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$1/f$ noise and plastic deformation

Lasse Laurson*
Helsinki University of Technology, Laboratory of Physics, HUT-02105 Finland
E-mail: lla@fyslab.hut.fi

Mikko J. Alava
Helsinki University of Technology, Laboratory of Physics, HUT-02105 Finland
E-mail: mja@fyslab.hut.fi

There is increasing evidence from experiments that plastic deformation in the micro- and mesoscopic scales is an intermittent and heterogeneous process, consisting of avalanches of dislocation activity with a power law distribution of sizes. This has been also discovered in many simulation studies of simplified models. In addition to direct studies of the avalanche statistics, interesting information about the dynamics of the system can be obtained by studying the spectral properties of some associated time series, such as the acoustic emission amplitude in an experiment. We discuss the generic aspects concerning the power spectra of such signals, e.g. the possibility of relating the exponent of the power spectrum to the avalanche exponents of the (dislocation) system.

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*Speaker.

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1. Introduction

Contrary to the traditional paradigm of plastic deformation under homogeneous loads being a spatially and temporally smooth and homogeneous process, recent evidence from both numerical simulations as well as experiments suggests that plastic deformation proceeds via coherent bursts of dislocation activity with long-range spatial and temporal correlations [1, 2, 3, 4, 5]. These bursts, or dislocation avalanches, can be characterized by apparently scale-free size distributions. This kind of "crackling noise" [6] response to slowly varying external conditions is observed also in many other non-equilibrium physical systems, such as earthquakes [7], Barkhausen noise in ferromagnets [8], vortex avalanches in superconductors [9] and martensitic shape memory alloys [10].

Such avalanching systems are typically studied by recording some time series $V(t)$, which could be e.g. the acoustic emission amplitude emitted from a plastically deforming crystal or the voltage recorded from a Barkhausen noise experiment. In ideal cases of certain simple slowly driven model systems (such as the sandpile models of self-organized criticality [11]), the signal $V(t)$ drops to zero between two consecutive avalanches, which can then be defined simply as connected sequences of non-zero values of $V(t)$. In the case of experiments and some more realistic model systems, however, the definition of a single pulse or avalanche presents some practical complications due to both the background noise and the fact that finite drive rates can lead to merging of avalanches in time. This forces one to threshold the signal $V(t)$ in some way to be able to identify the individual avalanches. As it is not a priori clear how the thresholding should be done, this process can lead to ambiguous definition of pulses.

An alternative approach is to study the spectral properties of the pulse train as a whole. Here we discuss the power spectrum (PS) $P(f)$ of a time series $V(t)$, defined as the absolute square of the Fourier transform of $V(t)$, or

$$P(f) \sim |\int e^{i2\pi ft}V(t)dt|^2. \tag{1.1}$$

In critical avalanching systems, the PS is typically of the power-law form $P(f) \sim 1/f^{\alpha}$, providing thus an example of "$1/f^{\alpha}$" or "flicker" noise [12]. Due to the relation between the time-time correlation function and the PS through the cosine transformation, the latter is a measure of temporal correlations in the system. Values less than two for the exponent α are an indication of the presence of complex time correlations in the system. Recently it has been found that under certain fairly general conditions it is also possible to relate the exponent α describing the power spectrum of a critical avalanching system to the scaling exponents characterizing the avalanche distributions. In particular this has been shown to be true for certain models of Barkhausen noise [13] and sandpile models of self-organized criticality [14].

In this paper we discuss briefly some aspects of noise in plastically deforming dislocation systems, present a scaling relation relating the scaling of the PS to the avalanche statistics and report some preliminary results on numerical simulations concerning the applicability of the scaling relation to a simple two-dimensional discrete dislocation dynamics model (to be presented in more detail elsewhere [15]). The paper is organized as follows: In the next Section the scaling argument for the scaling of the PS is presented, followed by a brief discussion on its applicability to plasticity as well as some other remarks on noise in plasticity in Section 3. The paper is finished with conclusions.

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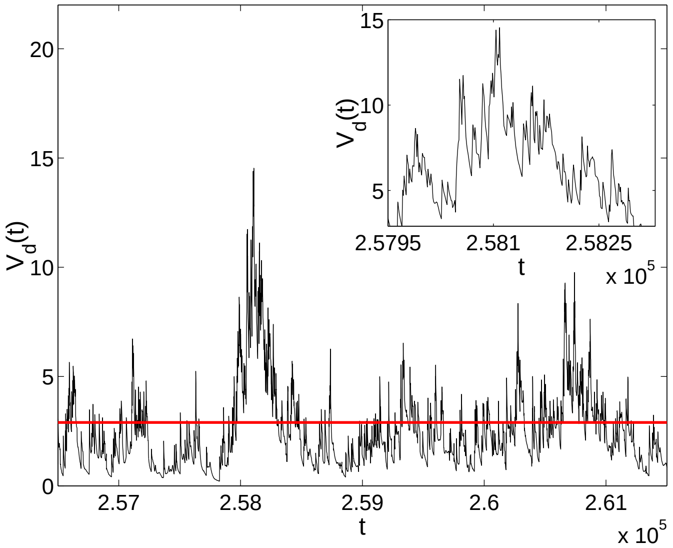

Figure 1: An example of the signal $V_d(t)$, obtained from a discrete dislocation dynamics simulation similar to that presented in Ref. [2]. The horizontal red line corresponds to the threshold level used to identify avalanches. The inset shows an example of a large avalanche identified after thresholding.

2. Power spectra and avalanche scaling

Consider a bursty time series $V(t)$ consisting of temporally separated avalanches. One possible definition of an avalanche is a connected sequence of values of $V(t)$ exceeding some threshold value V_{th}, corresponding to uncorrelated background noise. If this kind of avalanche starts at $t = 0$ and ends at $t = T$, the size s of an avalanche of duration T is defined as $s(T) = \int_0^T [V(t) - V_{th}]dt$. Assume that the average avalanche size $\langle s(T)\rangle$ of avalanches of a given duration T scales as a power law of the duration,

$$\langle s(T)\rangle \sim T^{\gamma_{st}}, \tag{2.1}$$

and that the avalanches are self-similar in the sense that average avalanche shapes $V(T,t)$ of avalanches of different durations T can be collapsed onto a single curve by using the ansatz

$$V(T,t) = T^{\gamma_{st}-1} f_{shape}(t/T), \tag{2.2}$$

where $f_{shape}(x)$ is a scaling function. By cosine transforming the time-time correlation function $C(\theta|s)$ of avalanches of a given size s, this leads to the scaling form $E(f|s) = s^2 g_E(f^{\gamma_{st}} s)$ for the corresponding energy spectrum. The scaling of the total power spectrum is then obtained by averaging $E(f|s)$ over the avalanche size probability distribution $D(s)$, assumed to be a power law $D(s) \sim s^{-\tau}$ cut off at $s = s^*$. The result will depend on the value of τ as well as on the form of the scaling function $g_E(x)$. Kuntz and Sethna [13] noticed that under conditions fairly generally applicable to critical avalanching systems, $g_E(x)$ should behave like $g_E(x) \sim 1/x$, which then implies that for $\tau < 2$ (which seems to be the case for dislocation avalanches [2]) the power spectrum scales as

$$P(f) \sim f^{-\gamma_{st}}. \tag{2.3}$$

One thus obtains a scaling relation $\alpha = \gamma_{st}$.

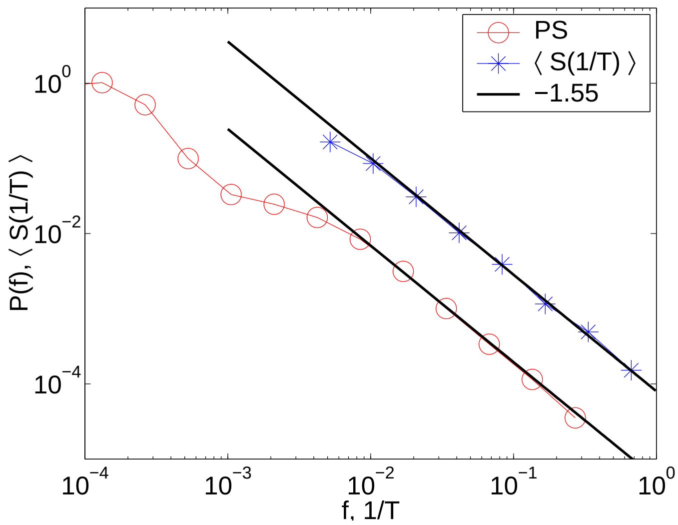

Figure 2: An example comparing the scaling of the PS of the signal $V_d(t)$ and that of $\langle s(1/T)\rangle$ from the two-dimensional discrete dislocation dynamics model. Both seem to scale with roughly the same exponent, in agreement with Eq. (2.3).

3. Noise in dislocation systems

The possible applications of the above scaling relation in studies of plasticity are twofold: One may consider either the motion of a single dislocation on its glide plane, or the collective motion of an ensemble of interacting dislocations, driven by an applied external stress.

A reasonable description of a single dislocation moving on its glide plane can be obtained by a 1+1 dimensional driven interface with the line tension approximation for the dislocation self-interaction and considering interactions with point obstacles [16]. It should be noted, however, that for certain properties (such as the roughness of the dislocation line [17]), long-range self-stresses and the fact that the pinning obstacles are in typical cases forest dislocations can be important. Regardless of the details of the model, such interfaces exhibit rich phenomenology close to and around the depinning transition, where the velocity of the dislocation becomes zero. Here the external stress or strain rate act as possible control parameters. Below the critical value one meets the creep regime, in which the dislocation undergoes thermal motion of a broad set of barriers set by the background pinning field, from other dislocations and solutes/impurities [5, 18]. Of particular interest is the case where the interface is driven in the constant *velocity* (strain rate) ensemble, which allows for the presence of avalanches, in the form of external force fluctuations [19]. By using the scaling relation $\gamma_{st} = (d+\chi)/z$, where χ is the roughness exponent, z the dynamical exponent and d the spatial dimension of the interface, together with numerical estimates for the exponents of a 1+1 -dimensional interface with the line tension approximation [20, 21], one obtains the value $\gamma_{st} = 1.57(5)$. Due to the different roughness exponent $\chi \approx 1.0$ obtained for the model with a nonlocal interaction kernel [17], a different value for γ_{st} is expected in that case.

In addition to quenched disorder, one may also consider interactions with mobile impurities, such as diffusing solute atoms in solid solutions. This could be done in particular in the context of a phase field model, where the interface of the phase field interacts self-consistently with a conserved field, the solute atmosphere [15].

Similar considerations are expected to apply to models with large number of interacting dislocations, as well as real plastically deforming crystals, but now the time series of interest arises from the collective motion of all the dislocations in the system. In this context it is important to notice that the above derivation assumes that correlations between avalanches are negligible. On the other hand it is known from experiments that dislocation avalanches tend to cluster in time, consisting of a "mainshock" followed by a sequence of few aftershocks [3]. This may modify the low frequency part of the PS, but the high frequency part, corresponding to correlations within individual avalanches, is still expected to scale according to Eq. (2.3). Otherwise the assumptions made in the derivation are such that one could expect them to hold for a wide class of systems with critical avalanche dynamics, possibly including dislocation avalanches in plastically deforming crystals.

We have studied these phenomena numerically in the context of constant external stress (creep) simulations of a simple two-dimensional discrete dislocation dynamics model similar to the one presented in Ref. [2]. The preliminary results concerning the signal $V_d(t) = \sum_i |v_i|$ (where v_i is the velocity of the i'th dislocation) indicate that the scaling $\alpha = \gamma_{st} \approx 1.6$ would hold within errorbars, see Figs. 1 and 2. A detailed presentation of this result will be published elsewhere [15].

4. Conclusions

In this paper we have briefly discussed the use of spectral tools in the study of dislocation avalanches in plastically deforming crystals. It would be interesting to test the scaling relation (2.3) presented here numerically for the various interface or line models for single dislocations as well as experimentally, e.g. in the creep experiments of ice single crystals [3] or in tensile tests of metallic single crystals [22]. If applicable, such a scaling relation would offer an alternative method for studying the avalanche statistics of the system, without the need to use possibly ambiguous thresholding methods.

Interestingly, Ananthakrishna *et al.* [23] have found, while studying the Portevin Le-Châtelier (PLC) effect in single crystals of Cu-10% Al, that the power spectrum of the stress time series for high strain rates decays as $f^{-1.55}$ for low frequencies, i.e. with roughly the same exponent as found by us for the constant stress simulation of a discrete dislocation dynamics model [15]. While the scaling arguments employed in Ref. [23] for the PS-scaling differ from the one presented here, it would be interesting to study the relation between spectral properties and avalanche statistics of the PLC time series in more detail. A possibility is to use a different definition of an avalanche, for instance by considering the stress drop from a certain threshold level and the subsequent increase of stress back to the same level as an "inverted pulse", instead of considering the stress drop statistics.

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