

## Observational constraints on a decaying cosmological term

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### Riou Nakamura\*

*Department of Physics, Graduate school of sciences, Kyushu University, Fukuoka 810-8560, Japan*

*E-mail: riou@gemini.rc.kyushu-u.ac.jp*

### Masa-aki Hashimoto

*Department of Physics, Faculty of sciences, Kyushu University Fukuoka 810-8560, Japan*

*E-mail: hashi@gemini.rc.kyushu-u.ac.jp*

### Kiyotomo Ichiki

*Research Center for the Early Universe, Graduate School of Science, The University of Tokyo Tokyo 113-0033, Japan*

*E-mail: ichiki@resceu.s.u-tokyo.ac.jp*

We investigate the evolution of a universe with a decaying cosmological term (vacuum energy) that is assumed to be a function of the scale factor. In this model, while the cosmological term increases to the early universe, the radiation energy density is lower than the model with the cosmological "constant". We find that the effects of the decaying cosmological term on the expansion rate at the redshift  $z < 2$  is negligible.

However, the decrease in the radiation density affects on the thermal history of the universe; e.g. the photon decoupling occurs at higher  $z$  compared to the case of the standard  $\Lambda$ CDM model. As a consequence, a decaying cosmological term affects on the cosmic microwave background anisotropy. We show the angular power spectrum in  $D\Lambda$ CDM model and compare with the Wilkinson Microwave Anisotropy Probe (WMAP) data.

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\*Speaker.

## 1. Introduction

Recent observations (e.g. type Ia supernovae [1], the cosmic microwave background (CMB) [2]) indicate that the cosmological term is necessary. If the cosmological term is constant from the Planck time to the present, there is the *cosmological constant problem*: the present value of the cosmological constant is extraordinarily small compared with an inferred vacuum energy during the Planck time. To solve this problem, it is natural to consider that the cosmological term decreases from the large value at the early epoch to the present value. Many functional form of the cosmological term has been suggested, e.g. the function of the scalar field [3]. On the other hand, more physically motivated researches of varying vacuum energy (e.g. cosmic quintessence) have been presented [4].

Cosmological constrains on and results of a decaying vacuum energy density have been investigated, where the ratio of vacuum to radiation energy was  $\sim 4 \times 10^{-4}$  [5]. We note that vacuum energy corresponds to a cosmological term in the present paper. The model with a decaying- $\Lambda$  term into the radiation has been found to affect the thermal evolution of the universe. Since the radiation temperature is lower compared with the standard  $\Lambda$ CDM (S $\Lambda$ CDM) model [6], the molecular formation occurs at earlier epoch compared to the case of the S $\Lambda$ CDM [7]. Furthermore the model is found to be consistent with the CMB temperature observations at  $z < 4$  if appropriate parameters are adopted [8].

In the present paper, we assume that the  $\Lambda$  term decays into the photon (hereafter we call this the DACDM model) and investigate the CMB temperature fluctuation in the DACDM model. Then, we constrain the parameters of the DACDM model.

## 2. A decaying $\Lambda$ cosmology

Using the Friedmann-Robertson-Walker metric, the Einstein equation and the energy-momentum conservation law are written as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad (2.1)$$

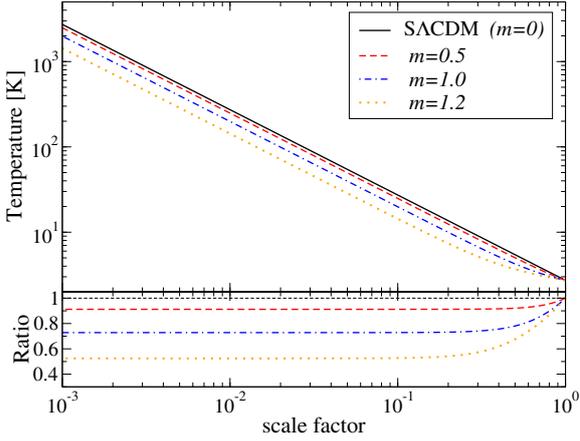
$$\dot{\rho} + \frac{\dot{\Lambda}}{8\pi G_N} = -3\frac{\dot{a}}{a}(\rho + p), \quad (2.2)$$

where  $a$  is the cosmic scale factor,  $k$  is the curvature and  $\Lambda$  is the cosmological term. The total energy density  $\rho$  and the pressure are written as

$$\rho = \rho_m + \rho_\gamma + \rho_\nu, \quad p = \frac{1}{3}(\rho_\gamma + \rho_\nu), \quad (2.3)$$

where the subscripts  $m$ ,  $\gamma$  and  $\nu$  are the non-relativistic matter (baryon and cold dark matter), photon, neutrino, respectively. Here the energy density of matter and neutrinos varies as  $\rho_m = \rho_{m0}a^{-3}$  and  $\rho_\nu = \rho_{\nu0}a^{-4}$ , where the subscript 0 means the present value. From eqs. (2.2) and (2.3), we get the evolution of the photon energy density:

$$\frac{d\Omega_\gamma}{da} + 4\frac{\Omega_\gamma}{a} = -\frac{d\Omega_\Lambda}{da}. \quad (2.4)$$



**Figure 1:** Upper panel: the evolution of the photon temperature  $T_\gamma$  as a function of the scale factor in a decaying  $\Lambda$  model. Lower panel: the ratio of  $T_\gamma$  for  $m = 0.5, 1.0, 1.2$  relative to  $m = 0$ .

with

$$\Omega_i \equiv \frac{\rho_i}{\rho_{\text{crit}}}, \quad \rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G_N}, \quad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

where  $H_0$  is the Hubble constant in unit of km/sec/Mpc.

We assume a functional form of  $\Lambda$  as follow [6, 7, 8, 9]:

$$\Omega_\Lambda = \Omega_{\Lambda 1} + \Omega_{\Lambda 2} a^{-m}. \quad (2.5)$$

Note the present value of  $\Omega_\Lambda$ :  $\Omega_{\Lambda 0} = \Omega_{\Lambda 1} + \Omega_{\Lambda 2}$ .

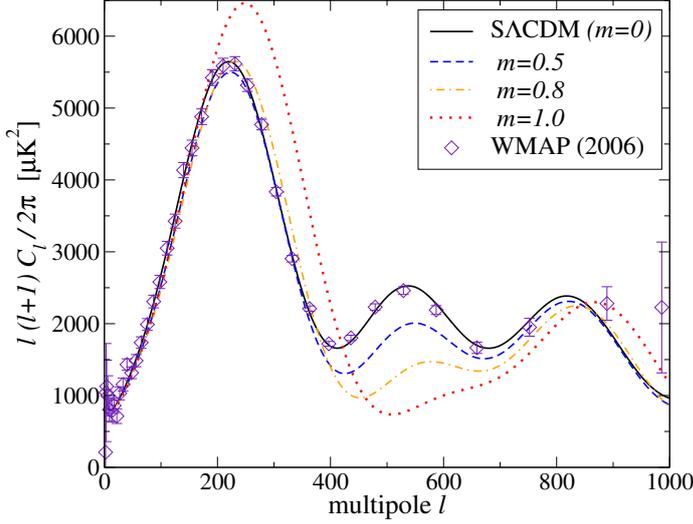
Following the Stefan-Boltzmann's law  $\rho_\gamma \propto T_\gamma^4$ , the photon temperature evolves as follows [8]:

$$T_\gamma = \frac{T_{\gamma 0}}{a} \left[ 1 + \frac{m\Omega_{\Lambda 2}}{(4-m)\Omega_{\gamma 0}} (a^{4-m} - 1) \right]^{1/4} \quad \text{for } m \neq 4 \quad (2.6)$$

$$T_\gamma = \frac{T_{\gamma 0}}{a} \left( 1 + 4 \frac{\Omega_{\Lambda 2}}{\Omega_{\gamma 0}} \ln a \right)^{1/4} \quad \text{for } m = 4 \quad (2.7)$$

where  $\Omega_{\gamma 0} = 2.471 \times 10^{-5} h^{-2} (T_{\gamma 0}/2.725 \text{ K})^4$  is the present photon energy density,  $h$  is the Hubble constant ( $H_0 \equiv 100h$  km/sec/Mpc). If  $m$  and/or  $\Omega_{\Lambda 2}$  are too large, the photon temperature is negative at some epoch of  $a < 1$ . By excluding this kind of solution, we obtain limits on  $\Omega_{\Lambda 2}$  and  $m$  from eq. (2.6):  $m\Omega_{\Lambda 2}/(4-m) < \Omega_{\gamma 0}$  for  $m < 4$ . In this parameter region, the first term in eq. (2.5) dominate the universe at low- $z$ . As the result, the effects of the second term in eq. (2.5) on the expansion rate is negligible. For  $\Omega_{\Lambda 2} < 0$  or  $m < 0$ , we find that  $T_\gamma$  becomes negative at  $a > 1$ . Therefore we calculate under the condition of  $\Omega_{\Lambda 2} \geq 0$  and  $m \geq 0$ .

Figure 1 illustrates the evolution of the photon temperature in the DACDM model. The adopted cosmological parameters are as follows:  $h = 0.73$ ,  $T_{\gamma 0} = 2.725$  K,  $\Omega_{\Lambda 0} = 0.763$  and  $k = 0$  (flat universe). The photon temperature evolves as  $T_\gamma \propto a^{-1}$  and  $T_\gamma \propto a^{-m/4}$  before and during the  $\Lambda$  dominant epoch, respectively. Changes in  $T_\gamma$  affects the cosmic thermal history significantly [7], which should be constrained by the observations such as CMB anisotropy as we shall show below.



**Figure 2:** The angular power spectrum in a decaying  $\Lambda$  cosmology and WMAP observation data [2]. The solid line is the result for the SACDM model. The dashed, the dot-dashed and the dotted lines are those of DACDM model with  $(\Omega_{\Lambda 2}, m) = (10^{-4}, 0.5)$ ,  $(10^{-4}, 1.0)$  and  $(10^{-4}, 1.2)$ , respectively.

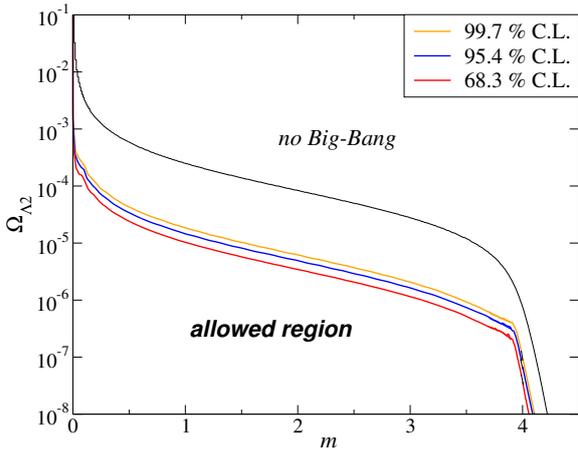
### 3. CMB constraint

The CMB anisotropy observed by the WMAP constrains the cosmological model with very high accuracy. In this section, we investigate the consistency of DACDM model with WMAP and give the limit to the model parameters.

We calculate the CMB power spectrum by modifying CMBFAST code [10]. Figure 2 shows the angular power spectrum in the DACDM model. We adopt the following cosmological parameters: the baryon density parameter  $\Omega_b h^2 = 0.0223$  and the CDM density parameter  $\Omega_{CDM} h^2 = 0.104$ . We neglect reionization. If  $m$  (and/or  $\Omega_{\Lambda 2}$ ) is small, the amplitude of the power spectrum decreases. If we take larger values of  $m$ , the first and third peaks of the CMB power spectrum increases because of the large baryon density relative to the photon energy density. Furthermore the CMB power spectrum shifts toward higher- $l$  because the photon last scattering occurs at an earlier epoch.

To obtain the upper limit of  $\Omega_{\Lambda 2}$  and  $m$ , we calculate the likelihood function given by Ref. [11]. Figure 3 shows 68.3%, 95.4% and 99.7% confidence limits on the  $m - \Omega_{\Lambda 2}$  plane from CMB. We obtain the constraint  $m\Omega_{\Lambda 2}/(4-m) < 4.9 \times 10^{-6}$  at 95 % confidence limit. With the value of this upper limit, the photon last scattering occurs at the earlier epoch by  $\Delta z \sim 30$  compared with that in the SACDM model. Our constraint is severer than that from the observed radiation temperature :  $|m| \leq 1, |\Omega_{\Lambda 2}| \leq 10^{-4}$  [8]. Therefore, the results indicate that a decaying- $\Lambda$  contribution to the cosmic thermal evolution should be small.

It should be noted that  $\Omega_b h^2$  has been fixed during the calculation for simplicity. If we operate CMBFAST with  $\Omega_b$  varying, we can get the more reasonable parameter regions for the DACDM model. Then the primordial abundances of He, D and Li would be different from those predicted by WMAP.



**Figure 3:** Constraint on  $m - \Omega_{\Lambda 2}$  plane from WMAP. Drawn lines correspond to 1, 2 and 3 $\sigma$  confidence limit. The labeled *no Big-Bang* region means that  $T_\gamma$  is negative at  $a < 1$ .

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