

Nuclear weak-interactions in stars

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Outline

- 1 Introduction
 - Why are weak interactions important?
- 2 Nuclear models
 - From QCD to Nuclear Structure
 - Approximated solutions
- 3 Weak interaction formalism
 - Basics
 - Allowed transitions
 - Electron capture and neutrino absorption
 - Neutral current neutrino scattering
- 4 Electron capture in Core-collapse supernovae
 - Presupernova phase
 - Collapse phase
- 5 Neutrino-nucleus interactions
 - Neutrinos and supernovae
 - Neutrinos in nucleosynthesis
 - Supernovae neutrino detection

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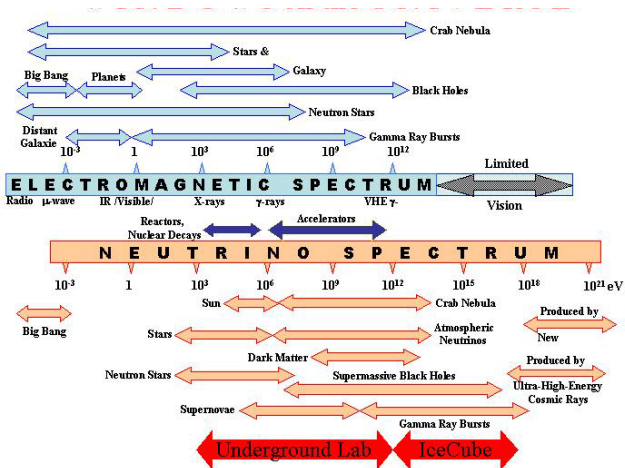
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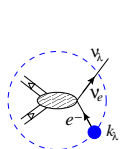
Why is important to understand weak-interactions?

- Weak interaction determines the duration at which many astrophysical processes occur:
 - Time scale for hydrogen burning in the sun (pp-chain) and massive stars (CNO).
 - The time scale for the r-process is determined by beta decays (and maybe neutrino absorption rates).
- In several astrophysical conditions all forces except weak interaction are in equilibrium and the dynamics is governed by weak interaction (core collapse supernovae).
- Neutrinos provide an additional “window” for the observation of the universe providing additional information to observations in the electromagnetic spectrum.

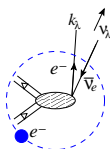
Windows on the Universe



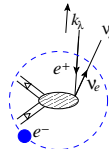
Semileptonic Weak Processes in Stars

orbital e^- capture

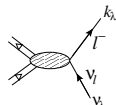
$$q_\lambda = v_\lambda - k_\lambda$$

 β^- decay

$$q_\lambda = v_\lambda + k_\lambda$$

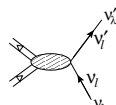
 β^+ decay

$$q_\lambda = v_\lambda + k_\lambda$$



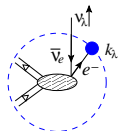
(anti)neutrino capture

$$q_\lambda = k_\lambda - v_\lambda$$

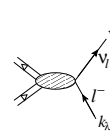


(anti)neutrino scattering

$$q_\lambda = v'_\lambda - v_\lambda$$

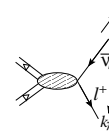
bound-state β^- decay

$$q_\lambda = v_\lambda + k_\lambda$$



continuum charged (anti)lepton capture

$$q_\lambda = v_\lambda - k_\lambda$$



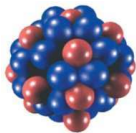
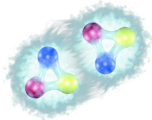
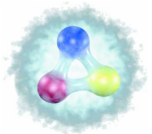
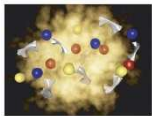
- What is the structure of the operators?
- How to calculate the relevant nuclear states?

From QCD to Nuclear Structure

better resolution / more fundamental

Quantum Chromo Dynamics

Nuclear Structure



- Finite nuclei
- Few-nucleon systems
- Nucleon-nucleon interaction
- hadron structure
- quarks and gluons
- deconfinement

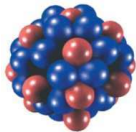
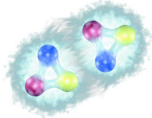
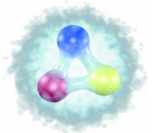
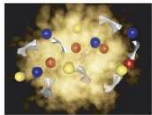
From QCD to Nuclear Structure

better resolution / more fundamental



Quantum Chromo Dynamics

Nuclear Structure



solve the interacting many-body problem

construct realistic nucleon-nucleon interaction from QCD

Realistic N-N potentials

- QCD motivated
 - symmetries, meson-exchange picture
 - chiral effective field theory
- Short-range phenomenology
 - short-range parametrization or contact terms
- Experimental two-body data
 - scattering phase-shifts & deuteron properties reproduced with high precision
- supplementary three-nucleon force
 - adjusted to spectra of light nuclei

Argonne V18

CD Bonn

Nijmegen I/II

Chiral N3LO

Argonne V18 +
Illinois 2

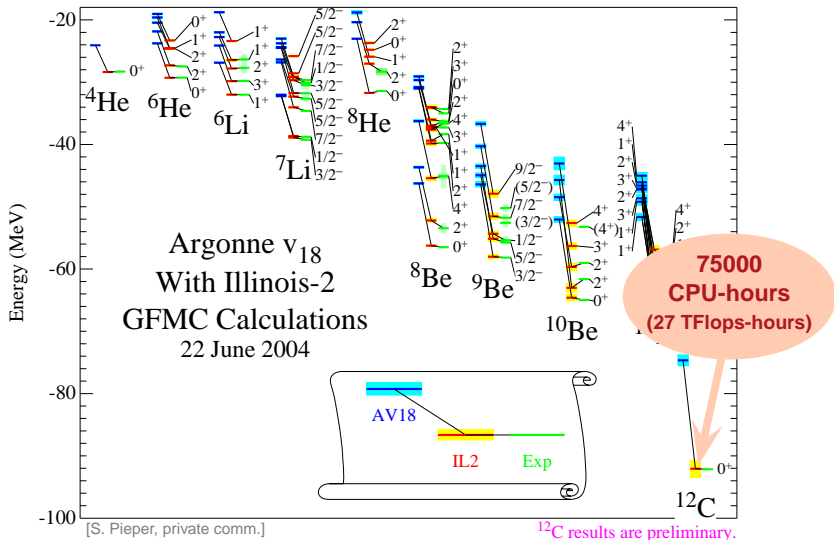
Chiral N3LO +
N2LO

Ab initio Methods

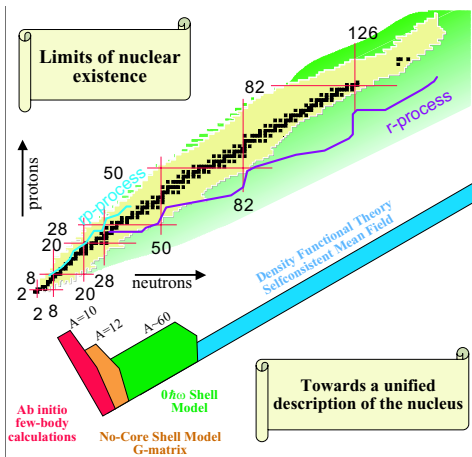
solve the quantum many-body problem for A nucleons interacting via a realistic NN-potential

- exact numerical solution possible only for small systems at an enormous computational cost
- Green's Function Monte Carlo: Monte Carlo sampling of the A -body wave function in coordinate space
- No-core Shell Model: large-scale diagonalization of the Hamiltonian in a harmonic oscillator basis

Ab initio Methods: GFMC



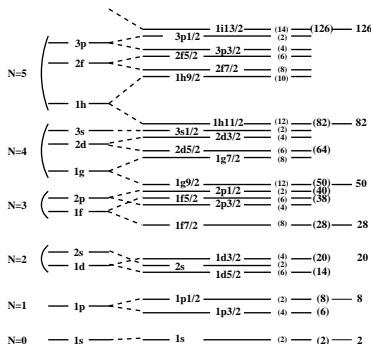
Theoretical models



Provide an approximate solution to the many-body problem

Theoretical models: basics

- They assume the existence of shells. Magic numbers are obtained when a shell is completely full.
- Shells results from the bunching (grouping) of levels coming from an independent particle average potential. Hartree-Fock method provides a way of obtaining this potential.



Independent-Particle Model

- Assume the existence of some single-particle wave functions that are the solution of a Schrödinger equation

$$h\phi(r) = \{T + U\}\phi_a(r) = \varepsilon_a\phi_a(r)$$

The independent-particle motion hamiltonian is then:

$$H_0 = \sum_{k=1}^A T(k) + U(r_k)$$

Eigenfunctions are the product of single-particle wave functions:

$$\Phi_{a_1 a_2 \dots a_A}(1, 2, \dots, A) = \prod_{k=1}^A \phi_{a_k}(r_k)$$

System identical particles

Wave function should be antisymmetric. For two particles:

$$\Phi_{ab}(1, 2) = \frac{1}{\sqrt{2}} [\phi_a(1)\phi_b(2) - \phi_a(2)\phi_b(1)] = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_a(1) & \phi_b(2) \\ \phi_a(2) & \phi_b(1) \end{vmatrix}$$

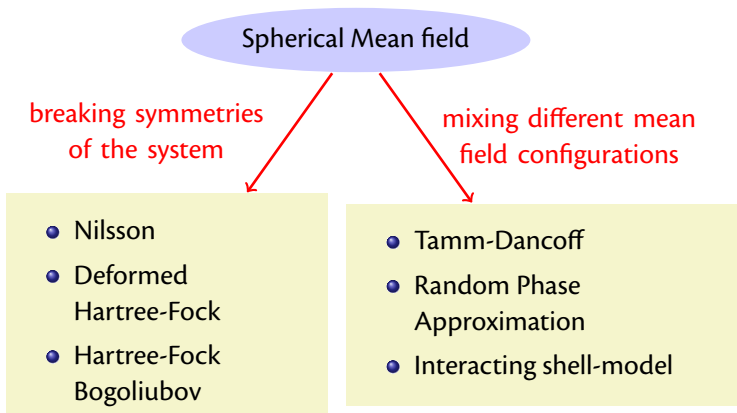
A-particle Wave function (Slater determinant):

$$\Phi_{a_1 a_2 \dots a_A}(1, 2, \dots, A) = \sqrt{\frac{1}{A!}} \begin{vmatrix} \phi_{a_1}(r(1)) & \phi_{a_1}(r(2)) & \dots & \phi_{a_1}(r(A)) \\ \phi_{a_2}(r(1)) & \phi_{a_2}(r(2)) & \dots & \phi_{a_2}(r(A)) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{a_A}(r(1)) & \phi_{a_A}(r(2)) & \dots & \phi_{a_A}(r(A)) \end{vmatrix}$$

In principle an infinite number of Slater determinants.

Residual interaction: Correlations

Residual interaction induces correlations between particles. In order to include them it is necessary to go beyond the mean-field.

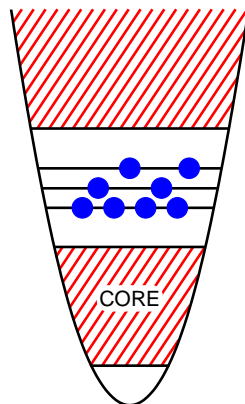


Shell Model

- Define a valence space
- Define an effective interaction

$$\mathbf{H}\Psi = E\Psi \rightarrow \mathbf{H}_{eff}\Psi_{eff} = E\Psi_{eff}$$

- Build and diagonalize the Hamiltonian matrix.



In principle, all the nuclear properties are described simultaneously.

Notation

In order to compute any transition mediated by the weak interaction we need to evaluate the matrix element of the relevant weak operator between some initial and final states:

$$|i\rangle = |J_i, T_i, T_{z_i}\rangle$$

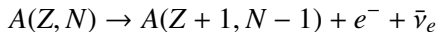
$$|f\rangle = |J_f, T_f, T_{z_f}\rangle$$

- J angular momentum of the state
- T isospin of the state
- T_z third component of the isospin ($\equiv (N - Z)/2$)

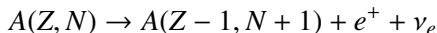
Nuclear beta decay, energetics

Q-value defined as the total kinetic released in the reaction

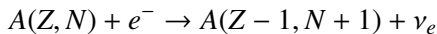
- β^- decay, $Q_{\beta^-} = M_i - M_f + E_i - E_f$



- β^+ decay, $Q_{EC} = Q_{\beta^+} + 2m_e = M_i - M_f + E_i - E_f$



- Electron capture,



Transition rates for β decay

Fermi's golden rule:

$$\lambda = \frac{2\pi}{\hbar} \int |\mathcal{M}_{if}|^2 (2\pi\hbar)^3 \delta^{(4)}(p_f + p_e + p_\nu - p_i) \frac{d^3 p_f}{(2\pi\hbar)^3} \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 p_\nu}{(2\pi\hbar)^3}$$

$$|\mathcal{M}_{if}|^2 = \frac{1}{2J_i + 1} \sum_{\text{lepton spins}} \sum_{M_i, M_f} |\langle f | \mathbf{H}_w | i \rangle|^2$$

Transition rates for β decay

$$\lambda = \frac{1}{2\pi\hbar^7} \int |\mathcal{M}_{if}|^2 \delta(M_f^{nuc} + E_e + E_\nu - M_i^{nuc}) p_e^2 p_\nu^2 dp_e dp_\nu \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

$$W = E_e / (m_e c^2)$$

$$W_0 = \frac{M_i^{nuc} - M_f^{nuc}}{m_e c^2} = \frac{Q}{m_e c^2} + 1$$

Transition rates for β decay

$$\lambda = \frac{m_e^5 c^4 G_V^2}{2\pi \hbar^7} \int_1^{W_0} C(W) F(Z, W) (W^2 - 1)^{1/2} W (W_0 - W)^2 dW$$

$$C(W) = \frac{1}{G_V^2} \int |\mathcal{M}_{if}|^2 \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

$F(Z, W)$ Fermi function, takes in account the distortion of the electron (positron) wave function due to Coulomb effects.

Transition rates for β decay

We need to compute shape factor,

$$C(W) = \frac{1}{G_V^2} \int \frac{1}{2J_i + 1} \sum_{\text{lepton spins}} \sum_{M_i, M_f} |\langle f | \mathbf{H}_w | i \rangle|^2 \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

between states:

$$|i\rangle = |J_i M_i; T_i T_{z_i}\rangle$$

$$|f\rangle = |J_f M_f; T_f T_{z_f}; e^-; \bar{\nu}\rangle$$

Weak Hamiltonian

Current-Current interaction:

$$\mathbf{H}_w = \frac{G_V}{\sqrt{2}} \int d^3r \mathcal{J}^\mu(\mathbf{r}) j_\mu(\mathbf{r})$$

Weak Hamiltonian

Current-Current interaction:

$$\langle f | \mathbf{H}_w | i \rangle = \frac{G_V}{\sqrt{2}} \int d^3 r \langle J_f M_f; T_f T_{z_f}; e, \nu | j_\mu \mathcal{J}^\mu | J_i M_i; T_i T_{z_i} \rangle$$

Assuming plane waves for electron and neutrino:

$$\langle e; \nu | j_\mu | 0 \rangle = e^{-i(\mathbf{p}_e + \mathbf{p}_\nu) \cdot \mathbf{r}} \bar{u} \gamma_\mu (1 - \gamma_5) v$$

Weak Hamiltonian

Current-Current interaction:

$$\langle f | \mathbf{H}_w | i \rangle = \frac{G_V}{\sqrt{2}} l_\mu \int d^3 r e^{-i\mathbf{q}\cdot\mathbf{r}} \langle J_f M_f; T_f T_{z_f} | \mathcal{J}^\mu | J_i M_i; T_i T_{z_i} \rangle$$

$$l_\mu = \bar{u} \gamma_\mu (1 - \gamma_5) v$$

Non relativistic reduction

Assuming one nucleon participates in the decay and that we can use the free current (impulse approximation):

$$\langle f | \mathbf{H}_w | i \rangle = \frac{G_V}{\sqrt{2}} l_\mu \int d^3 r e^{-i\mathbf{q}\cdot\mathbf{r}} \bar{\psi}_f \gamma^\mu (1 + g_A \gamma_5) \mathbf{t}_\pm \psi_i$$

$$\psi = \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M} \end{pmatrix} \phi \rightarrow \begin{pmatrix} \phi \\ 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}; \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\langle f | \mathbf{H}_w | i \rangle = \frac{G_V}{\sqrt{2}} \int d^3 r e^{-i\mathbf{q}\cdot\mathbf{r}} \phi_f (l_0 \mathbf{1} + g_A \mathbf{l} \cdot \boldsymbol{\sigma}) \mathbf{t}_\pm \phi_i$$

Non relativistic reduction

Generalization to A particles:

$$H_w = \frac{G_V}{\sqrt{2}} \sum_{k=1}^A e^{-i\mathbf{q}\cdot\mathbf{r}_k} (l_0 \mathbf{1}^k + g_A \mathbf{l} \cdot \boldsymbol{\sigma}^k) t_{\pm}^k$$

$$e^{-i\mathbf{q}\cdot\mathbf{r}} = \sum_l \sqrt{4\pi(2l+1)} (-i)^l j_l(qr) Y_{l0}(\theta, \varphi)$$

$$j_l(qr) \approx \frac{(qr)^l}{(2l+1)!!}$$

- Zero order: Allowed transitions (Fermi, Gamow-Teller)
- Higher orders: Forbidden transitions.

ft-value

$$\lambda = \frac{\ln 2}{K} \int_1^{W_0} C(W) F(Z, W) (W^2 - 1)^{1/2} W (W_0 - W)^2 dW$$

For Allowed transitions: $C(W) = B(F) + B(GT)$,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{K} [B(F) + B(GT)] f(Z, W_0)$$

$$ft_{1/2} = \frac{K}{B(F) + B(GT)}, \quad K = 6147.0 \pm 2.4 \text{ s}$$

Fermi Transitions

$$B(F) = \frac{1}{2J_i + 1} \sum_{M_i, M_f} |\langle J_f M_f; T_f T_{z_f} | \sum_{k=1}^A \mathbf{t}_{\pm}^k | J_i M_i; T_i T_{z_i} \rangle|^2$$

$$B(F) = [T_i(T_i + 1) - T_{z_i}(T_{z_i} \pm 1)] \delta_{J_i, J_f} \delta_{T_i, T_f} \delta_{T_{z_f}, T_{z_i} \pm 1}$$

Energetics:

$$E_{\text{IAS}} = Q_{\beta} + \text{sign}(T_{z_i}) [E_C(Z + 1) - E_C(Z) - (m_n - m_H)]$$

$$\Delta E_C = 1.4136(1) \bar{Z}/A^{1/3} - 0.91338(11) \text{ MeV}$$

Selection rule:

$$\Delta J = 0 \quad \Delta T = 0 \quad \pi_i = \pi_f$$

Sum rule (sum over all the final states):

$$S(F) = S_-(F) - S_+(F) = 2T_{z_i} = (N - Z)$$

Gamow-Teller Transitions

$$B(GT) = \frac{g_A^2}{2J_i + 1} |\langle J_f; T_f T_{z_f} \| \sum_{k=1}^A \sigma^k t_{\pm}^k \| J_i; T_i T_{z_i} \rangle|^2$$

$$g_A = -1.2695 \pm 0.0029$$

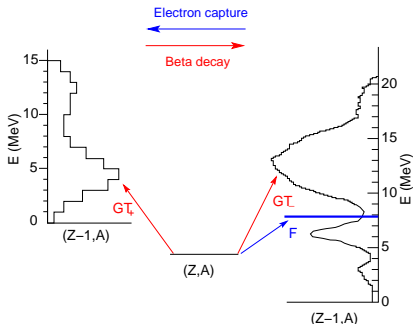
Selection rule:

$$\Delta J = 0, 1 \text{ (no } J_i = 0 \rightarrow J_f = 0) \quad \Delta T = 0, 1 \quad \pi_i = \pi_f$$

Ikeda sum rule:

$$S(GT) = S_-(GT) - S_+(GT) = 3(N - Z)$$

Summary



- In neutron rich nuclei GT_+ strength represent a small part of Ikeda sum rule [$3(N-Z)$].
- For neutron rich nuclei GT_- constitutes most of the Ikeda sum rule. Most of the strength is located in a resonance with a width of several MeV and at energy: $E_{GT} - E_{IAS} = 7.0 - 28.9(N - Z)/A$ MeV.
- Fermi transitions only contribute to the β^- direction. All the strength $(N-Z)$ is located at the IAS state at an energy with respect of the parent state: $Q_{IAS} = \Delta E_C - (m_n - m_p)$

Electron capture and Neutrino absorption

Fermi's golden rule:

$$\sigma = \frac{2\pi}{\hbar v_e} \int |\mathcal{M}_{if}|^2 (2\pi\hbar)^3 \delta^{(4)}(p_f + p_\nu - p_i - p_e) \frac{d^3 p_f}{(2\pi\hbar)^3} \frac{d^3 p_\nu}{(2\pi\hbar)^3}$$

Electron capture: $(Z, A) + e^- \rightarrow (Z - 1, A) + \nu_e$

$$\sigma_{i,f}(E_e) v_e = \frac{G_V^2}{2\pi\hbar^4 c} F(Z, E_e) [B(F) + B(GT)] p_\nu^2$$

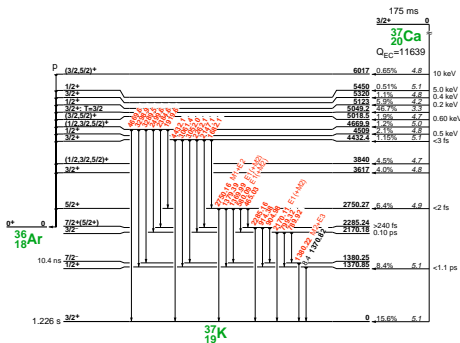
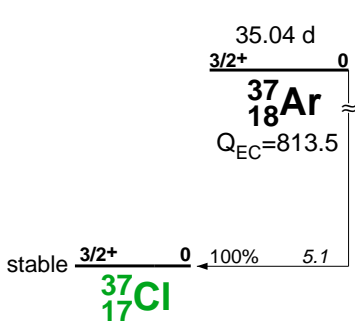
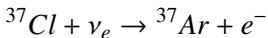
Neutrino absorption: $(Z, A) + \nu_e \rightarrow (Z + 1, A) + e^-$

$$\sigma_{i,f}(E_\nu) = \frac{G_V^2}{\pi\hbar^4 c^3} p_e E_e F(Z + 1, E_e) [B(F) + B(GT)]$$

$$\sigma_0 = \frac{6G_F^2 V_{ud}^2 (m_e c^2)^2}{\pi\hbar^4 c^4} = 2.505(2) \times 10^{-44} \text{ cm}^2$$

Example: Solar neutrino rate on Cl

Neutrinos detected via reaction:



Summing over all final states and integrating over ^8B neutrino spectrum the cross section is $\sigma = 1.14 \times 10^{-42} \text{ cm}^2$. Multiplying by the total ^8B flux ($5.69 \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1}$)

6.6 SNU (10^{-36} captures per target per second)

Neutrino scattering

Neutrinos can also interact via the neutral current.

- Vector part of the current describes elastic scattering (responsible for neutrino trapping in supernovae):

$$\sigma(E_\nu) = \frac{G_F^2}{4\pi\hbar^4 c^4} E_\nu^2 \left[N - (1 - 4 \sin^2 \theta_W) Z \right]^2$$

- Axial vector part describes neutrino scattering:
 $(Z, A) + \nu \rightarrow (Z, A)^* + \nu$

$$\sigma_{i,f}(E_\nu) = \frac{G_F^2}{\pi\hbar^4 c^4} (E_\nu - w)^2 B(GT_0)$$

with $w = E_f - E_i$. In general, multipoles beyond allowed transitions are necessary. See Donnelly and Peccei, Phys. Repts. **50**, 1 (1979).

Exercise: Neutrino trapping in supernovae

During the collapse of the core of a massive star the densities become so large that even neutrinos become dynamically trapped in the collapsing core at densities $\sim 10^{12} \text{ g cm}^{-3}$.

The neutrino mean free path ($\lambda_\nu = 1/n\sigma$) can be estimated from the previous expression for the cross section (assume matter composed of nuclei with $A = 110$ $Z = 40$).

$$1/\lambda_\nu = \frac{\rho N_A G_F^2}{4\pi(\hbar c)^4 A} E_\nu^2 N^2 \approx 2.5 \times 10^{-9} \rho_1 2 E_\nu^2 N^2 / A$$

$$\lambda_\nu \approx 220 \text{ m } (E_\nu = 20 \text{ MeV})$$

The diffusion time for a distance of 30 km is:

$$t = \frac{3L^2}{c\lambda_\nu} \approx 41 \text{ ms}$$

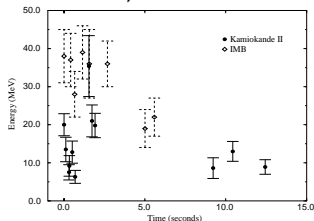
SN1987A

Type II supernova in LMC
(~ 55 kpc)

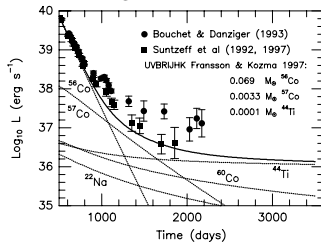


- $E_{\text{grav}} \approx 10^{53}$ erg
- $E_{\text{rad}} \approx 8 \times 10^{49}$ erg
- $E_{\text{kin}} \approx 10^{51}$ erg = 1 foe

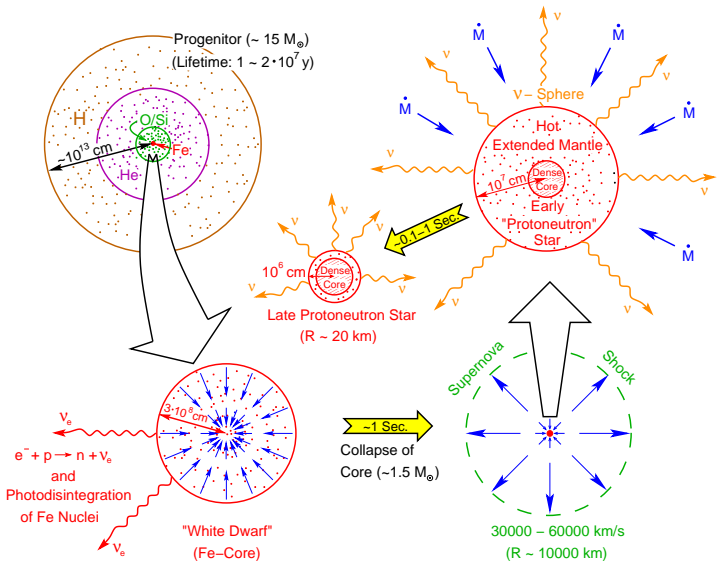
neutrinos $E_\nu \approx 2.7 \times 10^{53}$ erg



light curve



Schematical Evolution



(From H.-Th. Janka)

Presupernova Models

- Describe the massive star evolution through the various hydrostatic burning stages (H, He, ..., Si) and follows the collapse of the central iron core until densities $\sim 10^{10} \text{ g cm}^3$ are reached.
- Large nuclear networks are used to determine the nuclear energy generation and the associated nucleosynthesis. Transition to Nuclear Statistical Equilibrium takes place after Silicon burning (Iron core formation).
- Neutrinos produced by weak interactions can leave the star unhindered.

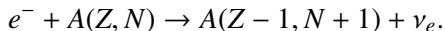
Early iron core

- The core is made of heavy nuclei (iron-mass range $A = 45-65$) and electrons. Composition given by Nuclear Statistical Equilibrium. There are Y_e electrons per nucleon.
- The mass of the core M_c is determined by the nucleons.
- There is no nuclear energy generation which adds to the pressure. Thus, the pressure is mainly due to the degenerate electrons, with a small correction from the electrostatic interaction between electrons and nuclei.
- As long as $M_c < M_{\text{ch}} = 1.44(2Y_e)^2 M_\odot$ (plus slight corrections for finite temperature), the core can be stabilized by the degeneracy pressure of the electrons.

Onset of collapse

There are two processes that make the situation unstable:

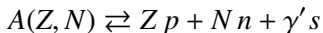
- 1 Silicon burning is continuing in a shell around the iron core. This adds mass to the iron core increasing M_c .
- 2 Electrons can be captured by protons (free or in nuclei):



This reduces the pressure and keep the core cool, as the neutrinos leave. The net effect is a reduction of Y_e and consequently of the Chandrasekhar mass (M_{ch})

Nuclear Statistical Equilibrium

- Processes mediated by the strong and electromagnetic interactions are in equilibrium. As neutrinos escape the weak interaction is not in equilibrium.
- Processes of creation and destruction are in equilibrium:



- Composition depends only on (T, ρ, Y_e) and its determined by the Entropy ($\sim T^3/\rho$). Large entropies (small ρ , large T) favors free nucleons. Small entropies (large ρ , small T) favors bound nuclei.

Nuclear abundances in NSE

NSE implies:

$$\mu(Z, A) = Z\mu_p + (A - Z)\mu_N$$

with the chemical potentials given by (Boltzmann)

$$\mu(Z, A) = m(Z, A)c^2 + kT \ln \left[\frac{n(Z, A)}{G(Z, A)} \left(\frac{2\pi\hbar^2}{m(Z, A)kT} \right)^{3/2} \right]$$

and the partition function:

$$G(Z, A) = \sum_i (2J_i + 1) e^{-E_i/kT}$$

Solution to NSE

Saha equation ($n_i = n(A_i, Z_i)$):

$$n(Z, A) = \frac{G(Z, A)A^{3/2}}{2^A} n_p^Z n_n^N \left(\frac{2\pi\hbar^2}{m_u kT} \right)^{3/2(A-1)} e^{B(Z,A)/kT}$$

with the constraints

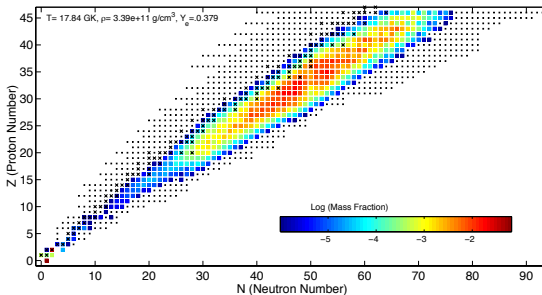
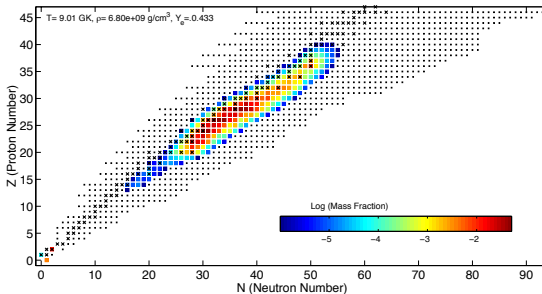
- $\sum_i n_i A_i = n$ (conservation number nucleons)
- $\sum_i n_i Z_i = n_e = n Y_e$ (charge neutrality)

Partition function determines composition during collapse (Bethe *et al.*, 1979)

$$G(T) \approx \frac{\pi}{6akT} \exp(akT)$$

Energy liberated during the collapse increases the internal excitation of the nuclei. Matter remains relatively cool and with low entropies ($\sim 1k$)

Abundances



Initial conditions

The dominant contribution to the pressure comes from the electrons. They are degenerate and relativistic:

$$P/\rho \approx \frac{Y_e \mu_e}{4}$$

μ_e is the chemical potential of the electrons:

$$\mu_e \approx 1.11(\rho_7 Y_e)^{1/3} \text{ MeV}$$

For $\rho_7 = 1$ ($\rho = 10^7 \text{ g cm}^{-3}$) the chemical potential is 1 MeV, reaching the nuclear energy scale. At this point is energetically favorable to capture electrons by nuclei.

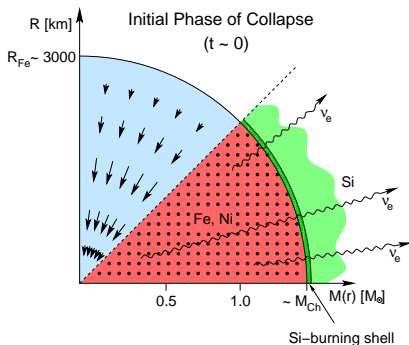
How to determine the evolution

- Composition determined by NSE, function of temperature, density and Y_e .
- Weak interactions are not in equilibrium. Change of Y_e has to be computed explicitly ($Y_i = n_i/n$):

$$Y_e = \sum_i Y_i Z_i$$

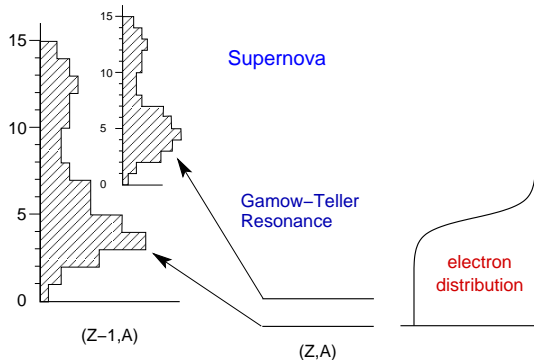
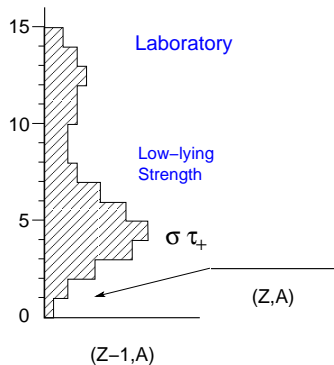
$$\dot{Y}_e = - \sum_i \lambda_{ec}^i Y_i + \sum_i \lambda_{\beta^-}^i Y_i$$

Presupernova evolution



- $T = 0.1\text{--}0.8$ MeV,
 $\rho = 10^7\text{--}10^{10}$ g cm $^{-3}$.
Composition of iron group nuclei.
- Important processes:
 - electron capture:
 $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$
 - β^- decay:
 $(N, Z) \rightarrow (N-1, Z+1) + e^- + \bar{\nu}_e$
- Dominated by allowed transitions (Fermi and Gamow-Teller)
- Evolution decreases number of electrons (Y_e) and Chandrasekar mass ($M_{\text{ch}} \approx 1.4(2Y_e)^2 M_{\odot}$)

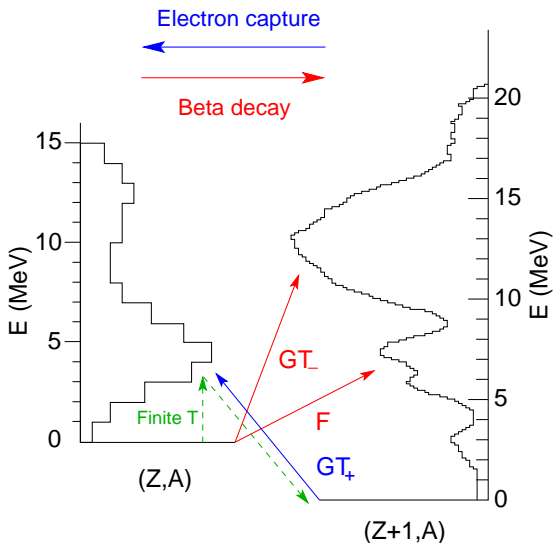
Laboratory vs. stellar electron capture



Capture of K-shell electrons to tail of GT strength distribution. Parent nucleus in the ground state

Capture of electrons from the high energy tail of the FD distribution. Capture to states with large GT matrix elements (GT resonance). Thermal ensemble of initial states.

Beta-decay



GT in charge exchange reactions

GT strength could be measured in Charge-Exchange reactions:

- GT_- proved in (p, n) , $({}^3\text{He}, t)$.
- GT_+ proved in (n, p) , $(t, {}^3\text{He})$, $(d, {}^2\text{He})$.

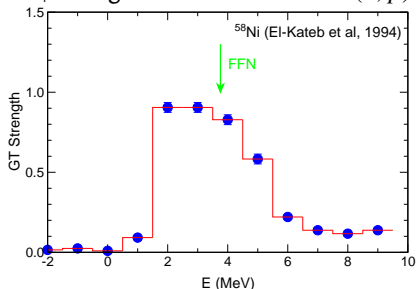
Mathematical relationship ($E_p \geq 100$ MeV/nucleon):

$$\frac{d\sigma}{d\Omega dE}(0^\circ) \approx S(E_x)B(GT)$$

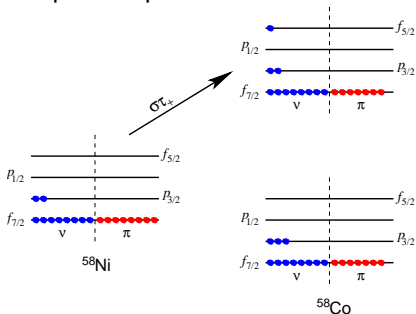
$$B(GT) = \left(\frac{g_A}{g_V}\right)^2 \frac{\langle f || \sum_k \sigma^k \mathbf{t}_\pm^k || i \rangle^2}{2J_i + 1}$$

Independent Particle Model

GT₊ strength in ⁵⁸Ni measured in (n, p).



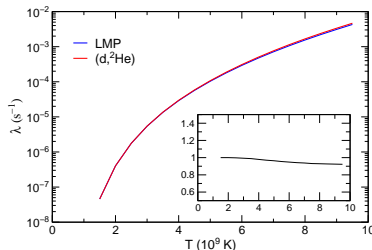
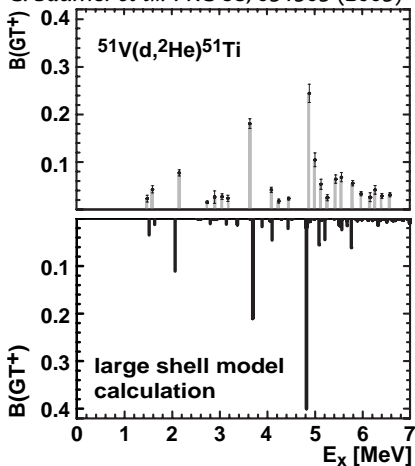
Independent particle model.



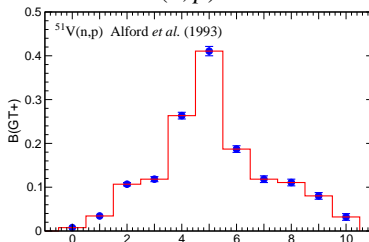
The IPM allows for a single transition ($f_{7/2} \rightarrow f_{5/2}$). It does not correctly reproduce the fragmentation of GT strength (correlations).

Shell-Model vs experiment

C. Bäumer *et al.* PRC **68**, 031303 (2003)

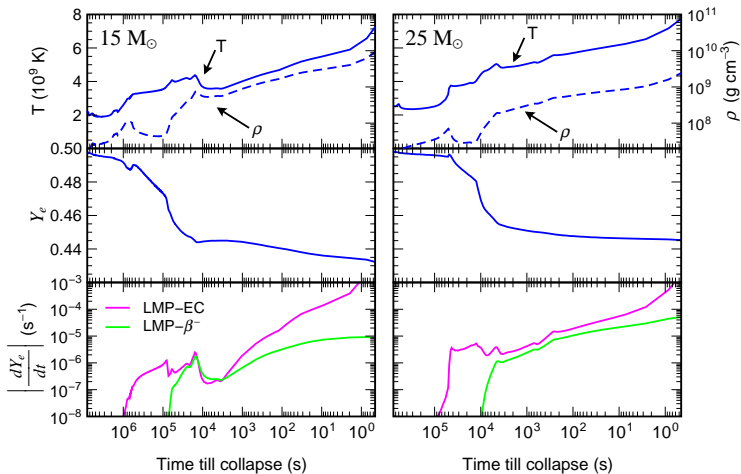


Old (*n*, *p*) data

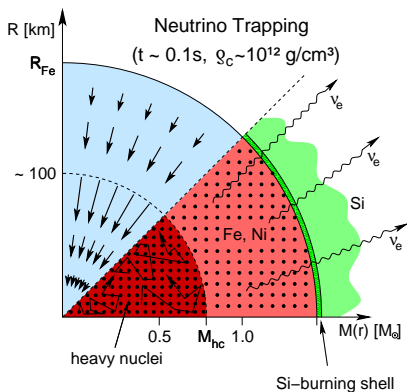


Consequences weak rates

(A. Heger *et al.*, 2001)



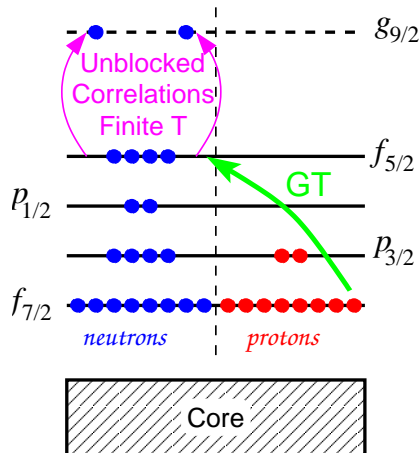
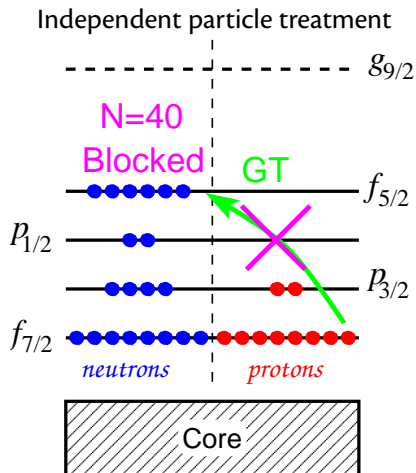
Collapse phase



Important processes:

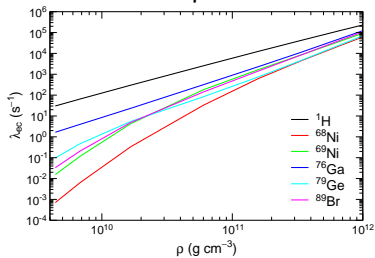
- Neutrino transport
(Boltzmann equation):
 $\nu + A \rightleftharpoons \nu + A$ (trapping)
 $\nu + e^- \rightleftharpoons \nu + e^-$ (thermalization)
 cross sections $\sim E_{\nu}^2$
- electron capture on protons:
 $e^- + p \rightleftharpoons n + \nu_e$
- electron capture on nuclei:
 $e^- + A(Z, N) \rightleftharpoons A(Z-1, N+1) + \nu_e$

(Un)blocking electron capture at N=40

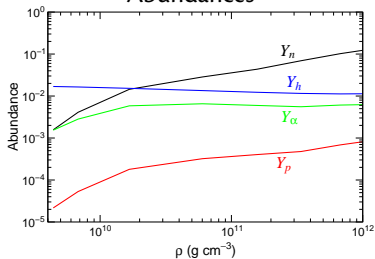


Electron capture: nuclei vs protons

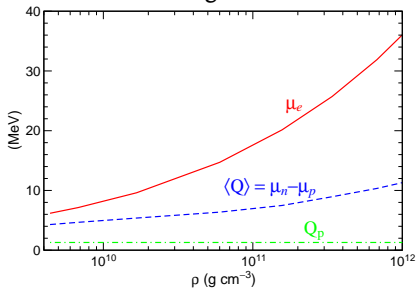
Electron capture rates



Abundances



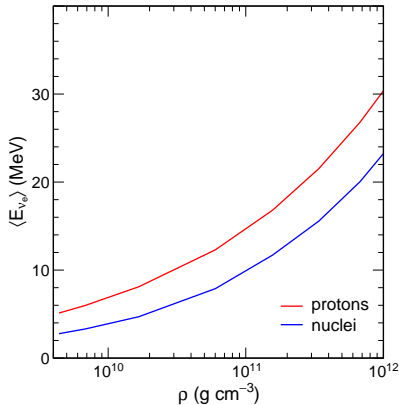
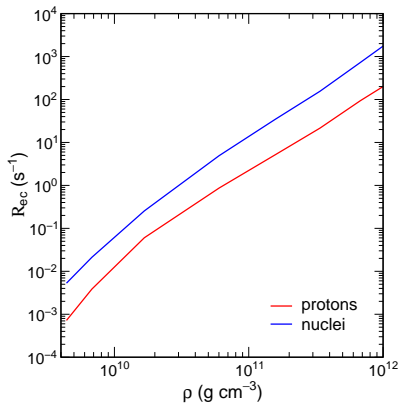
Energetics



$$R_h = \sum_i Y_i \lambda_i = Y_h \langle \lambda_h \rangle$$

$$R_p = Y_p \lambda_p, \quad Y_i = n_i/n$$

Reaction rates



Electron capture on nuclei dominates over capture on protons

Neutrino-nucleus interactions

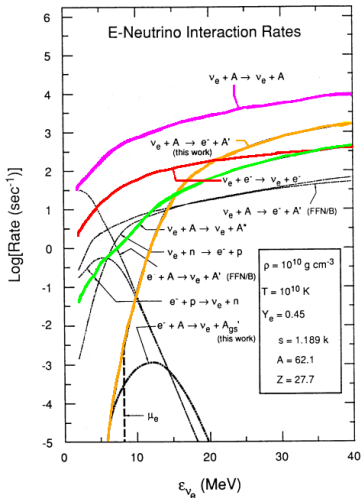
Neutrino-nucleus interactions are necessary for several applications

- During the collapse of a massive star neutrino-nucleus inelastic scattering can play a role in the dynamics.
- Neutrinos emitted from the exploding core can contribute to the nucleosynthesis of several key isotopes (^{11}B , ^{19}F , ^{138}La , ^{180}Ta) (ν -process).
- The r-process is thought to occur in the neutrino-driven wind from a proto-neutron star. Neutrino-nucleus interactions will compete with beta-decays.
- The detection of neutrinos from astrophysical sources requires the knowledge of neutrino-cross sections on the detector material.

Neutrino interactions in the collapse

Bruenn and Haxton (1991)

Based on results for ^{56}Fe



- **Elastic scattering:**
 $\nu + A \rightleftharpoons \nu + A$ (trapping)
- **Absorption:**
 $\nu_e + (N, Z) \rightleftharpoons e^- + (N - 1, Z + 1)$
- **ν - e scattering:**
 $\nu + e^- \rightleftharpoons \nu + e^-$ (thermalization)
- **Inelastic ν -nucleus scattering:**
 $\nu + A \rightleftharpoons \nu + A^*$

Neutrino absorption on ^{56}Fe

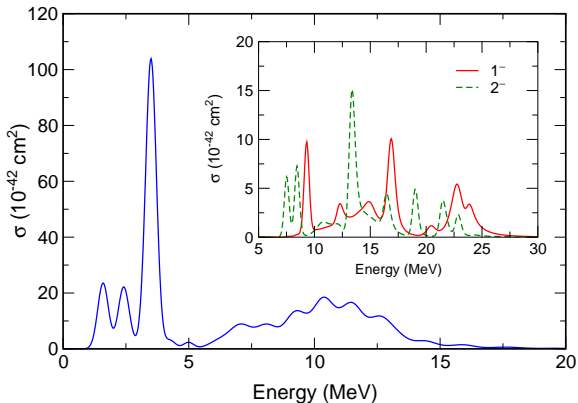
Neutrino-nucleus cross sections are difficult to measure. ν_e absorption is measured in ^{12}C and ^{56}Fe . No data for inelastic scattering exists.

$^{56}\text{Fe}(\nu_e, e^-)^{56}\text{Co}$ measured by KARMEN collaboration

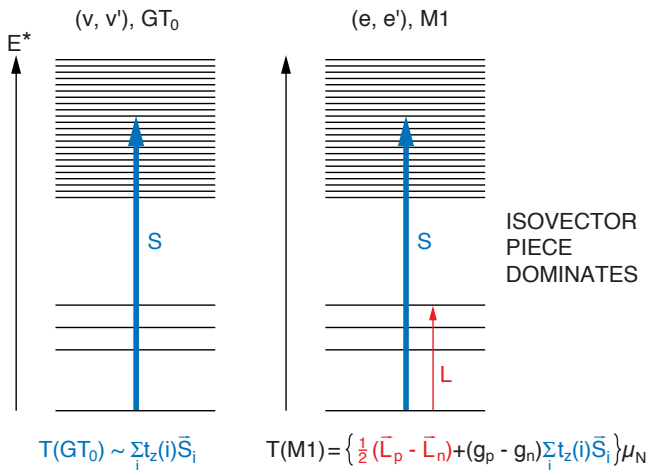
(ν_e from muon decay):

$$\sigma_{\text{exp}} = 2.56 \pm 1.08(\text{stat}) \pm 0.43(\text{syst}) \times 10^{-40} \text{ cm}^2$$

$$\sigma_{\text{th}} = 2.38 \times 10^{-40} \text{ cm}^2$$



Neutrino scattering from (e, e')

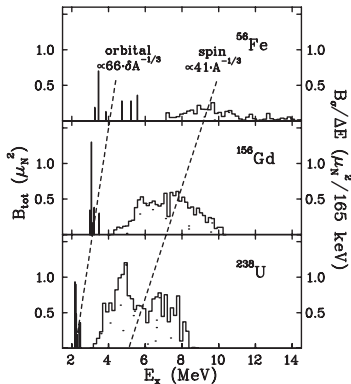


M1 data give GT_0 information

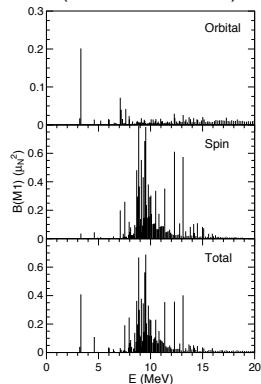
if **Orbital contribution can be removed**

Neutrino scattering from (e, e')

M1 DATA
(A. Richter)

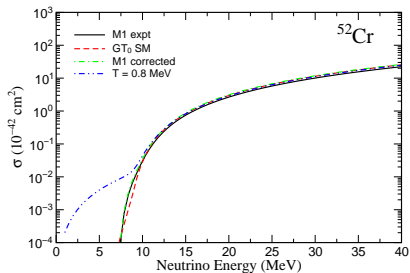
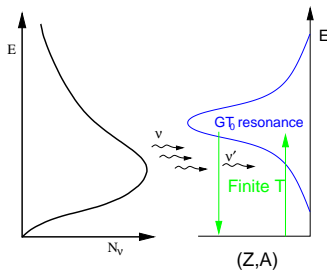
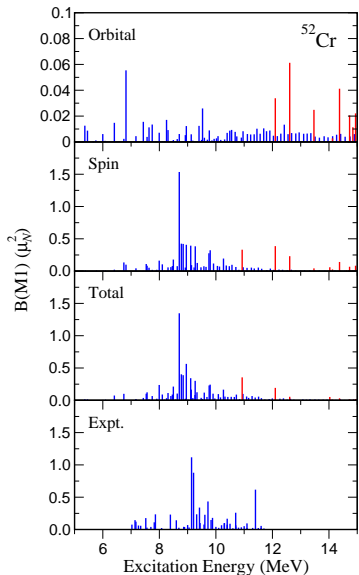


DECOMPOSITION OF
M1 STRENGTH
(SHELL MODEL)

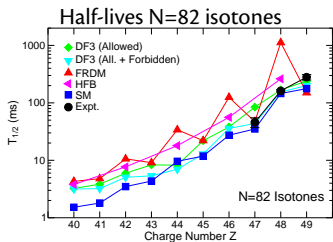


Usually orbital and spin parts well separated.
Spherical nuclei: Orbital part strongly suppressed.

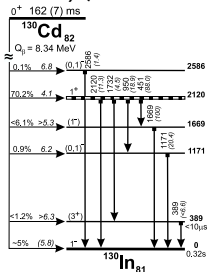
Neutrino Scattering from (e, e')



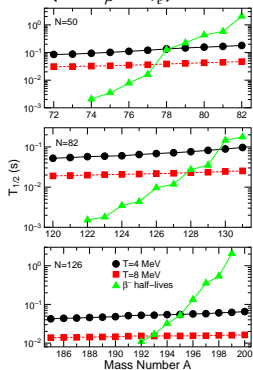
Weak processes in the r-process



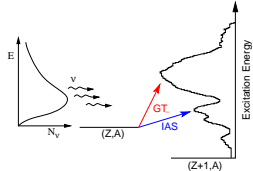
^{130}Cd decay (I. Dillmann *et al*)



ν_e charge-current interactions can accelerate the flow of matter ($\lambda = \lambda_{\beta} + \lambda_{\nu_e}$)

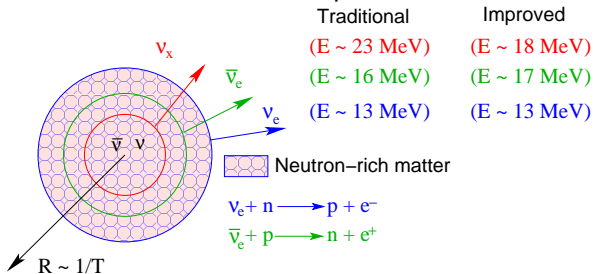


Neutrino rates are not sensitive to shell-effects

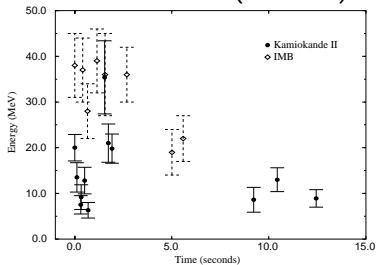


Neutrinos from supernovae

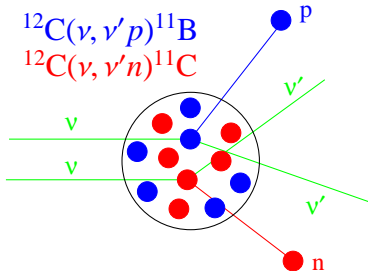
Raffelt *et al.*, astro-ph/0303226



neutrino detection (SN1987A)

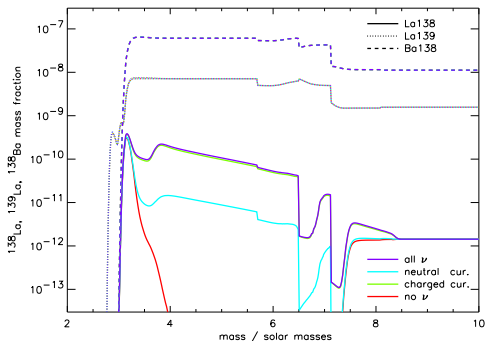
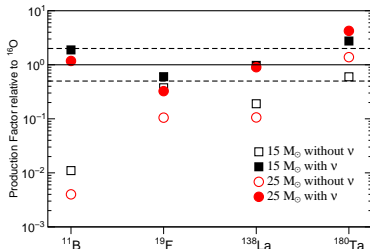


Neutrino nucleosynthesis



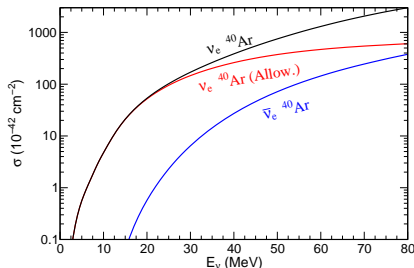
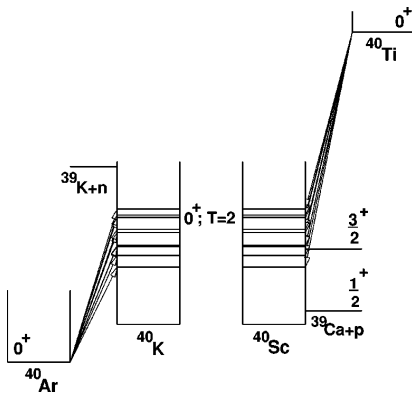
Neutrino nucleosynthesis

Product	Parent	Reaction
^{11}B	^{12}C	$(\nu, \nu' n), (\nu, \nu' p)$
^{19}F	^{20}Ne	$(\nu, \nu' n), (\nu, \nu' p)$
^{138}La	^{138}Ba	(ν_e, e^-)
	^{139}La	$(\nu, \nu' n)$
^{180}Ta	^{180}Hf	(ν_e, e^-)
	^{181}Ta	$(\nu, \nu' n)$



Neutrino detection on Earth

ICARUS (3 kton liquid ^{40}Ar detector)



At supernovae neutrino energies large contribution of forbidden transitions.