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Xpol,
CMB angular power spectra estimator using cross-correlation

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We present a method to estimate the angular power spectrum of the CMB anisotropies for both temperature and polarization. We compute cross power spectra from a set of maps coming from different detectors or different instruments. Each of these cross power spectra is an unbiased estimate of the C(l) as long as the detector noises are uncorrelated. Then the spectra are optimally combined into a final power spectrum and analytical estimates of the power spectrum error bars are computed. This method presents three main advantages : (1) no estimation of the noise power spectrum is needed, (2) error bars can be computed very fast as no Monte-Carlo simulations are involved, (3) weighting scheme including the covering factor can be individually taken into account for each input map. We show with simulations of Planck HFI-like surveys that the power spectra and their error bars are recovered without bias at all angular scales. In addition, we apply this method to simulations of WMAP high frequency channels, which are comparable to Planck-LFI.

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1. Introduction

We describe a method derived from XSPECT [1] extended to polarization. XPOL estimate the C_ℓs of temperature (TT), polarization (EE, BB and EB) and temperature-polarization correlation (TE and TB), by computing the cross-power spectra between a collection of input maps coming either from multiple detectors of the same experiment or from different instruments. Analytical error bars are derived for each of them. The cross-power spectra, that do not include the *auto*-power spectra, can be then combined using a Gaussian approximation of the likelihood function.

2. Cross-power spectra

XPOL belong to the so-called 'pseudo-C_ℓ's estimators [2] that compute directly the *'pseudo'* angular power spectra from the data. Then, they correct them for sky coverage, beam smoothing, data filtering, pixel weighting and noise biases (eq. 2.1).

$$\widehat{D_\ell} = \sum_{\ell'} M_\ell |p_{\ell'}|^2 B_{\ell'}^2 F_{\ell'} \langle C_{\ell'} \rangle + \langle N_\ell \rangle \tag{2.1}$$

Since the first description of this method given by [3], several approaches have been developed ([4], [5], [6], [7]). These estimators can be evaluated using fast spherical harmonic transforms $\mathscr{O}\left(N_{pix}^{3/2}\right)$ and therefore provide fast and accurate estimates of the C_ℓs. However, they require an accurate knowledge of the instrumental setup and noise in order to correct them for the biases discussed previously. In fact, they use an estimation of the power spectrum of the noise in the map, generally computed via Monte-Carlo simulations, which is subtracted from the original power spectrum. This is also used to estimate the error bars in the power spectrum by calculating the variance of the C_ℓs over the set of simulations.

In [1], authors show that, assuming no correlation between the noise contribution from two different maps A and B, $\langle N_\ell^{AB} \rangle = 0$, each of the corrected cross-power spectra is an unbiased estimate of the C_ℓs. The 'pseudo' cross-power spectra are explicitly corrected for incomplete sky coverage, beam smoothing, filtering and pixelization (eq. 2.2).

$$\widehat{D_\ell^{AB}} = \sum_{\ell'} M_\ell^{AB} |p_{\ell'}|^2 B_{\ell'}^A B_{\ell'}^B F_{\ell'}^{AB} \langle C_{\ell'}^{AB} \rangle \equiv \mathscr{M}_{\ell\ell'}^{AB} \langle C_{\ell'}^{AB} \rangle \tag{2.2}$$

Extension to polarization leads to a more complex kernel matrix $\mathscr{M}_{\ell\ell'}^{pol}$ corresponding to the six angular power spectra $C_\ell^{AB} = \left(C_\ell^{T_A T_B} C_\ell^{E_A E_B} C_\ell^{B_A B_B} C_\ell^{T_A E_B} C_\ell^{T_1 B_B} C_\ell^{E_A B_B} \right)$. $\mathscr{M}_{\ell\ell'}^{pol}$ is band diagonal and take into account the correlation between pure E and B modes [8].

3. Cross-correlation matrix - analytical error bars and covariance matrix

From N input maps and for each polarization mode, we can obtain $N(N-1)/2$ cross-power spectra C_ℓ^{AB} ($A \neq B$) which are unbiased estimates of the angular power spectrum but which are obviously not independent. The cross-correlation matrix describes the correlations between cross-spectra and between multipoles. We show how we can construct an analytical estimate of the this matrix using cross- (and auto-) power spectra of the data. Error bars and covariance matrix can be deduced for each cross-power spectra. The correlation matrix between sets of independent I, Q or U maps A, B, C and D reads

$$\Xi^{AB,CD}_{\ell\ell'} = \mathscr{M}^{AB\,-1}_{\ell\ell_1}\left[\frac{\mathscr{M}^{(2)}_{\ell_1\ell_2}\left(W^{AC,BD}\right)C^{AC}_{\ell_1}C^{BD}_{\ell_2}}{2\ell_2+1} + \frac{\mathscr{M}^{(2)}_{\ell_1\ell_2}\left(W^{AD,BC}\right)C^{AD}_{\ell_1}C^{BC}_{\ell_2}}{2\ell_2+1}\right]\left(\mathscr{M}^{CD\,-1}_{\ell'\ell_2}\right)^T \tag{3.1}$$

In the case of large sky coverage, $\mathscr{M}_{\ell\ell'}$ can be approximated as diagonal so that the effect of a non-homogeneous sky coverage is represented by a simple function ν_ℓ which can be associated to the effective number of degrees of freedom in the $\chi^2_{\nu_\ell}$ distribution of the C_ℓs over the sky [4] $\nu_\ell = (2\ell+1)\Delta_\ell \frac{w_2^2}{w_4}$ where w_i is the i-th moment of the mask. Thus, from Ξ, we can deduce estimates of the covariance matrix containing the error bars for each cross-power spectra

$$Cov^{AB}(\ell,\ell') = \Xi^{AB,AB}_{\ell\ell'} \simeq \frac{1}{\nu_{\ell'}}\left[C^{AB}_\ell C^{AB}_{\ell'} + C^{AA}_\ell C^{BB}_{\ell'}\right] \tag{3.2}$$

Furthermore, combined angular power spectra can be obtained by, for example, maximizing a Gaussian approximated quadratic likelihood function for all modes (TT, EE, BB, TE, TB, EB).

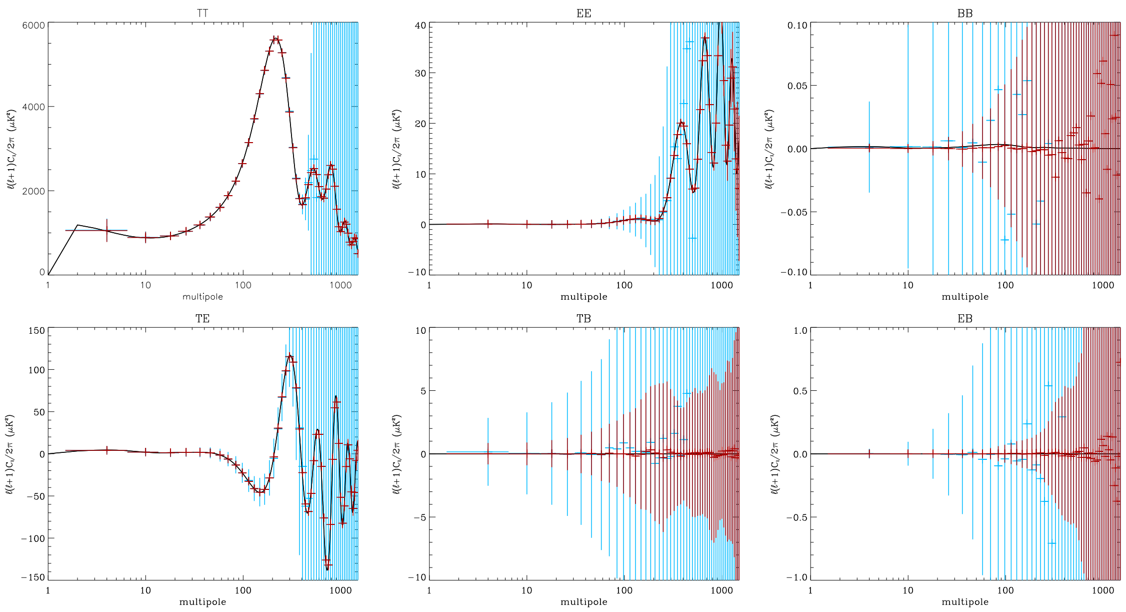

Figure 1: Angular power spectra estimated with XPOL on simulations from WMAP (*blue*) and Planck-HFI (*red*) compared to the input CMB model (*black*). *upper left to lower right* : TT, EE, BB, TE, TB and EB.

4. Xpol applied on simulations

We have applied XPOL to simulations based on WMAP and Planck-HFI on one year mission. For WMAP, we used the five frequency bands at 23, 33, 41, 61 and 94 GHz respectively, whereas we used the 100, 143, 217 and 353 GHz channels for Planck-HFI. For a given CMB sky, we simulate a set of maps in temperature and polarization (I,Q,U) with realistic inhomogeneous independent noise per pixel at 7 arcmin resolution approximatively ($N_{side} = 512$). Beam smoothing effect has been modeled by the beam transfer functions for each detector. The galactic mask used by WMAP team (Kp0) has been applied for a total sky coverage of $\sim$76%.

We apply XPOL to each simulated data set and compute the mean angular power spectrum from the 1000 simulations for each mission as well as the error bars associated to it (fig. 1). We use the same multipole binning than the one used by the WMAP team [9] for polarization power spectra. For both missions, we observe that XPOL produces an unbiased estimate of the six angular power spectra.

5. Conclusion

We have presented a method, XPOL, for estimating the CMB angular power spectra in temperature (TT), polarization (EE, BB, EB) and correlation temperature-polarization (TE, TB) with analytical error bars. XPOL is based on the computation of 'pseudo' cross-power spectra obtained from input maps which are corrected for beam smoothing, timeline filtering and inhomogeneous sky coverage effects. XPOL can deal with different and complex weighting schemes for each of the detectors involved. It produces analytical estimates of the correlation matrix (and thus of the error bars) associated to the angular power spectra. This property is of great interest when producing Monte Carlo simulations in the process of testing and improving the estimation of the C_ℓs or when checking the robustness of the data analysis with respect to the various choices of mask, filtering or binning. Finally, the corrected cross-power spectra are optimally combined into a single angular power spectrum.

XPOL has been used for the estimation of the dust angular power spectra with Archeops [10]. A similar method also based on the combination of a set of cross-power spectra has been used to obtain results from WMAP data [9]. Whereas, WMAP team estimates the cross-correlation matrix from a model, XPOL gives an analytical estimate based on the data which allows us to include naturally the mode coupling in this matrix.

As it has been shown by [2], a better and hybrid estimation of the power spectrum could be obtained by combining a 'pseudo' power spectrum estimator, for example XPOL, at large multipoles with a quadratic maximum likelihood estimator at low multipoles.

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