Neutrino Oscillations

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Neutrinos come in different flavors $\nu_e, \nu_\mu, \nu_\tau, \ldots$

Flavor eigenstates $\neq$ mass eigenstates   Pontecorvo (1957)

The rest is just quantum mechanics . . .

Consider two flavors ν_e, ν_μ

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \end{aligned}$$

Flavor eigenstates $|\nu_{e,\mu}\rangle$ $\leftrightarrow$ Mass eigenstates $|\nu_{1,2}\rangle$

What does that have to do with oscillations?

Consider Schrödinger equation

$$i\hbar \frac{d}{dt}\psi = H\psi \quad \Rightarrow \quad i\hbar \frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = H \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

What is the Hamiltonian H?

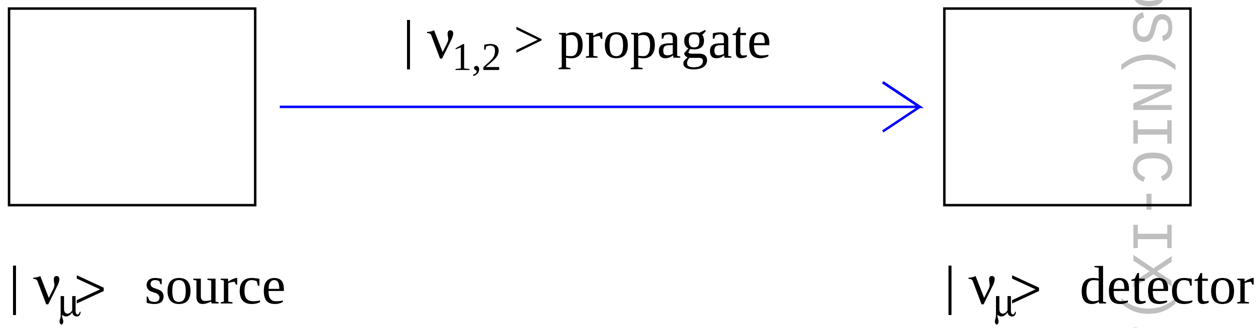
$$\begin{aligned} H|\nu_1\rangle &= (p^2 + m_1^2)^{1/2}|\nu_1\rangle \\ H|\nu_2\rangle &= (p^2 + m_2^2)^{1/2}|\nu_2\rangle \end{aligned}$$

H is diagonal in basis of mass eigenstates

Write $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \ldots$. The rest is algebra.

Wave equation

$$i\hbar c \frac{d}{dr}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \frac{\delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$



$$P(|\nu_\mu\rangle \rightarrow |\nu_\mu\rangle) = 1 - \sin(2\theta)\sin^2\left(\frac{1.27\delta m^2 L}{E}\right)$$

θ	mixing angle	L	path length
δm^2	mass difference	E	neutrino energy

Real world: 3×3 (or more) mixing $\rightarrow$ MNS matrix

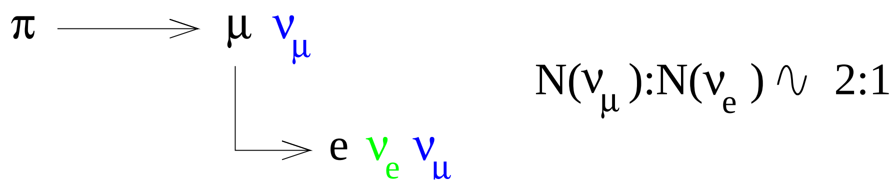
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

3 mixing angles, CP violating phase

$$\begin{aligned} U_{e1} &= \cos\theta_{12}\cos\theta_{13} \\ U_{e2} &= \sin\theta_{12}\cos\theta_{13} \\ U_{e3} &= \sin\theta_{13}e^{-i\delta} \\ &\ldots \end{aligned}$$

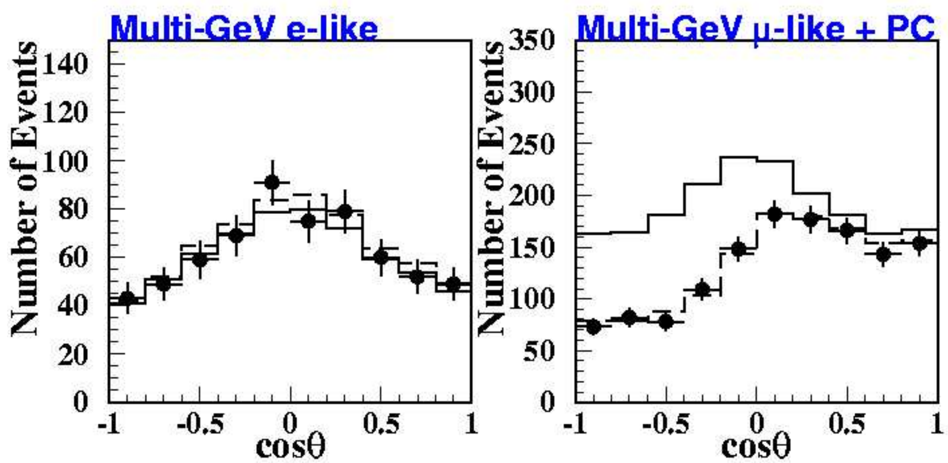
Atmospheric Neutrinos

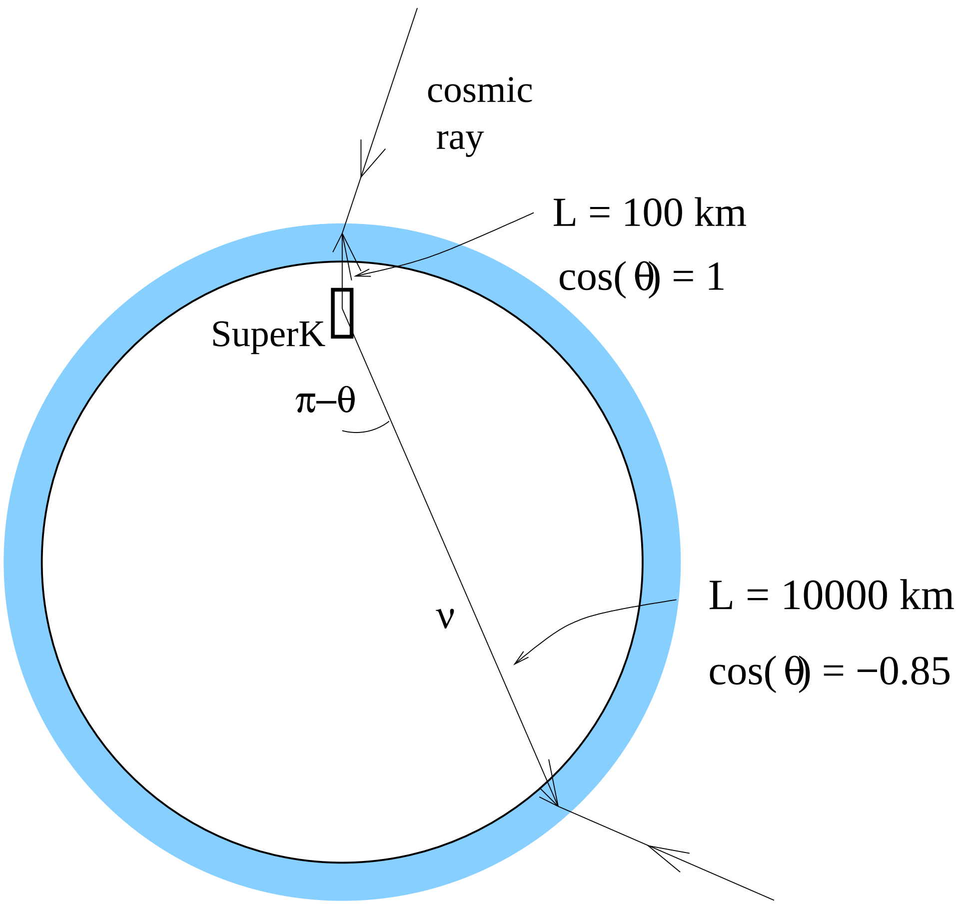
Cosmic rays collide with O, N and produce $\pi, K, \ldots$ which decay to ν_e, ν_μ



SuperK finds a deficit of ν_μ, no enhancement of ν_e.

SuperK also finds azimuthal dependence of ν_μ suppression





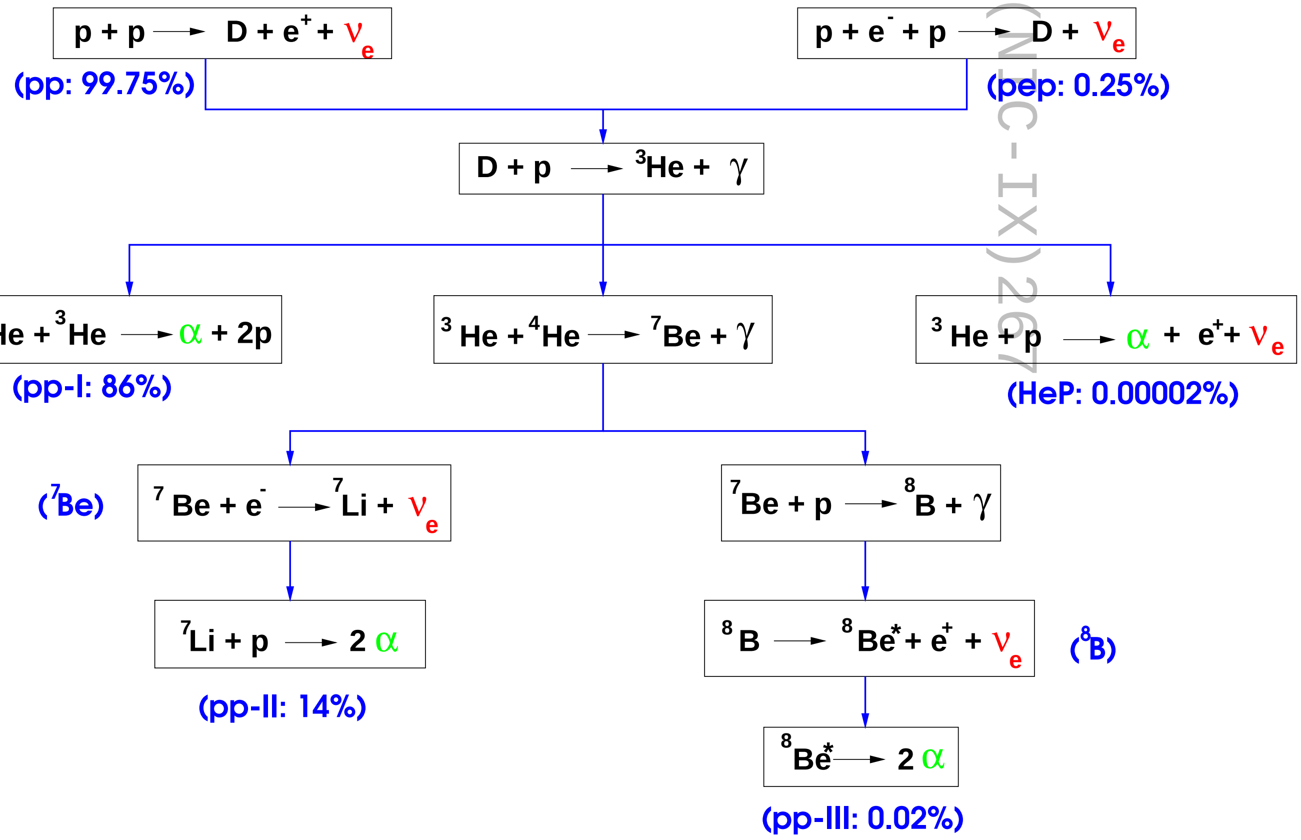
Explanation: $\nu_\mu \to \nu_\tau$ oscillations

$$\delta m^2_{23} \text{ or } \delta m^2_{32} \simeq 2 \times 10^{-3}\,\text{eV}^2$$

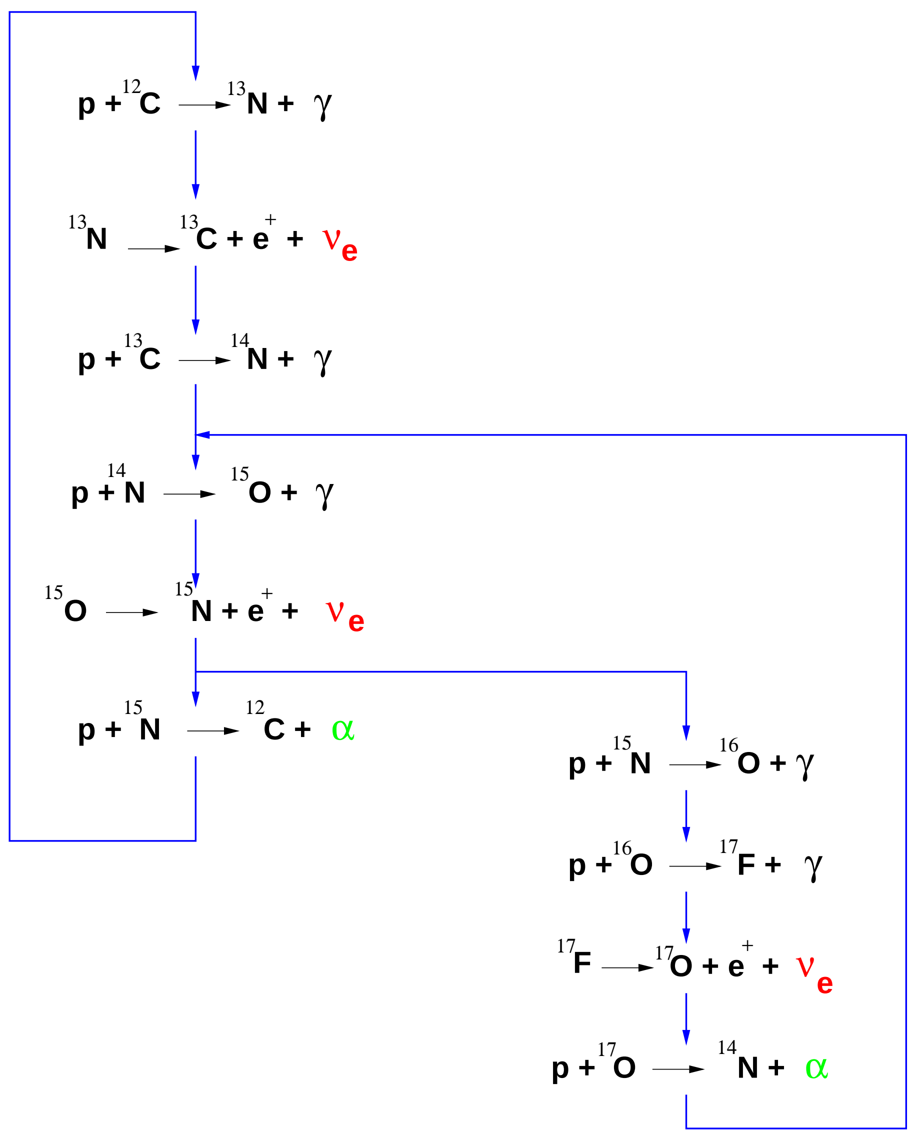
$$\sin^2 2\theta_{23} \simeq 1$$

Solar Neutrinos

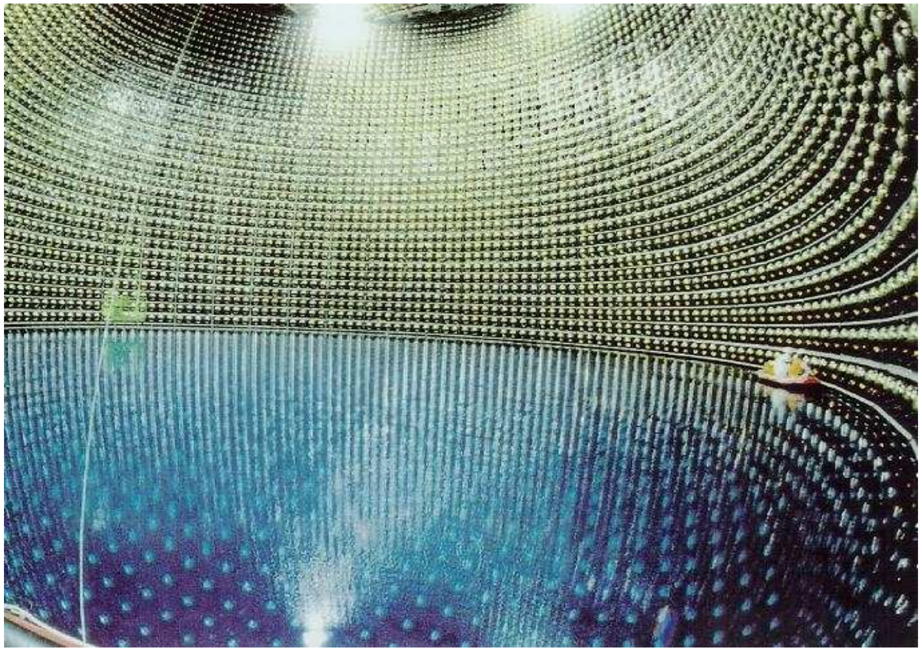
Sun produces ν_e through nuclear burning: pp cycle

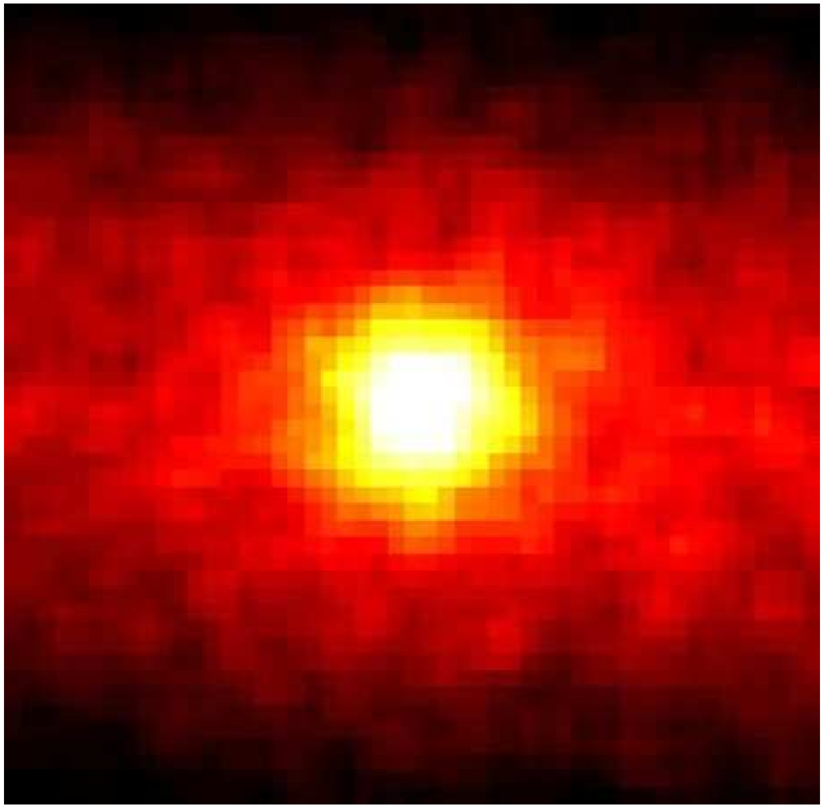


CNO cycle (small)



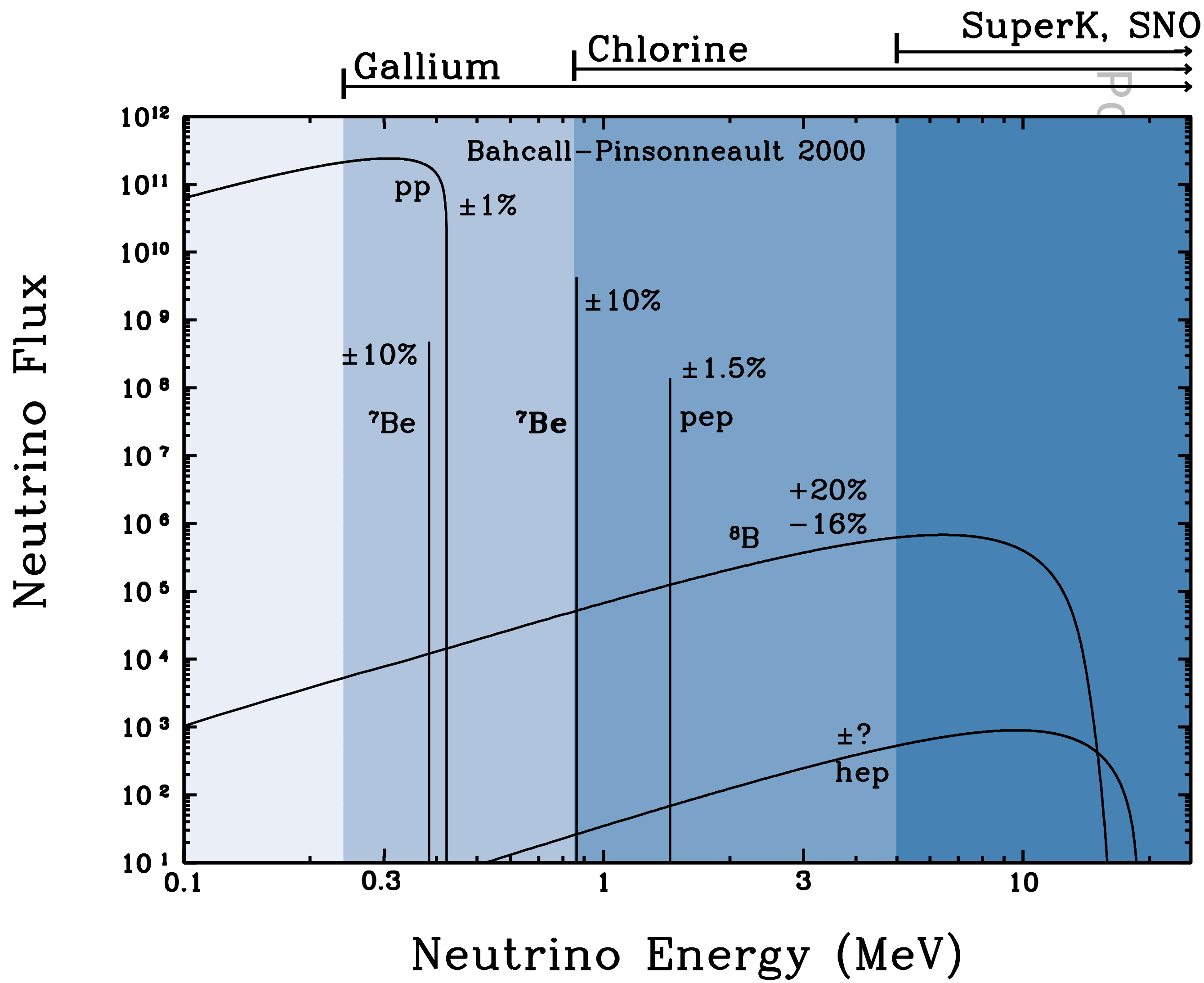
SuperK can see the Sun in Neutrinos





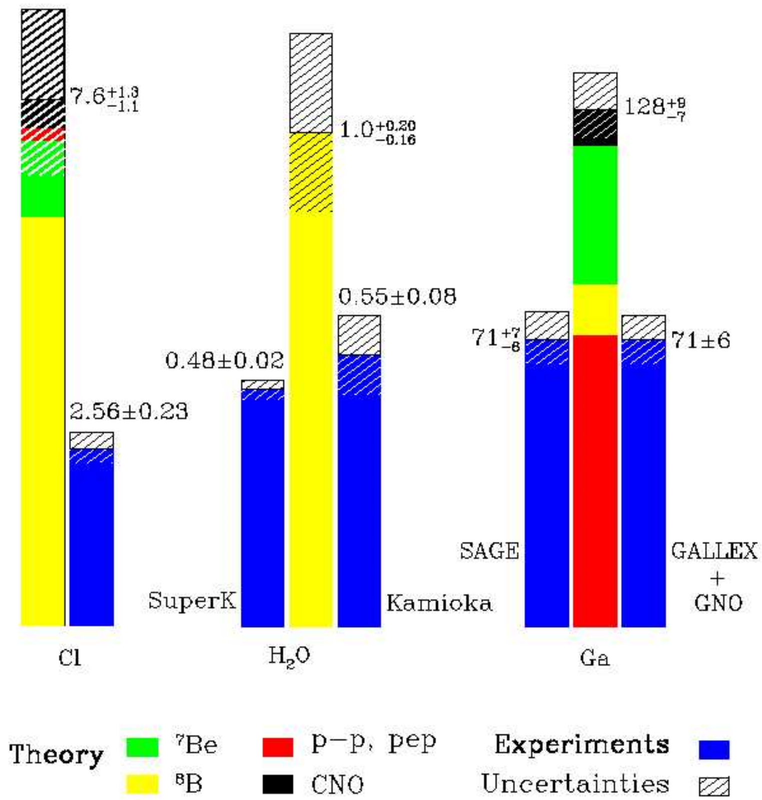
$$\nu_e + e^- \rightarrow \nu_e + e^-$$

ν_e flux predicted by standard solar model (Bahcall and Pinsonneault, pp chain only)



Experiments measure Deficit of ν_e

(Homestake, Gallex, Sage, Kamiokande, SuperK, SNO)



Homestake

$$\nu_e + Cl \to e^- + Ar$$

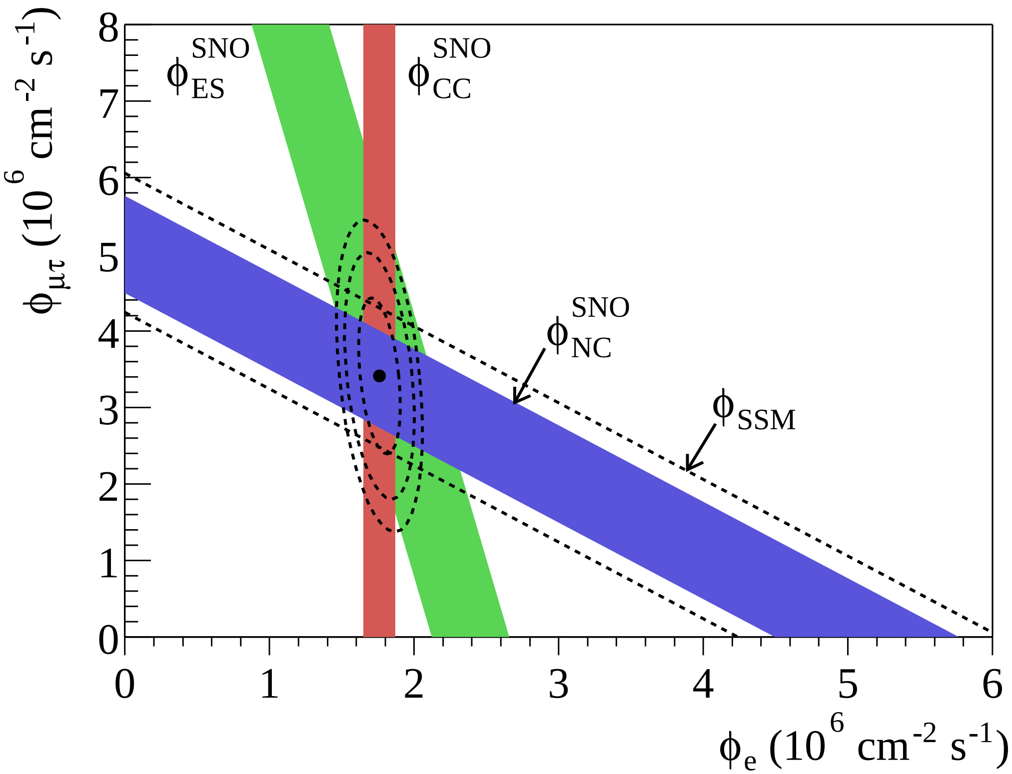
SAGE, GALLEX

$$\nu_e + Ga \to e^- + Ge$$

SuperK

$$\nu_e + e^- \to \nu_e + e^-$$

SNO can measure both the total $\nu_e + \nu_\mu + \nu_\tau$ flux and the ν_e flux



(CC)

$$\nu_e + D \rightarrow p + p + e^-$$

(NC)

$$\nu_x + D \rightarrow n + p + \nu_x$$

Total flux matches prediction!

Explanation: $\nu_e \rightarrow \nu_\mu, \nu_\tau$

Survival probability

$$P(\nu_e) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 c^4 L}{4\hbar c E}\right)$$

Oscillation length $L_0 = \frac{4\hbar c E}{\delta m^2 c^4} \ll L$ (if $\delta m^2 c^4 > 10^{-9}\,\mathrm{eV}$)

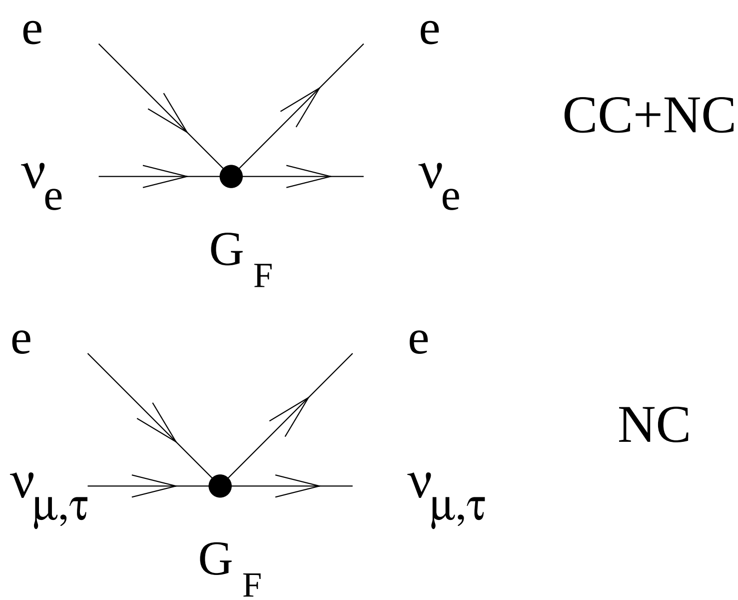
^8B νs have broad spectrum $\rightarrow$ oscillations average to 1/2

$$P(\nu_e) = 1 - \frac{1}{2}\sin^2(2\Theta)$$

But: suppression is factor 3

Matter Enhanced (MSW) Oscillations

Neutrino propagation in matter: forward scattering on electrons leads to effective potential



$$V = \frac{V_e - V_x}{2} = 2\sqrt{2} G_F N_e(r)$$

electron density $N_e(r)$

Wolfenstein (1978)

Mikheyev-Smirnov (1985)

Modified wave equation

$$i\hbar c \frac{d}{dr} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} V - \frac{\delta m^2}{4E}\cos(2\theta) & \frac{\delta m^2}{4E}\sin(2\theta) \\ \frac{\delta m^2}{4E}\sin(2\theta) & -V + \frac{\delta m^2}{4E}\cos(2\theta) \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

Consider eigenstates of RHS ("matter eigenstates")

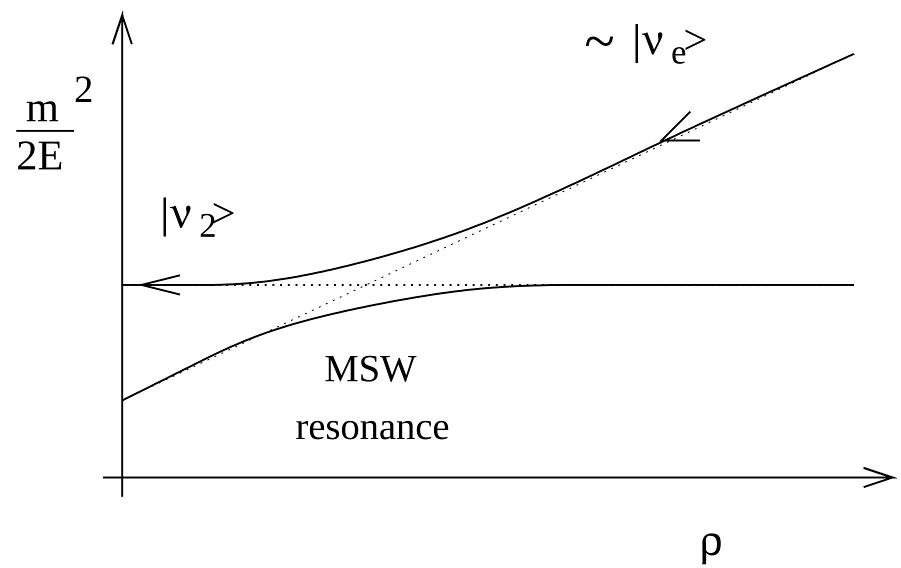
Start at high density

matter eigenstates $\simeq$ flavor eigenstates

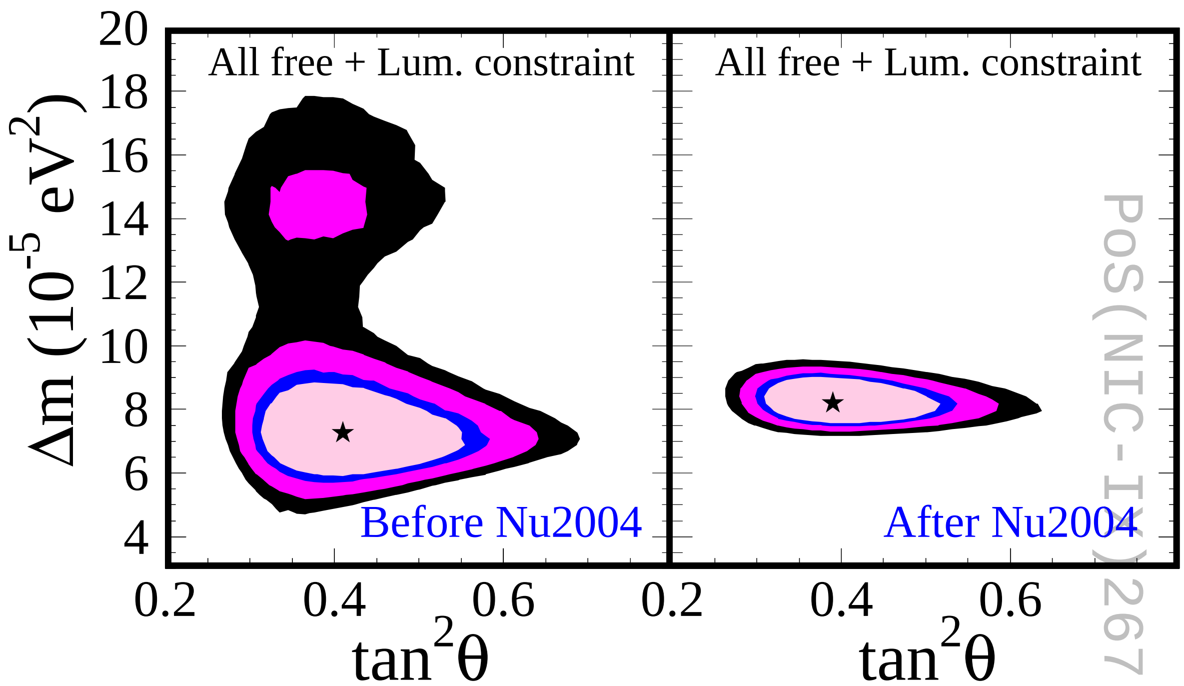
Resonance occurs if diagonal element vanishes. Possibilities

$$1)\quad \nu_e \to \nu_1 \qquad \text{non}-\text{adiabatic} \qquad P(\nu_e) \sim \cos^2\theta,\ P(\nu_\mu) \sim \sin^2\theta$$

$$2)\quad \nu_e \to \nu_2 \qquad \text{adiabatic} \qquad P(\nu_e) \sim \sin^2\theta,\ P(\nu_\mu) \sim \cos^2\theta$$



Global Fit



Cl and Ga experiments (Homestake, Gallex, SAGE)

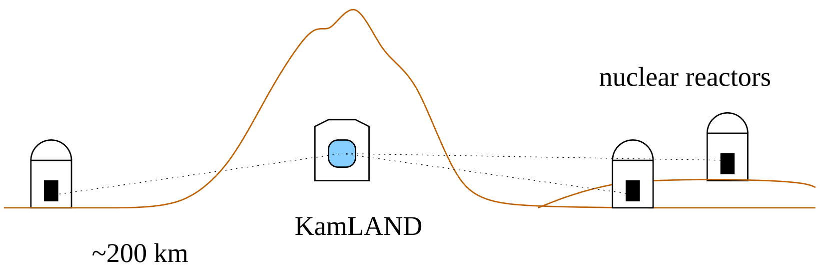
SuperK zenith angle-recoil energy spectra

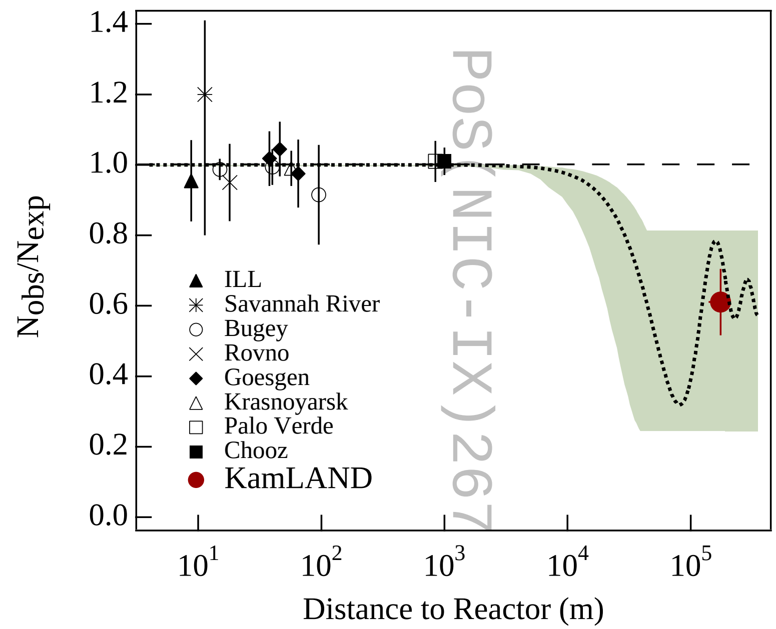
Kamland

M. C. Gonzalez-Garcia (2004)

Checking Solar ν Oscillations at KamLAND

KamLAND looks for $\bar{\nu}_e$ disappearance





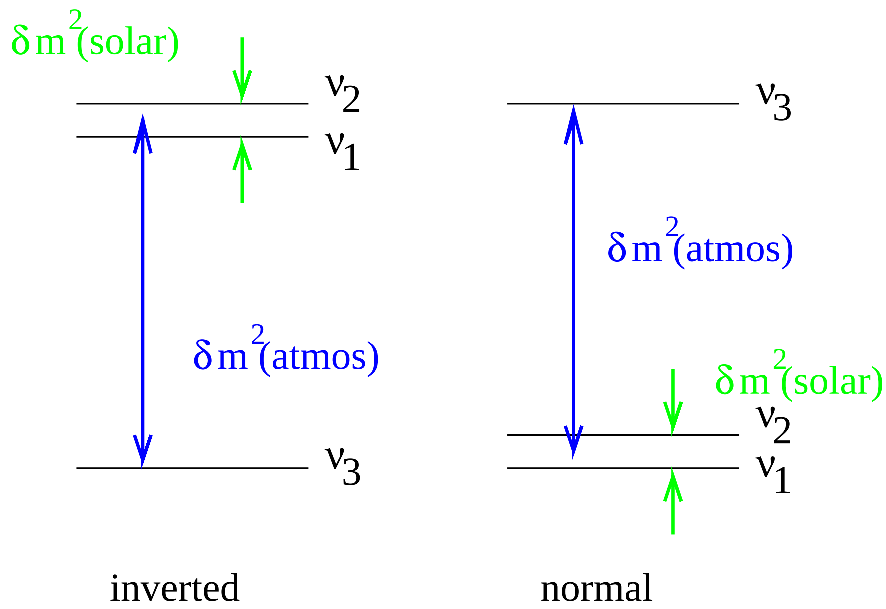
Results favor LMA solution to solar neutrino problem

$$\delta m^2 = 8 \cdot 10^{-5}\,\mathrm{eV}^2, \qquad \tan^2(\theta) = 0.4$$

Checking atmospheric ν oscillations: K2K, MINOS

Global Fit - Which Hierarchy?

Have to account for solar and atmospheric neutrino oscillations (ignoring LSND). Two possible schemes



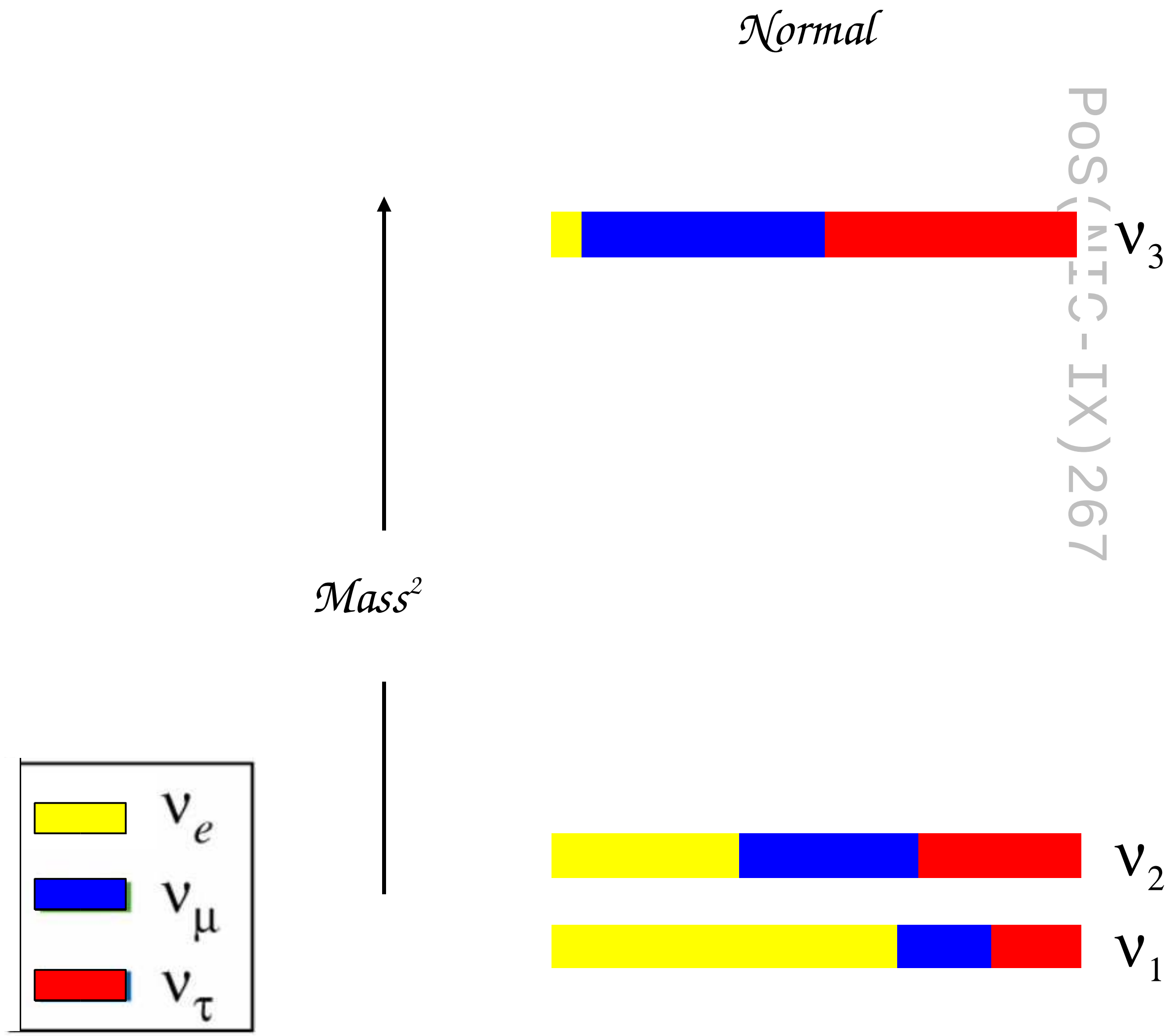
Experimental results determine

$$\theta_{12}, \quad \theta_{23}, \quad \text{all } |\delta m^2|$$

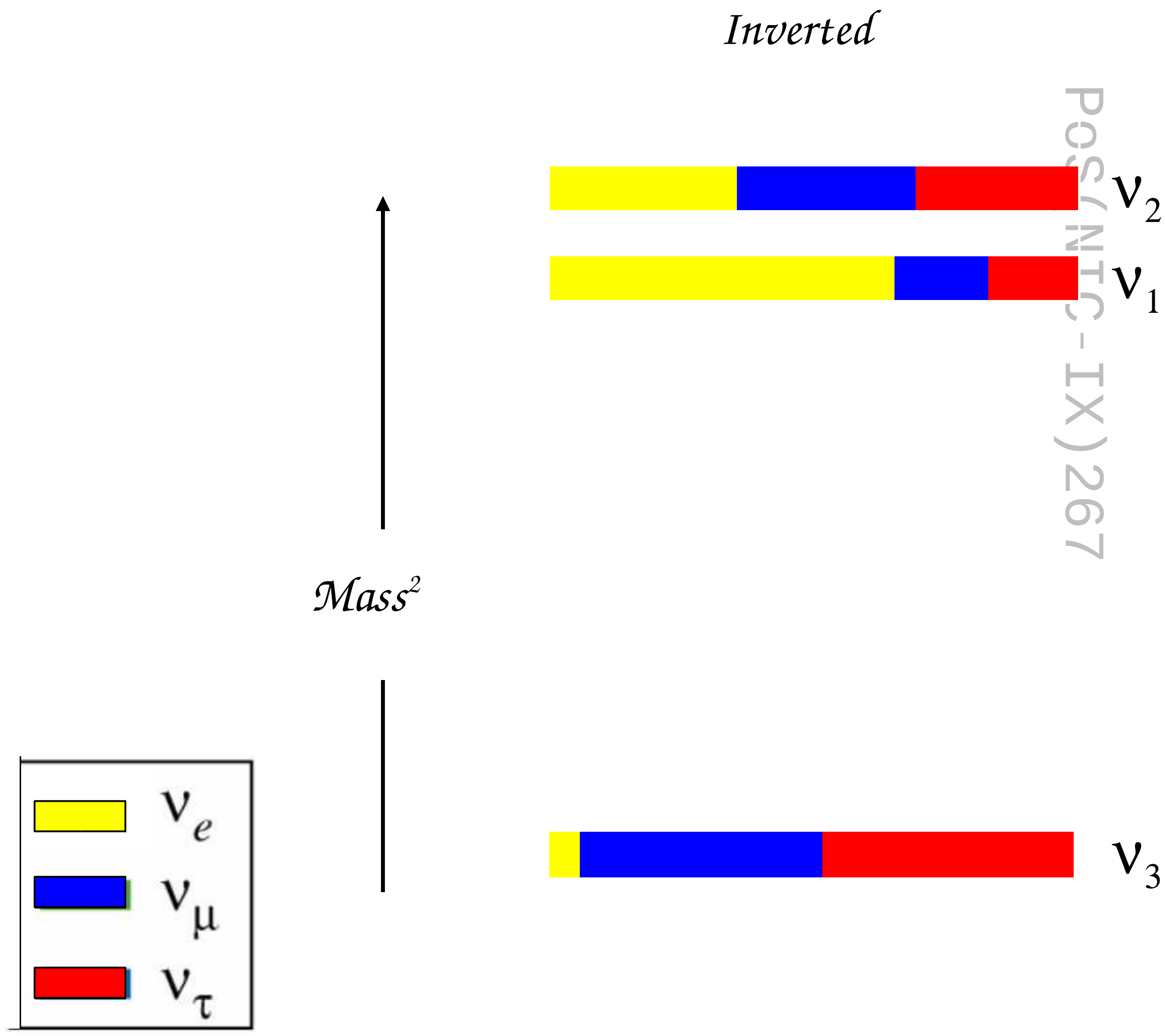
Unknown parameters

$$\text{hierarchy}, \quad \theta_{13}, \quad \text{phases}$$

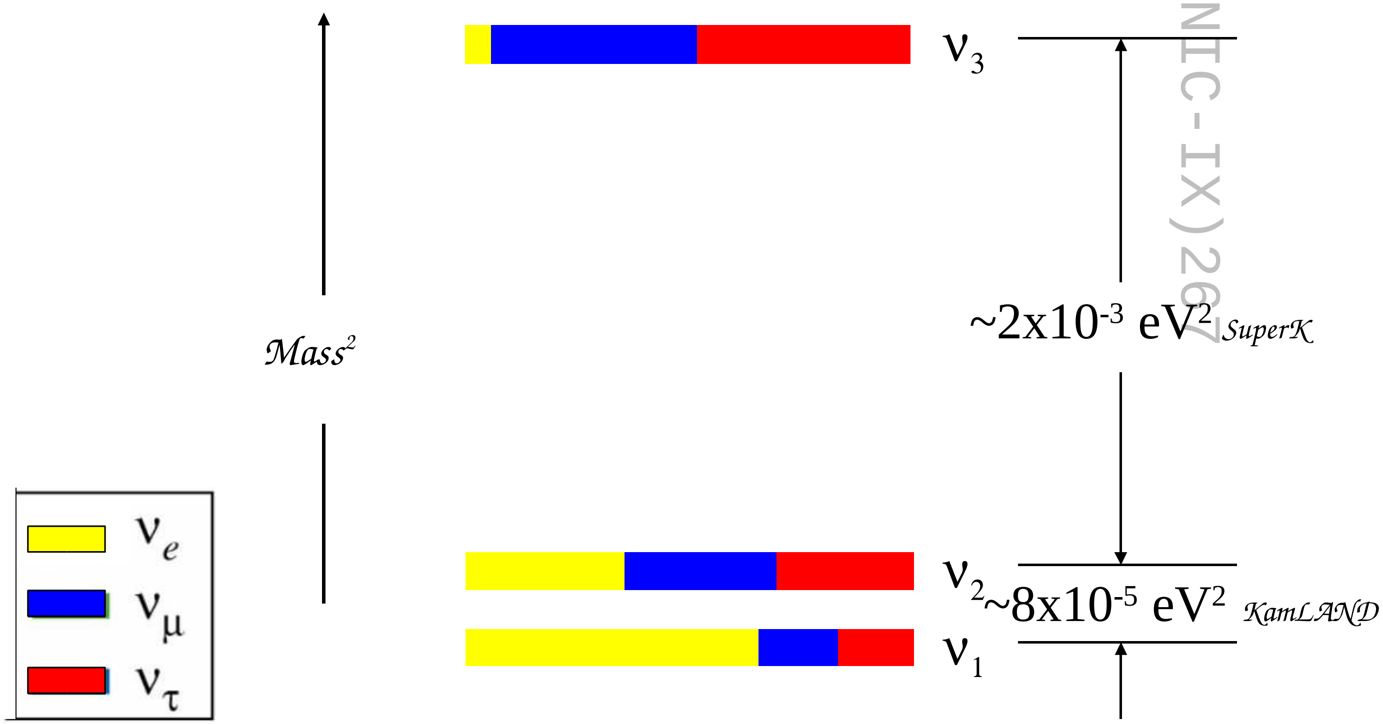
Neutrino Mass



Neutrino Mass



Neutrino Mass



What neutrino oscillations tell us about absolute neutrino mass

One neutrino is at least as large as $\sqrt{(\delta m^2_{atmos})} \approx 0.05$ eV.

Another is at least as large as $\sqrt{(\delta m^2_{solar})} \approx 0.01$ eV

They could be larger: but not larger than the $^3\mathrm{H}$ limit of 2.2 eV.

Recall: astrophysics says $\sum m_\nu \lesssim 1\,\mathrm{eV}$