=0.2 (fm^{-3})$ 238 0.01 Etot/ng(MeV) $(in m_a^4)$ 236 0.005 234 0 232 $n_q=0.1 (fm^{-3})$ -0.005 230 228 -0.01 226 -0.015 0.1 0.2 0.3 0.4 0.5 0.7 0.8 0.9 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.6 n_{α} (fm⁻³) р

Figure 3: Total energy is given as a function of the polarization parameter $p = (n^+ - n^-)/2n_q$. **Figure 4:** Critical lines as functions of quark-number density.

Then, the total energy of quark matter ε_{tot} is given as a function of the polarization parameter $p = (n^+ - n^-)/2n_q$ with particle numbers n^{\pm} for different polarizations. In Figs. 3 and 4 we demonstrate some results for the standard spin configuration (2.5) by using the MIT bag model parameters. Fig. 3 clearly shows the first order phase transition at low densities. Interestingly we can see a metamagnetic state even in the paramagnetic phase. The critical lines are depicted in Fig. 4 as the functions of density: $\delta\varepsilon$ denote the energy difference between p = 0 and p = 1 cases, and χ is inversely proportional to magnetic susceptibility s.t. $\delta\varepsilon = \chi^{-1}p^2 + O(p^4)$, which specifies the second order phase transition. Then the negative region of $\delta\varepsilon$ or χ indicates ferromagnetism. We can see that the critical densities given by both lines result in the similar value to each other, which implies the phase transition is *weakly first order*. The remaining line denoted by η shows the derivative of ε_{tot} at p = 1, $\eta = \partial \varepsilon_{tot}/\partial p|_{p=1}$. Its negative value suggests that the metamagnetic state exists in a wide density regime.

Here we have discussed only the lowest order contribution, but a recent paper has also suggest the phase transition at low densities by including the higher-order effects [12].



2.2 Self-consistent approach

By way of the mean-field approximation, we have the effective action instead of the original OGE action (2.1),

$$I_{MF} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \bar{\psi}(p) G_A^{-1}(p) \psi(p).$$
(2.9)

The inverse quark Green function $G_A^{-1}(p)$ involves various self-energy (mean-fi eld) terms, of which we only keep the color singlet particle-hole mean-fi eld V(p),

$$G_A(p)^{-1} = \not p - m + V(p).$$
(2.10)

Taking into account the lowest diagram, we can then write down the self-consistent equations for the mean-field, *V*:

$$-V(p) = (-ig)^2 \int \frac{\mathrm{d}^4 k}{i(2\pi)^4} \{-iD^{\mu\nu}(p-k)\} \underbrace{\gamma_\mu \frac{\lambda_\alpha}{2} \{-iG_A(k)\}\gamma_\nu \frac{\lambda_\alpha}{2}}_{(A)}.$$
 (2.11)

Applying the Fierz transformation for the OGE action (2.1) we can see that there appear the color-singlet scalar, pseudo-scalar, vector and axial-vector self-energies by the Fock exchange interaction. Taking the Feynman gauge for the gluon propagator, some manipulation gives

$$(A) = \frac{N_c^2 - 1}{4N_c^2} \frac{1}{N_f} \left\{ \operatorname{Tr}(G_A) + i\gamma_5 \operatorname{Tr}(G_A i\gamma_5) - \frac{1}{2} [\gamma^{\mu} \operatorname{Tr}(G_A \gamma_{\mu}) + \gamma_5 \gamma^{\mu} \operatorname{Tr}(G_A \gamma_5 \gamma_{\mu})] \right\} + \{ \text{color non-singlet or flavor non-singlet terms} \},$$
(2.12)

where we can see the correct prefactor $(N_c^2 - 1)/4N_c^2N_f$ (c.f. Eq. (2.7)). When we restrict the ground state to be an eigenstate with respect to color and flavor, there is only left the first term which is color singlet and flavor singlet. Still we must take into account various mean-fields in V, $V = U_s + i\gamma_5 U_{ps} + \gamma_\mu U_\nu^\mu + \gamma_\mu \gamma_5 U_{a\nu}^\mu$ with the mean-fields U_i . Here we only retain $\mathbf{U}_{a\nu} (\equiv \mathbf{U}_A)$ for simplicity and suppose that others to be vanished;

$$V(k) = \gamma \gamma_5 \cdot \mathbf{U}_A(\mathbf{k}), \tag{2.13}$$

with the static axial-vector mean-field $U_A(\mathbf{k})$.

The poles of $G_A(p)$, det $G_A^{-1}(p_0 + \mu = \varepsilon_n) = 0$, give the single-particle energy spectrum:

$$\varepsilon_n = \pm \varepsilon_{\pm}$$
 (2.14)

$$\boldsymbol{\varepsilon}_{\pm}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + \mathbf{U}_A^2(\mathbf{p}) + m^2 \pm 2\sqrt{m^2 \mathbf{U}_A^2(\mathbf{p}) + (\mathbf{p} \cdot \mathbf{U}_A(\mathbf{p}))^2}},$$
(2.15)

where the subscript in $\varepsilon_{\zeta}(\mathbf{p}), \zeta = \pm$ represents spin degrees of freedom, and the dissolution of the degeneracy corresponds to the *exchange splitting* of different "spin" states; the spectrum is reduced to a familiar form $\varepsilon_{\pm} \sim m + \frac{p^2}{2m} \pm |\mathbf{U}_A|$ in the non-relativistic limit. Note that the energy spectrum is no more rotation symmetric, and thereby Fermi seas are deformed (Fig.5). We have two Fermi seas with different shapes: the Fermi sea of the majority particle is deformed in the "prolate" shape, while that of the minority particle in the "oblate" shape [13]. It would be interesting to see that the



Figure 5: Modification of the Fermi sea as U_A (=const.) is increased from left to right. The larger Fermi sea (F^-) takes a prolate shape, while the smaller one (F^+) an oblate shape for a given U_A . In the large U_A limit (completely polarized case), F^+ disappears as in the right panel.

eigenspinors $u^{(\zeta)}(p)$, corresponding to ε_{ζ} , are reduced to those given by the maximal-spin choice (2.6) in the limit, $U_A \to 0$.

If we consider an effective model with the zero-range interaction,

$$H_{\rm int} = G^2 \bar{\psi} \gamma_\mu \gamma^5 \psi \bar{\psi} \gamma^\mu \gamma^5 \psi, \qquad (2.16)$$

the mean-fi eld U_A becomes momentum independent and the self-consistent equation becomes very simple,

$$U_A = 2G^2 \sum_{\zeta = \pm 1} \int \frac{d^3 p}{(2\pi)^3} \theta(\mu - \varepsilon_{\zeta}(\mathbf{p})) \frac{U_A + \zeta \sqrt{m^2 + p_z^2}}{\varepsilon_{\zeta}(\mathbf{p})} + \text{VAC}, \qquad (2.17)$$

where "VAC" means the contribution from the Dirac sea. The Dirac sea contribution is obviously divergent and we have to regularize it in a proper way. Since the energy spectrum have lost rotation symmetry, the usual momentum cut-off is not relevant. Instead, we must use the one which is not directly related to the form of the energy spectrum. We have used the proper time regularization with the cut-off Λ [4]. The critical density is easily obtained from Eq. (2.17). For an infi nitesimally small U_A and $m/\Lambda \ll 1$, the critical chemical potential is given as

$$\mu_c = \mu_c^F \Lambda / m e^{-\gamma_E/2} (\gamma_E : \text{Euler's constant}), \qquad (2.18)$$

where μ_c^F is the critical chemical potential given only by the Fermi seas,

$$\mu_{c}^{F} = \frac{m}{2} \exp\left(\frac{4\pi^{2}}{3N_{f}N_{c}G^{2}m^{2}}\right).$$
(2.19)

Here we can clearly see the non-perturbative nature with respect to the coupling constant G and quark mass m. We can also see that the vacuum contribution works against the spin polarization. Note that the critical chemical potential given by Eq. (2.18) should be overestimated, since the zero-range interaction is too simple; n ear the critical point the interaction effectively works only for particles near the Fermi surface and the vacuum contribution is suppressed by the Pauli principle.

Extension of the framework to the finite temperature case is straightforward. We show the phase diagram on the temperature-density plane in Fig. 5 for the three-flavor case under two conditions: the chemical equilibrium condition (CEC) $\mu_u = \mu_d = \mu_s$ and the charge neutral condition without electrons (CNC) $\rho_u = \rho_d = \rho_s$, where quark masses are taken as $m_u = m_d = 5$ MeV and



Figure 6: Phase diagram for two cases of strange quark masses, 250, 350MeV.

 $m_s = 150 - 350$ MeV, i.e., $\mu_s = \sqrt{\mu_{u,d}^2 + m_s^2 - m_{u,d}^2}$ for T = 0. In both conditions, since the spin polarization caused by the axial-vector mean-field is fully enhanced by the quark mass for given density or temperature, choice of the current quark mass seriously affects the results; especially, largeness of the strange quark mass has an essential effect on spin polarization.

3. Astrophysical implications

There are two possibilities about the existence of quark matter in compact stars: one is in the quark stars, which are composed of low density strange matter, and the other is in the core region of neutron stars, which are called hybrid stars. Consider magnetars as quark stars or hybrid stars. Then we can easily estimate their magnetic field near the surface, if ferromagnetism is realized in quark matter. The magnetic field generated on the surface r = R by quark matter with density n_Q can be written as

$$\mathbf{B}_{\text{surface}} = \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}}{R^3},\tag{3.1}$$

with the total magnetic moment **m**,

$$\mathbf{m} = \frac{4\pi r_Q^3}{3} \mathbf{M}, \mathbf{M} = \mu_Q \hat{\mathbf{z}}, \tag{3.2}$$

where r_Q is the radius of quark lump and μ_Q the single magnetic moment. We can see then that he maximum magnitude of **B**_{surface} amounts to $O(10^{15-17})$ G for $n_Q = 0.1$ fm⁻³, which might be enough for magnetars.

There may be left an interesting problem about hierarchy of magnetic field in compact stars (Table 1). However, the idea of ferromagnetism may not be sufficient for explaining it, and we need the global magnetic structure and some dynamical mechanisms, e.g. formation of magnetic domain or existence of metamagnetism, besides it.

We might also consider a scenario about the cosmological magnetic field in the galaxies and extra galaxies. It is well known that magnetic fields are present in all galaxies and galaxy clusters,

	millisecond pulsars	usual radio pulsars	magnetars
Magnetic fi eld [G]	109	10 ¹²	10 ¹⁵
Period [sec]	10^{-2}	10^{0}	10^{1}
Age [year]	10 ⁹	10 ⁶	10 ³

Table 1: Hierarchy of magnetic field in compact stars.

which are characterized by the strength, $10^{-7} - 10^{-5}$ G, with the spatial scale, ≤ 1 Mpc [14]. The origin of such magnetic fi elds is still unknown, but the first magnetic fi elds may have been created in the early universe. If magnetized quark lumps are generated during the QCD phase transition, they can give seed fi elds.

4. Summary and Concluding remarks

We have discussed a possibility of ferromagnetism in quark matter. The spin configuration in the relativistic ferromagnetism is not trivial, different from the non-relativistic case, because the spin operator cannot be commutable with the Dirac operator.

We have seen that ferromagnetism is induced by the one-gluon-exchange interaction at low densities through the first order phase transition. In the self-consistent treatment with the effective zero-range interaction ferromagnetism will be developed at high densities through the second order phase transition. These opposite features are originated from the difference of the interaction range. One may think ,e.g., the Debye screening in the gluon propagator and the zero-range effective interaction may be regarded as a limit case, like in the Stoner model. Anyway further study is needed about the range of the interaction. We may apply the Fermi liquid theory or take the renormalization group approach to study this problem.

One may wonder the relation of ferromagnetism to color superconductivity, since their possible density region is overlapped. We have discussed the coexistence of ferromagnetism with color superconductivity by assuming the *P*-wave type pairing [13]. We have found that they can coexist with little interference. However, it is not unique possibility. One may consider other interesting types of pairing in the ferromagnetic phase.

Besides ferromagnetism, there is another magnetic aspect of quark matter. We may expect spin density wave at moderate densities, where chiral symmetry is expected to be restored. In the recent paper we have discussed its possibility to see some region where spin density wave appears [4]. We have also suggested a hadron-quark continuity that magnetic properties in pion condensation are succeeded by spin density wave in quark matter.

These magnetic aspects should have some implications on compact star phenomena. As an example, we have suggested a microscopic origin of magnetic fi eld in compact stars by ferromagnetism of quark matter. We have roughly estimated the strength of the magnetic fi eld in compact stars by assuming the existence of the core of quark matter, and found that it may give enough strength even for magnetars. It would be ambitious to explain the hierarchy of the magnetic fi eld in compact fi eld in compact stars by using the idea of ferromagnetism in quark matter.

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