PROCEEDINGS OF SCIENCE

Hadronic Sources of the Event-by-event Fluctuations of Particle Yield Ratios

PoS(CFRNC2006)017

Dmytro Kresan*
Gesellschaft für Schwerionenforschung (GSI), Darmstadt, Germany
E-mail: `D.Kresan@gsi.de`

Volker Friese
Gesellschaft für Schwerionenforschung (GSI), Darmstadt, Germany
E-mail: `V.Friese@gsi.de`

We present a study of dynamical fluctuations of the kaon to pion ratio in central heavy ion collisions, using the hadron-string model UrQMD and a generic resonance decay model. We study the effect of detector acceptance and investigate the influence of resonance decays on the fluctuations. It is shown that hadronic sources contribute significantly to the data measured by NA49, but cannot reproduce the observed energy dependence of the kaon to pion ratio fluctuations. However, the model results are sensitive to the resonance yields, thus detailed modelling of resonances is necessary for the interpretation of data.

2nd International Workshop on Correlations and Fluctuations in Relativistic Nuclear Collisions
July 7-9 2006
Galileo Galilei Institute, Florence, Italy

*Speaker.

1. Motivation

One of the basic questions of the study of properties of matter at high net baryon densities and moderate temperatures is the existence and exact location of the critical point of QCD that is supposed to be the end-point of the first order phase transition line from hadronic to partonic degrees of freedom [1 , 2] (see Figure 1). Experimentally, large event-by-event fluctuations are expected to be the signatures of this critical point. As one candidate for such fluctuations, the ratio of the yields of strange to nonstrange mesons[1], in particular the kaon to pion ratio, have been proposed [3].

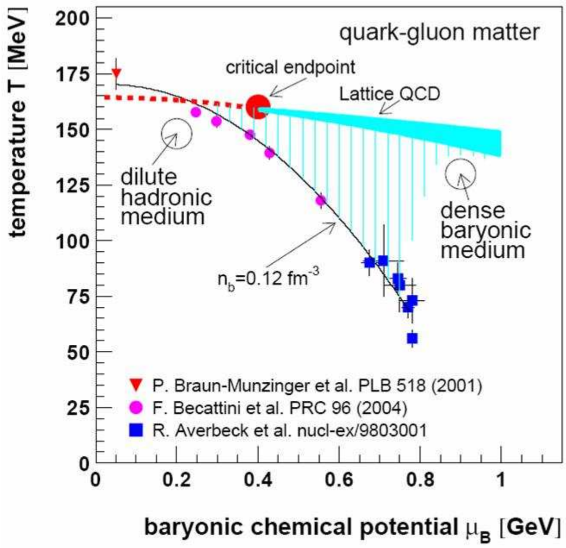

Figure 1: Phase diagram of Quantum Chromo Dynamics

A measurement of this observable has been performed at CERN SPS by the NA49 Collaboration [4] for central lead on lead collisions in the beam energy range 20 - 158 *A*GeV [5]. Figure 2 shows the measured excitation functions of the event-by-event fluctuations of the K/π and p/π ratios together with results of UrQMD calculations. The proton to pion ratio fluctuations are negative and agree well with the transport model results. They have been thus attributed to the correlations between protons and pions due to resonance, in particular Δ decays, which are properly modelled in UrQMD. In contrast, the kaon to pion ratio fluctuations are positive throughout the covered energy range and show an increase towards lower beam energies. The string-hadronic model also predicts positive values for the fluctuations but cannot reproduce the measured excitation function. In particular, the model shows no energy dependence.

From Figure 2 it is clear that hadronic effects like resonance decays and secondary interactions result in non-vanishing fluctuations. In order to interpret the results obtained by NA49, it is thus necessary to study such influences in detail. In particular, one would like to understand why the string-hadronic model UrQMD, not containing any critical phenomena, predicts the kaon to pion ratio fluctuations to be positive. This study is devoted to an investigation of the hadronic

[1] Throughout this article, we denote the particle yield ratio shortly by particle ratio.

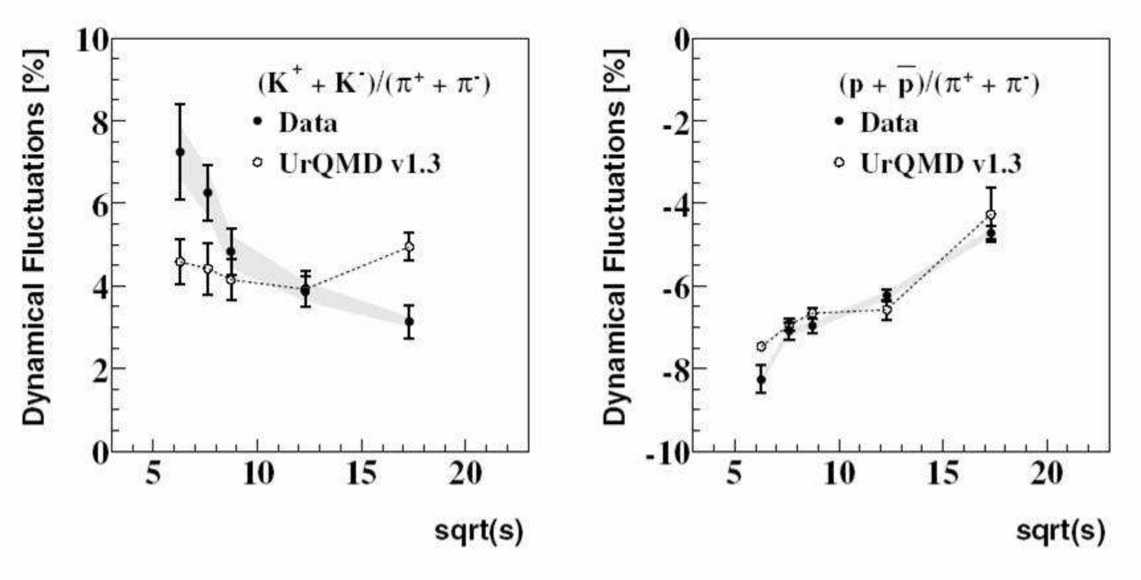

Figure 2: Preliminary results on the energy dependence of the dynamical fluctuations of the particle ratios from the NA49 Collaboration and comparison with UrQMD calculations [5]

sources of fluctuations. After introducing the definition of fluctuations and the analysis technique, we present results obtained with UrQMD [6], [7] as well as with a generic resonance model.

2. Definition of the Dynamical Fluctuations

Let N_π and N_K denote the numbers of measured charged pions and kaons, respectively, in an event, then $\frac{N_K}{N_\pi}$ is the kaon to pion ratio for this event. By measuring a set of events, we can numerically calculate the mean and RMS for the distribution of the event-by-event particle ratio and define the corresponding event-by-event fluctuations as:

$$\sigma = \frac{RMS}{MEAN} \tag{2.1}$$

The statistical error of this variable is, neglecting the error in the mean

$$\delta(\sigma) = \frac{\sigma}{\sqrt{2N_{ev}}}, \tag{2.2}$$

where N_{ev} is the number of analysed events.

Defined in this way, σ contains a "static" contribution from finite-number statistics and detector effects (resolution, acceptance) and a "dynamical" contribution which may be connected with critical phenomena:

$$\sigma^2 = \sigma^2_{stat} + \sigma^2_{dyn}, \tag{2.3}$$

The static contribution can experimentally be determined by a careful event mixing technique, which keeps the total multiplicity per event unchanged, but due to random selection of each track from a different real event destroys all correlations between tracks. In addition, in order to reproduce the phase space population of the real data, each track is used only once in the mixed events.

The mixed events are analysed in the same way as the real events. The dynamical fluctuations are then defined as

$$\sigma > \sigma_{mixed} \ : \ \sigma_{dyn} = \sqrt{\sigma^2 - \sigma^2_{mixed}},$$
$$\sigma < \sigma_{mixed} \ : \ \sigma_{dyn} = -\sqrt{\sigma^2_{mixed} - \sigma^2}, \tag{2.4}$$

where the first equation corresponds to anti-correlation (broadening of the distribution compared to the background) and the second to correlation (narrowing). The statistical error of this quantity is obtained by error propagation and eq. 2.2 as:

$$\delta(\sigma_{dyn}) = \frac{1}{|\sigma_{dyn}|\sqrt{2N_{ev}}} \sqrt{\sigma^4 + \sigma^4_{mixed}} \tag{2.5}$$

3. UrQMD simulations

The results in this section have been obtained with UrQMD version 1.3 [8] by analyzing the freeze-out configuration. Consequently, weak decays are not included in the analysis. This is justified by the fact that the NA49 experiment is able to exclude secondary particles efficiently from the analysis by a cut on the track impact parameter at the event vertex.

3.1 Influence of detector acceptance

Still after the subtraction of the static background, a limited detector acceptance may have effects on the measured fluctuations. In the context of resonance decays, the acceptance influences the mean multiplicities of independently produced particles serving as normalisation (see sections 4.1 and 4.2) and may destroy correlations of decay products.

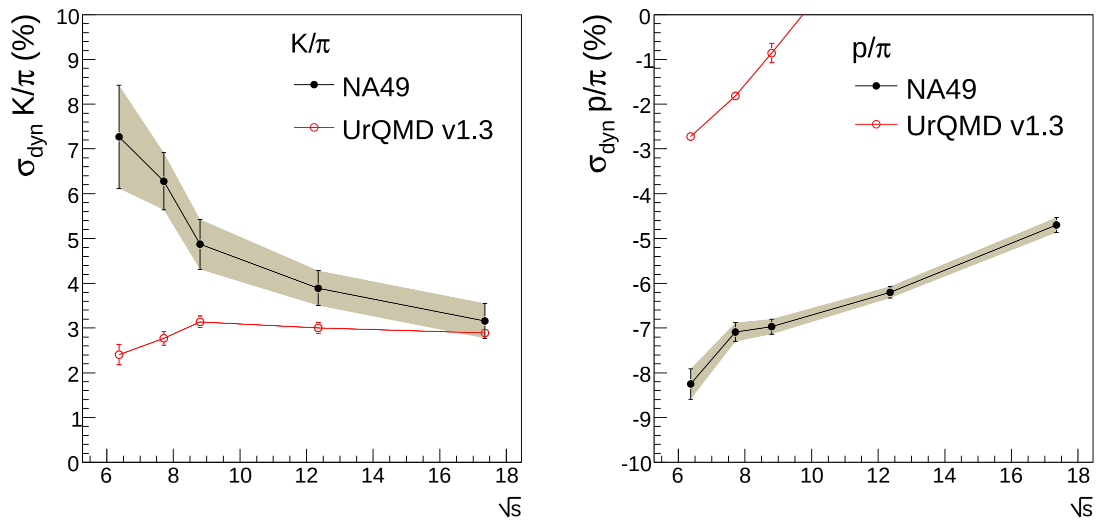

Figure 3: Dynamical event-by-event fluctuations of the kaon to pion (left) and proton to pion (right) ratios as the function of $\sqrt{s}$. The open circles show values obtained by UrQMD simulations in 4π, the closed ones NA49 data.

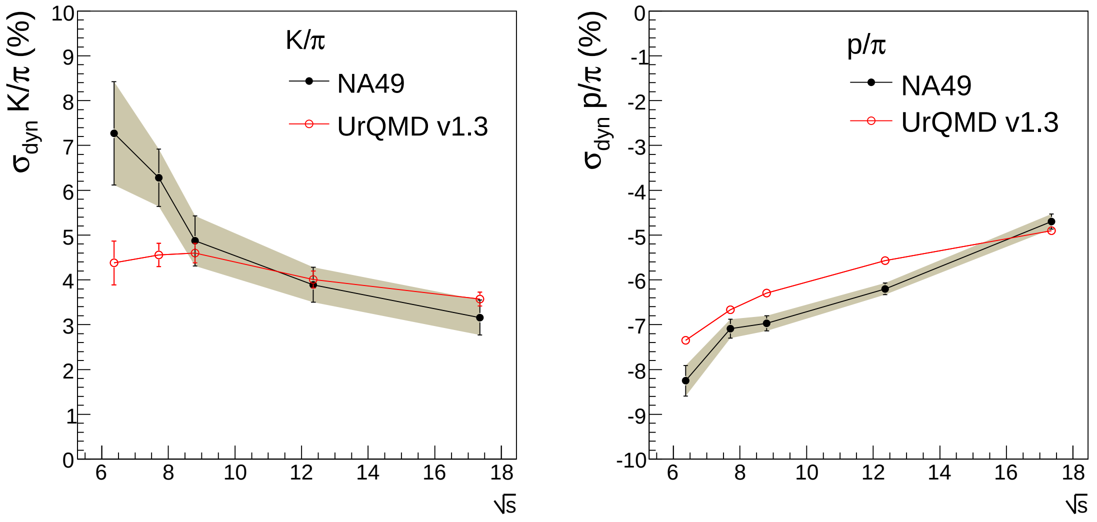

Figure 4: Dynamical event-by-event fluctuations of the kaon to pion (left) and proton to pion (right) ratios as the function of $\sqrt{s}$. The open circles show values obtained by UrQMD simulations using NA49 acceptance cuts, the closed symbols show NA49 data.

In order to approximate the NA49 acceptance, we restrict our analysis to tracks in the forward acceptance and with momenta large than 3 GeV [5]. In addition, tracks near beam rapidity are cut to suppress projectile spectators. The model results for the kaon to pion fluctuations in 4π at the five energies measured by NA49 are shown in Figure 3 , those after the acceptance cuts in Figure 4 , both together with NA49 data. The comparison of the two figures shows that the acceptance has little effect on the kaon to pion ratio; in particular it does not introduce a significant energy dependence. In contrast, the acceptance strongly influences the proton to pion ratio fluctuations, which can be attributed to the fact that the proton spectators are cut out. The numerical values of the fluctuations of various particle ratios in 4π and after acceptance cuts are compared in Table 1 for the system central Au+Au at 25 *A*GeV as relevant for the future experiment CBM at FAIR.

Our model results agree reasonably well with the model calculations obtained by NA49 [5] (see Figure 2), showing that the analysis algorithms are consistent. Small remaining differences can be explained by our rough acceptance cuts neglecting the incomplete azimuthal acceptance of NA49.

3.2 Single Particle Ratios

The K/π fluctuations measured by NA49 refer to the sum of charged kaons over the sum of charged pions. As resonances are expected not to feed only in the $p\pi$ channel but also into $K\pi$ like the K^* resonance, we studied the single particle ratios K^+/π^+, K^+/π^- and vice-versa. Figure 5 shows the distributions of these ratios in central Au+Au at 25 *A*GeV both for same and mixed events together with the numerical value of the dynamical fluctuations. Remarkably, all of these fluctuations are negative, signalling the correlation due to resonance decays. This holds also for the ratios K^+/π^+ and K^-/π^- which are fed by the decay channels $K_1 \to K\rho \to K\pi\pi$ and $K_1 \to K^*\pi \to K\pi\pi$.

Particle ratio	Dynamical fluctuations (%)	
	4π	after acceptance cuts
$(K^{+}$+$K^{-})/(\pi^{+}$+$\pi^{-})$	2.5 $\pm$ 0.3	3.4 $\pm$ 0.6
$(p$+$\overline{p})/(\pi^{+}$+$\pi^{-})$	-5.56 $\pm$ 0.04	-7.0 $\pm$ 0.1
K^{+}/π^{+}	-6.1 $\pm$ 0.1	-6.1 $\pm$ 0.5
K^{+}/π^{-}	-8.0 $\pm$ 0.1	-8.6 $\pm$ 0.3
K^{-}/π^{+}	-8.7 $\pm$ 0.3	-10.7 $\pm$ 0.7
K^{-}/π^{-}	-7.3 $\pm$ 0.3	-6.7 $\pm$ 1.2

Table 1: Dynamical fluctuations of the particle ratios in 4π and within approximated NA49 acceptance. Simulations with UrQMD for central gold on gold collisions at 25 AGeV.

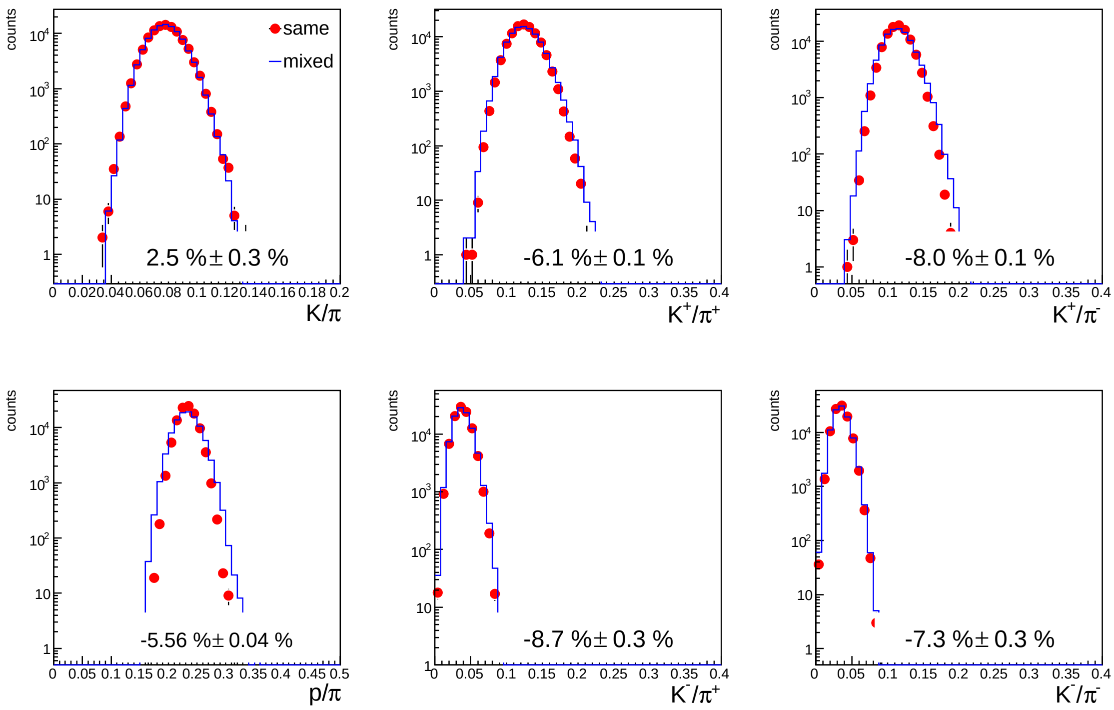

Figure 5: Distributions of the event-wise particle ratio for same and mixed events together with the values of dynamical fluctuations. Simulations done with UrQMD for central gold on gold collisions at 25 AGeV. Left column: sum of kaons over sum of pions (top), sum of protons and anti protons over sum of pions (bottom). Middle and right columns: single particle ratios.

To understand the relative importance of resonance feeddown to the fluctuations, the sources of kaons in UrQMD have been studied using the full collision history. Figure 6 shows the contributions of various resonances to the final state kaons. About half of the K^{+} and one third of K^{-} originate from the decay of K^{*}, while the K_1 contributes less to the kaon yields. We thus can qualitatively understand the source of the negative fluctuations in the single particle K/π ratios seen in UrQMD.

The positive fluctuation in the sum K/π ratio remains to be understood. A possible source is the ϕ meson, decaying alternatively into a kaon pair or into three pions, either directly or via the

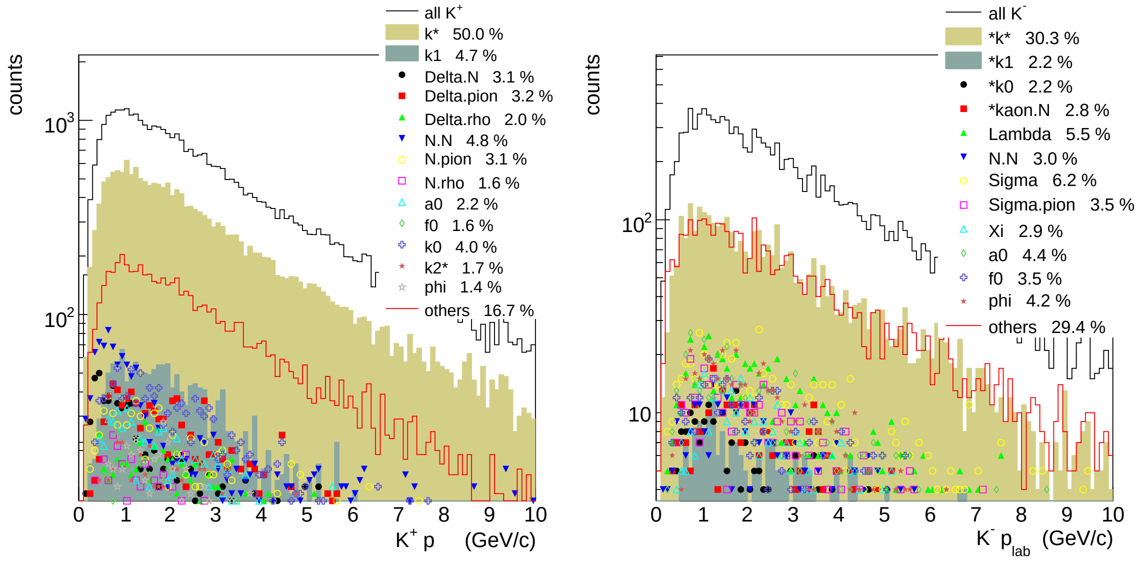

Figure 6: Momentum distribution of K^+ (left) and K^- (right) generated by UrQMD for central gold on gold collisions at 25 AGeV. Black curves show the momentum spectra of all $K^{+/-}$, other distributions - momentum spectra of kaons from different sources, which are named in the legends. The results were obtained by analysing the UrQMD collision history file.

ρ. This introduces an anticorrelation for the pairs which would reflect as a positive fluctuation. However, the ϕ meson contributes only 1.5 % to the K^+ and 4.3 % to the K^- yield.

3.3 Suppression of resonance contributions

In order to check these qualitative considerations, UrQMD calculations were performed suppressing all excited kaon and ϕ decays. The simulations are again done for central Au+Au collisions at 25 AGeV. Table 2 summarises the results obtained in 4π. As expected, the proton to pion ratio fluctuations are not affected at all by the supression of the strange resonance decays. As can be seen from the numbers, the excited kaons indeed drive the K/π ratios towards smaller numbers. This also holds for the sum ratio. The supression of the ϕ decay leaves the single particle ratios untouched while it significantly reduces the sum kaon over sum pion ratio fluctuations. Again, this agrees qualitatively with the expectations. However, other sources of correlations are obviously present in particular for the single particle ratios.

It must be noted that the suppression of the K^* decay significantly influences the mean kaon yields, thus making other correlations more visible (see sections 4.1 and 4.2). It is hence difficult to get a more quantitative insight into the resonance decay dynamics by using UrQMD in this manner.

4. Generic Model Simulations

In order to study the influence of resonance decays on the dynamical particle ratio fluctuations, we generate events containing kaons, pions and resonances with given mean multiplicities and phase space distributions. As in this section we restrict ourselves to the 4π acceptance, the latter

Particle ratio	Dynamical fluctuations (%)		
	Nominal	no K^*	no ϕ
$(K^+$+$K^-)/(\pi^+$+$\pi^-)$	2.5 ± 0.3	6.7 ± 0.2	1.3 ± 0.5
$(p$+$\overline{p})/(\pi^+$+$\pi^-)$	-5.56 ± 0.04	-5.59 ± 0.04	-5.51 ± 0.04
K^+/π^+	-6.1 ± 0.1	-3.6 ± 0.6	-6.3 ± 0.1
K^+/π^-	-8.0 ± 0.1	-6.3 ± 0.3	-8.2 ± 0.1
K^-/π^+	-8.7 ± 0.3	-8.1 ± 0.3	-8.8 ± 0.3
K^-/π^-	-7.3 ± 0.3	-6.6 ± 0.4	-7.3 ± 0.4

Table 2: Dynamical fluctuations of the particle ratios calculated with UrQMD. Values are shown in 4π acceptance for standard conditions ("Nominal"), with K^* decay suppressed and ϕ-meson decay suppressed.

are of no importance. Resonance decays are performed for predefined decay channels until stable daughters are reached. Thus, their influence on the fluctuations can be studied in an environment of a fixed number of independently produced particles.

The simulations were performed with mean multiplicites of pions and kaons taken from UrQMD calculations for central Au+Au at 25 *A*GeV. The resonance yields were treated as free parameters.

4.1 Influence of the $K^*(892)$ decay

For a given mean number of independently produced pions and kaons, the fluctuations induced by the $K^* \to K^+\pi^-$ decay can be obtained analytically (see also [9]). Assuming a Poissonian distribution of N_{π^+} and N_{K^+}, the relative width of the event-wise K^+/π^- ratio distribution for same events consists of three terms :

$$\sigma^2_{same} = \frac{1}{< N_{K^+} >} + \frac{1}{< N_{\pi^-} >} - 2\frac{Cov(N_{K^+}, N_{\pi^-})}{< N_{K^+} >< N_{\pi^-} >} \tag{4.1}$$

In the mixed events all correlations are destroyed, and the third terms vanishes. Note that by construction, the mean multiplcities remain the unchanged:

$$\sigma^2_{mixed} = \frac{1}{< N_{K^+} >} + \frac{1}{< N_{\pi^-} >} \tag{4.2}$$

So we have $\sigma_{same} < \sigma_{mixed}$ (correlation) and according to (2.4), the dynamical fluctuations are

$$\sigma_{dyn} = -\sqrt{\sigma^2_{mixed} - \sigma^2_{same}} = -\sqrt{2\frac{Cov(N_{K^+}, N_{\pi^-})}{< N_{K^+} >< N_{\pi^-} >}} \tag{4.3}$$

The number of kaons consists of independently produced particles ($N^{(p)}_{K^+}$) as well as products from resonance decays ($< K^* > \cdot BR$ on average, where $BR = 50\%$). The same holds for pions. After calculating the covariance we get

$$\sigma_{dyn} = -\sqrt{\frac{2\cdot < K^* > \cdot BR}{(< N^{(p)}_{K^+} > + < K^* > \cdot BR)(< N^{(p)}_{\pi^-} > + < K^* > \cdot BR)}} \tag{4.4}$$

Figure 7 shows the results of our generic simulations together with the analytical formula 4.4 . The agreement confirms the analytical considerations outlined above. The absolute value of the fluctuations induced by the K^* decay increases with the square root of the relative amount of K^*.

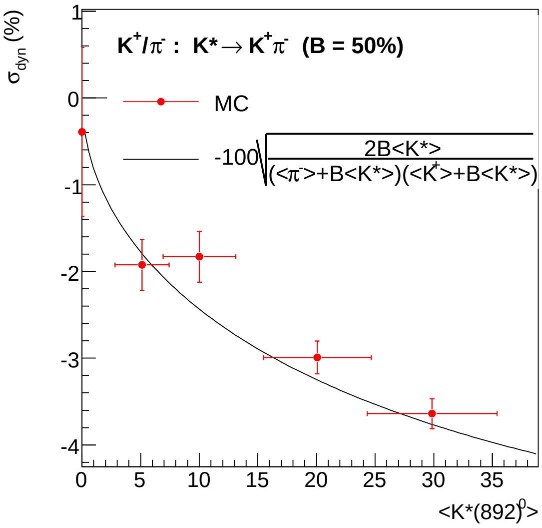

Figure 7: Dynamical fluctuations of kaon to pion ratio calculated with the generic model as function of the $K^*(892)$ yield. Multiplicities of primary pions and kaons are taken from UrQMD calculations for central gold on gold collisions at 25 AGeV.

4.2 Influence of the $\phi(1020)$ decay

With the same technique, the influence of the ϕ meson on the $(K^+ + K^-)/(\pi^+ + \pi^-)$ ratio fluctuations was studied. The simulated decay scheme of the resonance is shown in Figure 8 , where only strong decays are considered.

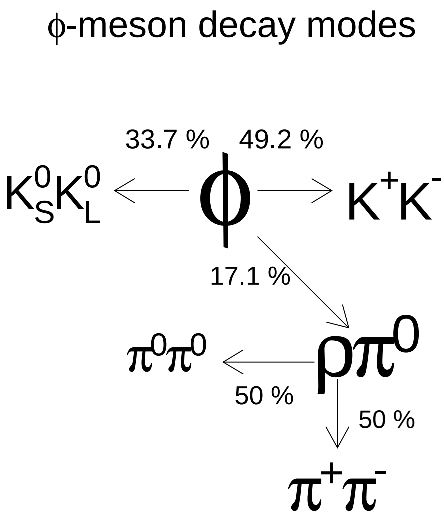

Figure 8: The three most probable decay modes of the $\phi(1020)$ meson

Again, an analytical expression can be derived for the fluctuations induced by the ϕ decay. Because of the correlations between K^+ and K^- and between π^+ and π^-, the first two terms of the relative width of the same-event distribution must be expressed in a more general way:

$$\sigma^2_{same} = \frac{Var(N_{K^+}) + Var(N_{K^-}) + 2Cov(N_{K^+}, N_{K^-})}{< N_K >^2} + \frac{Var(N_{\pi^+}) + Var(N_{\pi^-}) + 2Cov(N_{\pi^+}, N_{\pi^-})}{< N_\pi >^2} - 2\frac{Cov(N_K, N_\pi)}{< N_K >< N_\pi >}, \tag{4.5}$$

Since N_{K^+} and N_{K^-} are distributed Poissonianlike, $Var(N) = N$ for all particles involved, so that the width can be written as

$$\sigma^2_{same} = \frac{1}{< N_K >} + 2\frac{Cov(N_{K^+}, N_{K^-})}{< N_K >^2} + \frac{1}{< N_\pi >} + 2\frac{Cov(N_{\pi^+}, N_{\pi^-})}{< N_\pi >^2} - 2\frac{Cov(N_K, N_\pi)}{< N_K >< N_\pi >}, \tag{4.6}$$

For the mixed events the width is driven by statistics only:

$$\sigma^2_{mixed} = \frac{1}{< N_K >} + \frac{1}{< N_\pi >} \tag{4.7}$$

Thus we have $Cov(N_K, N_\pi) < 0$ and $\sigma_{same} > \sigma_{mixed}$. Now we can derive the dynamical fluctuations:

$$\sigma_{dyn} = \sqrt{\sigma^2_{same} - \sigma^2_{mixed}} = \sqrt{\frac{Cov(N_{K^+}, N_{K^-})}{< N_K >^2} + \frac{Cov(N_{\pi^+}, N_{\pi^-})}{< N_\pi >^2} - 2\frac{Cov(N_K, N_\pi)}{< N_K >< N_\pi >}} \tag{4.8}$$

Analogously to the case with K^* we calculate the covariances and simplify the last equation:

$$\sigma_{dyn} = \sqrt{\frac{2\cdot < \phi > \cdot BR_{KK}}{< N_K >^2} + \frac{2\cdot < \phi > \cdot BR_{\pi\pi}}{< N_\pi >^2} - 2\frac{Cov(N_K, N_\pi)}{< N_K >< N_\pi >}} \tag{4.9}$$

In 4π acceptance we have $N_\pi >> N_K$ and for the ϕ resonance $BR_{\pi\pi} << BR_{KK}$, thus the two last terms can be neglected:

$$\sigma_{dyn} \approx \sqrt{\frac{2\cdot < \phi > \cdot BR_{KK}}{(< N_K^{(p)} > +2\cdot < \phi > \cdot BR_{KK})^2}} \tag{4.10}$$

Again, the simulation results obtained by the generic resonance model agree well with the derived formula as shown in Figure 9 , proving the validity of the approximations performed in the derivation of 4.10 . As expected, the ϕ decay results in positive fluctuations of the sum kaon over sum pion ratio. The fluctuations increase with the square root of the ϕ/K ratio. Inspection of eq. 4.8 and 4.10 shows that these fluctuations are not primarily due to the anticorrelation of pion with kaon pairs but to the covariance of K^+ and K^-.

It should be noted that for ϕ multiplicities around unity, as is the case for central collisions at 20 - 30 AGeV, the magnitude of the dynamical fluctuations induced by the ϕ decay is a steep function of the ϕ multiplicity. Transport models should therefore be checked whether they reproduce the measured yield. This holds in principle for all resonances. However, for the range of ϕ multiplicities allowed by the experimental uncertainties, the fluctuations obtained by our model are well below the NA49 measurements.

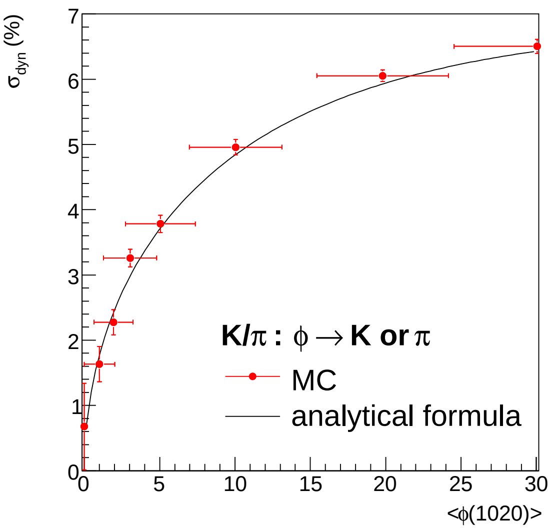

Figure 9: Dynamical fluctuations of the kaon to pion ratio obtained by the generic model as function of the $\phi(1020)$ yield. Multiplicities of primary pions and kaons are taken from UrQMD calculations for central gold on gold collisions at 25 AGeV.

5. Summary and outlook

We have shown that UrQMD simulations reproduce the fluctuations of the eventwise proton to pion ratio measured by NA49, but fail to describe the fluctuations in the $(K^+ + K^-)/(\pi^+ + \pi^-)$ ratio. This agrees with the UrQMD simulations performed by NA49 itself. The detector acceptance has a moderate influence on the dynamical fluctuations but does not introduce a significant energy dependence as seen in the data.

In contrast to the sum ratio of kaons and pions, the single particle ratios show negative fluctuations in the UrQMD model. It was shown that a large fraction of these fluctuations is due to resonance decays, in particular that of the $K^*(892)$. However, other sources of correlation are present in the model. Similarly, positive fluctuations in the sum kaon over sum pion ratio can be attributed to the ϕ decay. In 4π, analytical formulas have been derived to describe the influences of these resonance decays. A generic model has been developed to simulate fluctuations caused by resonance decays. The model agrees with the analytical expressions in 4π, but can also be applied to a limited detector acceptance.

Since hadronic models agree with the measured proton to pion fluctuations and show the same sign for the kaon to pion fluctuations, it is crucial to disentangle the hadronic contribution to the measured fluctuations by careful investigations. Accurate modelling of resonance yields and phase space distributions is indispensible for such studies. We will thus apply our generic resonance model to all beam energies measured by NA49, using experimental data on pion, kaon and resonance yields and distributions and taking into account the detailed detector acceptance.

References

[1] S. Ejiri et al. hep-lat/0312006

[2] Z. Fodor, S. Katz JHEP 0404:050 (2004)

[3] S. Jeon, V. Koch Phys. Rev. Lett. 85 (2000) 2076-2079

[4] S. Afanasiev et al. Nucl. Instr. Meth. A 430 (1999) 210-244

[5] C. Roland (for the NA49 Collaboration) J. Phys. G: Nucl. Part. Phys. 31 (2005) 1075-1078

[6] S. A. Bass et al. Prog. Part. Nucl. Phys. 41 (1998) 225-370

[7] M. Bleicher et al. J. Phys. G: Nucl. Part. Phys. 25 (1999) 1859-1896

[8] http://th.physik.uni-frankfurt.de/ urqmd/

[9] S. Jeon, V. Koch Phys. Rev. Lett. 83 (1999) 5435-5438