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Color screening in a constituent quark model of hadronic matter

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The nuclear medium is studied using a quark-based model wich properly incorporates clustering at low densities and deconfinement at high densities. First, we characterize the system by identifying the transition to quark matter and the strange quark content as a function of density. Then we compute the potential and wave function for a heavy quark-antiquark pair in this medium to study the color response of the system. It is shown that in the present model abrupt changes in the properties of the heavy meson correlate strongly with the confining phase transition and therefore confirming it as a robust signature.

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1. Introduction

The vast and rich phenomena of the strong interaction offers a window to the search for novel states of matter. States whose properties are very subtle and may need of different sets of observables for their proper identification. One of these is the so-called quark-gluon plasma (QGP), the deconfined state of quarks and gluons, predicted from QCD to be reached at high temperatures and/or high baryon densities[1, 2]. Devoted experiments have being established to produce such a state, from the earliest SPS at CERN and AGS at Brookhaven to RHIC and the currently under construction LHC, by colliding heavy ions at high energies and therefore exploring the high temperature regime [3]. On another hand, in Neutron Stars (NS), the high baryon density at the core is expected to have such a phase where the quarks may behave as a free gas, which could lead out to changes in the properties of the star, like the radius-mass relation. Moreover, the features of the system at high densities may be of such relevance that the nature itself of the star can be modified to allow a strange content [4, 5]. The so-called strange stars, whose mass may be of the same magnitude of a normal NS but with a radius of around 40 percent smaller.

One of the possible evidences for the formation of the QGP is the response of this state to a heavy quark-antiquark pair produced by primary collisions, where a suppression to the bound state formation is expected [6], although it has being also argued that statistical recombination may be partially offset [7] the suppresion. In the present work we are concern with the description and response of such system using a QCD-inspired model, whose main virtue is that it allows to study the evolution from low to high density without relaying in matching models with diferent degrees of freedom at the inter-phase, and therefore it provides important information on the clustering drop out upon the appearence of quark matter. Variational monte carlo techniques are employed to simulate the system behavior in a wide range of densities and therefore fixing the free parameters (variational parameter of the wave function and the strangeness content) as will be discussed along the paper. Our approach is based on a high baryon density description rather than high temperature and therefore it complements lattice QCD studies [8] that, while succesful at finite temperature, are unable yet to simulate hadronic systems at finite baryon densities. The work is organized as follow: First, we study the nucleon to quark transition and how the strange quarks content becomes energetically favorable to be included, defining so the strangeness fractions on the system. Then, the quark and gluonic rearrangement upon the heavy-quark pair appearance is studied through the quark-antiquark potential, which essentially reflects the energetic cost of such a modification and then we determine the quark-antiquark bound system wave function. At the end, based on our results, we conclude that abrupt changes in the properties of the heavy pair in the hadronic medium correlate strongly with the deconfining phase transition.

2. The model

We use the *string flip model* [9, 10] to describe the nuclear matter. It includes quarks and colors as degrees of freedom and the gluons are considered by optimally pairing the quarks via a truly many body potential, which at low densities confines quarks building nucleons while allowing cluster separability, and at high densities the interaction becomes negligible and the system resem-

Figure 1: Determination of optimal pairing, building color singlets.

bles a Fermi gas of quarks. The transition between both regimes is dynamically generated and thus it allows tracking observables as they evolve in density, without the need of matching two sets of degrees of freedom. Namely, nucleons and quarks.
The wave function to describe the system is taken as a variational one as follows:

$$\Psi = e^{-\lambda V}\psi_{FG} \tag{2.1}$$

where ψ_{FG} is the Fermi gas wave function given by a product of Slater determinants, λ is the variational parameter and V is the many body potential. The procedure to find V is to choose the minimum pairing configuration between two different sets of colored particles, building A pairs.

$$V_{Q_1Q_2} = \min_P \sum_{i=1}^{A} v\Big(\mathbf{r}_{iQ_1}, P(\mathbf{r}_{iQ_2})\Big) , \tag{2.2}$$

where $\mathbf{r}_{iQ_1}$ denotes the spatial coordinate of the *ith* quark of color 1 and $P(\mathbf{r}_{iQ_2})$ is the coordinate of the optimal mapped *ith* quark with color 2 ($\mathbf{r}_{iQ_1} \mapsto P(\mathbf{r}_{iQ_2}) \equiv \mathbf{r}_{jQ_2}$).
This procedure avoids the problem of the appearance of long range Van Der Waals forces and is easily generalized to more degrees of freedom, for example color 1 and 2 can be color-anticolor in a quark-antiquark nucleon, or red, blue and green in a 3-quarks nucleon, the colorless of nucleons must be imposed in any case. The confining potential v is assumed harmonic with a spring constant k. That is,

$$v(\mathbf{r}_{iQ_1}, \mathbf{r}_{jQ_2}) = \frac{1}{2}k(\mathbf{r}_{iQ_1} - \mathbf{r}_{jQ_2})^2 \tag{2.3}$$

In Figure 1, we exemplify the optimal grouping of two possible configurations for a set of colored quarks making clusters of three. Note that in our algorithm the gluonic link is Δ-shaped because it is based in the two body pairing as discussed.

2.1 Variational Monte Carlo simulation

As a baseline, let us consider a single nucleon built up of 3 quarks interacting via the harmonic potential, as in the non-relativistic quark model, then we can compute the corresponding values for the energy and variational parameter for an isolated nucleon in its ground state, given by [10]: $E_0 = \sqrt{3}$ and $\lambda_0 = 1/\sqrt{3}$ (in $\hbar = m = k = 1$ units).
To describe the many body generalization for $3N = 3(N_u + N_d + N_s)$ quarks, where $N_u = N_d$ is the number of u and d quarks of mass m, N_s is the number of s quarks of mass m_s of a single color, we require the use of the variational wave function (2.1) where the potential includes 3 pairing

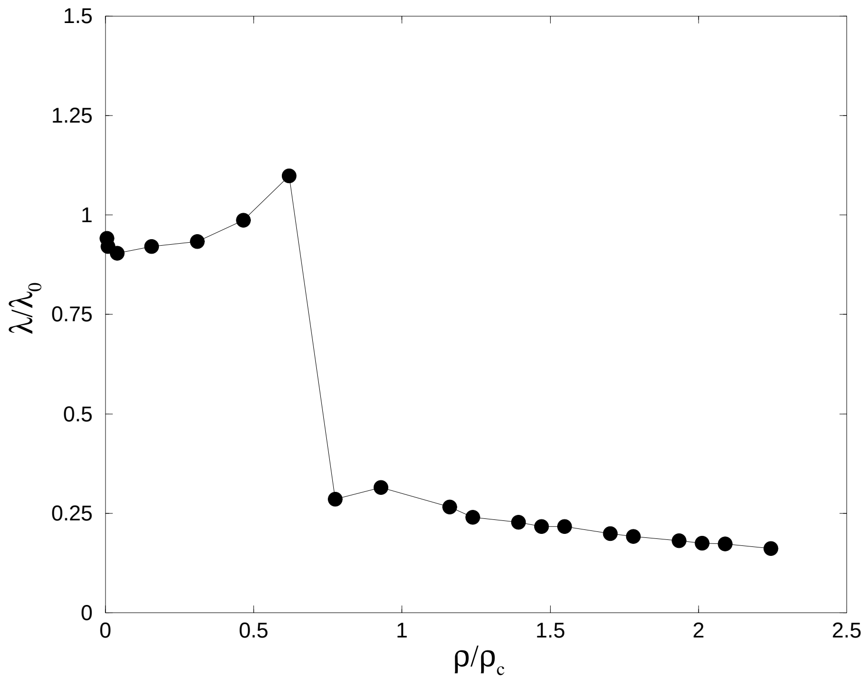

Figure 2: Variational parameter as a function of density.

terms, one for each color combination $V = V_{RB} + V_{RG} + V_{GB}$, and the Fermi gas wave function is a product of 9 Slater determinants, one for each color-flavor combination. At very low densities the system must reproduce the E_0 and λ_0 values, with no strange quark content, and then it has to evolve dynamically to a Fermi gas ($\lambda \approx 0$) with strangeness. The energy evolution and strange quark content as a function of density for this kind of systems was considered in a previous work [10]. Here we give a short description of the procedure: The variational wave function allows us to compute the ground state properties of the system at a given density ρ. To begin with, the expectation value of the Hamiltonian can be set as:

$$E_\lambda =< H >_\lambda = T_{FG} + \lambda^2 < W >_\lambda + < V >_\lambda, \tag{2.4}$$

where the kinetical energy develops an interaction term W, which represents the average position of the two quarks connected to the ith quark, and T_{FG} is the Fermi gas kinetic energy for the three flavors, which in terms of the Fermi momentum k_F takes the following form:

$$\frac{T_{FG}(\rho,\sigma)}{3N} = (1-\sigma)m + \sigma m_s + \frac{3k_F^2}{10m}(1-\sigma)^{5/3} + \frac{3k_F^2}{20m_s}(2\sigma)^{5/3} \tag{2.5}$$

where $\sigma = N_s/N$ is the strangeness per quark.
Metropolis Monte Carlo algorithm was used to compute the expectation values of W and V, using $3N = 120$ quarks in a cubic box, whose size is changed to account for the different densities. Note that it involves computing 9N integrals and finding the optimal clustering of quarks in color singlets by searching among $(N!)^2$ configurations. The variational approach ask us to consider λ such that

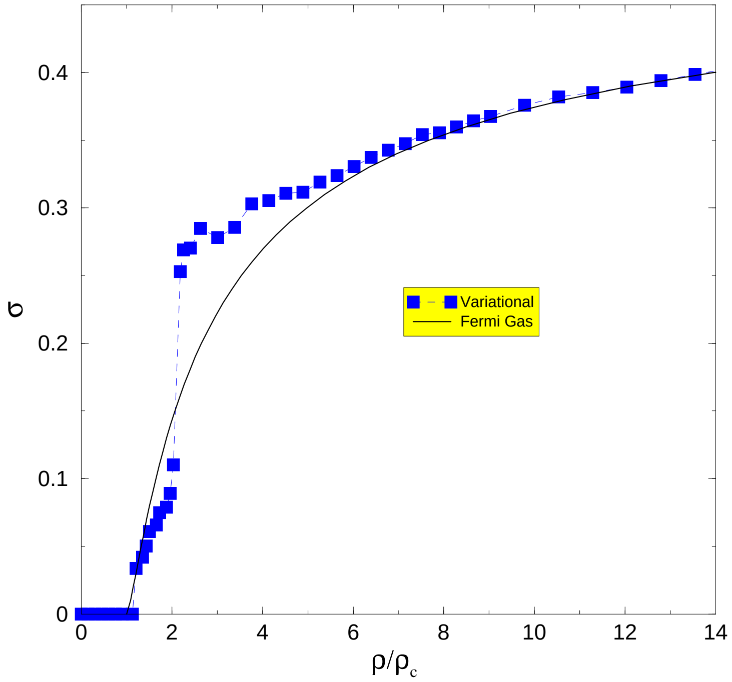

Figure 3: Strangeness fraction as a function of density.

it minimizes the energy at a given density and strangeness,

$$\frac{\partial E_\lambda}{\partial \lambda} = 0 \tag{2.6}$$

In order to determine, simultaneously, the transition to strange matter we leave the strangeness fraction σ as a free parameter, which is also fixed by minimization conditions. The Fermi gas approach can be used to define a critical density for such transition by requiring:

$$\frac{\partial T_{FG}/N}{\partial \sigma} = \left[m_s + \frac{k_F^2}{2m_s}(2\sigma)^{2/3} \right] - \left[m + \frac{k_F^2}{2m}(1-\sigma)^{2/3} \right] = 0, \tag{2.7}$$

and setting the strangeness content equal to zero.

3. Medium characterization

The minimization procedure determines the optimal value of λ at a given density. Figure 2 shows the evolution of the λ parameter obtained for a set of density values. We observe that at very low densities the isolated nucleon value $\lambda/\lambda_0 = 1$ is approached. A deep change occurs around 0.7 signing the onset of the percolation phase transition. The density normalization ρ_c corresponds to the transition density to strange matter for a free Fermi gas eqn. (2.7). For $m = 300$ MeV and $m_s = 1.6m$ it takes the value $\rho_c = 0.468\ fm^{-3}$. At high densities, although small, λ is not negligible and therefore reflects the fact that interactions still being sizable. In Figure 3 we have plotted the strangeness content as a function of density at the optimal λ. The square symbols are the results from the simulation as it is modified from (u,d) to (u,d,s) which happens at a value slightly bigger

than 1, and we compare it with the Free Fermi approach represented by the solid line. A discontinuity is observed around 2 and from there on both results approaches to each other.
This complete our characterization of the medium, regarding the interaction behavior and strangeness content. In the following we explore the color response of the system.

4. Quark-antiquark potential and wave function

The system in the ground state configuration will react to the appearance of a quark-antiquark pair by modifying its gluon flux-tubes arrangement. Using the quark-antiquark potential we can characterize this behavior [11]; it consist on introducing a fast $q\bar{q}$ pair at a relative distance r from each other into the system. We assume that upon the sudden introduction, the quarks of the medium do not change their relative positions and thus the modification in the many-body potential comes entirely from the introduced extra pair. Then, we can write the quark-antiquark potential as:

$$V_{Q\bar{Q}}(r,\rho) \equiv V_{A+1}(r) - V_A = \int dx_1...dx_{2A}\Psi^2_\lambda(x_1,...,x_{2A}) \min_{[P]} \sum_{i=1}^{A+1} v(\mathbf{r}_{iq},\mathbf{r}_{j\bar{q}}) - \int dx_1...dx_{2A}\Psi^2_\lambda(x_1,...,x_{2A}) \min_{[P]} \sum_{i=1}^{A} v(\mathbf{r}_{iq},\mathbf{r}_{j\bar{q}}), \tag{4.1}$$

where V_A and $V_{A+1}(r)$ are the potentials before and after the introduction of the extra pair at a relative distance $\mathbf{r}$, respectively. The low density behavior, shown in Figure 4, reflects the similarity to the isolated square potential (solid line) which we have drawn to compare, also there is a drop of the potential at higher densities that is in correspondence with the density region where the λ parameter drops significantly. At short distances the potential is negative for all densities which means that the potential energy is reduced upon the pair introduction. At larger densities, it arrives to the screening region and the changes are less dramatic.

Once the potential have being computed we can find the wave function for the quark-antiquark pair, by numerically solving the corresponding Schrödinger's equation:

$$\left[-\frac{\hbar^2\partial^2}{m_Q\partial r^2} + V_{Q\bar{Q}}(r,\rho)\right]\varphi(r) = 0. \tag{4.2}$$

where m_Q is the mass of a heavy quark, and the full solution is then given by $\Phi(r) = \varphi(r)/r$. The solution for a pure harmonic potential, as expected in the isolated case, is useful as a baseline to compare.
In figure 5 we plot the ground state density weighted by $r^2/4$ for a set of medium densities. In brackets we also display the corresponding mean square radii, which are obtained by integrating the area under the curves. For densities below the transition, the wave function clusters around the free space value (black line) and, as soon as the transition density is crossed, a significantly spread in the wave function develops.

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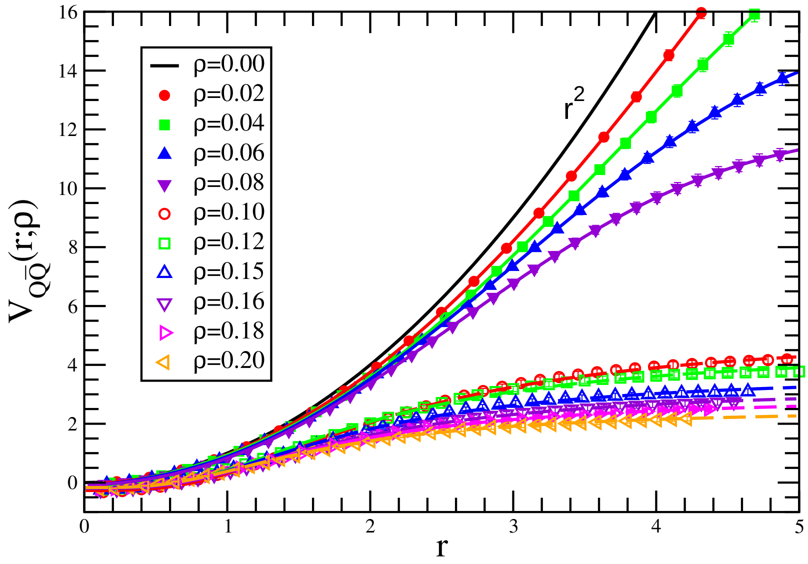

Figure 4: Quark-antiquark potential for a set of densities.

5. Conclusions

We have computed the ground state of a quark-made system for a wide range of densities via a variational procedure. At low densities we reproduced the nuclear medium where optimal color-neutral clusters were formed and then we identify the transition to the deconfined quark matter state by observing a drop in the clustering efficiency, given by the variational parameter. Simultaneously, we established the strange quark content evolution on density. Upon these results, we studied the response of this medium to a heavy quark-antiquark pair by determining its binding potential, which showed an important decrease at long distances as density increased. The screening by the medium yields an effective potential that is drastically different from its free-space value above the transition density. The wave function for the bound state also reflected that behavior. Our results confirm that the use of the medium modifications to a heavy meson is a robust signature for the onset of the deconfining phase transition.

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References

[1] T. Matsui, Prog. Theor. Phys. Suppl. **151** (2003) 11, nucl-th/0305096.

[2] G. Baym, Physica **96A** (1979) 131; T. Celik, F. Karsch, H. Satz, Phys. Lett. **B97** (1980) 128.

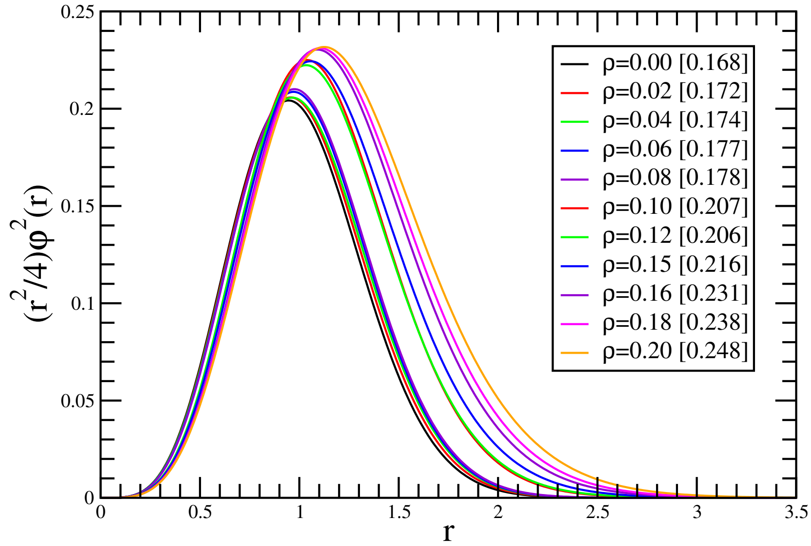

Figure 5: Weighted quark-antiquark wave function for a set of densities

[3] G. Baym, Nucl. Phys. **A 698** (2002) 23.

[4] A. R. Bodmer, Phys. Rev. D **4** (1971) 1601; E. Witten, Phys. Rev. D **30** (1984) 272.

[5] N. K. Glendenning *Compact Stars* (Springer-Verlag, New York, 1997).

[6] T. Matsui and H. Satz, Phys. Lett. B **178** (1986) 416.

[7] R. L. Thews, M.chroedter and J. Rafelski, Phys. Rev. C **63** (2001) 054905.

[8] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. **B605**, (2001) 579; T. Umeda, H. Matsufuru, O. Miyamura, and K. Nomura, Nucl. Phys. **A721**(2003) 922; R. Rapp and L. Grandchamp, J. Phys. G **30** (2004) S305.

[9] C. J. Horowitz, E. J. Moniz and J. W. Negele Phys. Rev. D **31** (1985) 1689; C. J. Horowitz and J. Piekarewicz, Phys. Rev. C **44** (1991) 2753; Nucl. Phys. **A536** (1992) 669.

[10] G. Toledo Sanchez and J. Piekarewicz, Phys. Rev. C **65** (2002) 045208.

[11] G. Toledo Sanchez and J. Piekarewicz, Phys. Rev. C **70** (2004) 035206.