Infrared behavior of the gluon and ghost propagators in Yang-Mills theories

Silvio Paolo Sorella[*][†], **Marcio Capri, Vitor Lemes, Rodrigo Sobreiro, Ronaldo Thibes**
UERJ - Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20550-013
Maracanã, Rio de Janeiro, Brasil
E-mail: `sorella@uerj.br,marcio@dft.if.uerj.br,`
`vitor@dft.if.uerj.br,sobreiro@dft.if.uerj.br,thibes@dft.if.uerj.br`

John Gracey
Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool,
P.O. Box 147, Liverpool, L69 3BX, United Kingdom
E-mail: `jag@amtp.liv.ac.uk`

David Dudal[‡]**, Henri Verschelde**
Ghent University, Department of Mathematical Physics and Astronomy, Krijgslaan 281-S9,
B-9000 Gent, Belgium
E-mail:`david.dudal@ugent.be, henri.verschelde@ugent.be`

We provide a short discussion of the dimension two condensate $\langle A^2 \rangle$ and its influence on the infrared behaviour of the gluon propagator in the Landau gauge. Simultaneously, we pay attention to the issue of Gribov copies in the Landau gauge. We also briefly discuss a local, gauge invariant non-Abelian action with mass parameter, constructed from the dimension 2 operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$.

Fifth International Conference on Mathematical Methods in Physics
24 – 28, April 2006
Rio de Janeiro, Brazil

1. Introduction

In recent years we have witnessed an intensive research activity aiming at improving our understanding of the behaviour of Yang-Mills theories in the low energy (infrared) regime. This is a rather complicated issue, closely related to the confinement of quarks and gluons. We consider pure Euclidean $SU(N)$ Yang-Mills theories with action

$$S_{YM} = \frac{1}{4} \int d^4x \, F_{\mu\nu}^a F_{\mu\nu}^a \,, \tag{1.1}$$

where A_μ^a, $a = 1,...,N^2 - 1$ is the gauge boson field, with associated field strength

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \,. \tag{1.2}$$

The theory (1.1) is invariant with respect to the local gauge transformations

$$\delta A_\mu^a = D_\mu^{ab} \omega^b \,, \tag{1.3}$$

with

$$D_\mu^{ab} = \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c \,, \tag{1.4}$$

denoting the adjoint covariant derivative.

Thanks to the asymptotic freedom, the coupling constant of the theory turns out to be small at very high energies, where perturbation theory then becomes reliable [1, 2]. However, in the infrared region, the coupling constant grows and nonperturbative effects have to be taken into account.

The introduction of condensates, i.e. the (integrated) vacuum expectation value of certain operators, allows one to parametrize certain nonperturbative effects arising from the infrared sector of e.g. the theory described by (1.1). Via the Operator Product Expansion (OPE) (viz. short distance expansion), which is applicable to local operators, one can relate these condensates to power corrections which give nonperturbative information in addition to the perturbatively calculable results.

Possible causes of nonperturbative effects are given by Gribov ambiguities which affect the Faddeev-Popov quantization procedure and hence the propagators [3], the existence of (topologically) nontrivial field configurations like instantons [4], etc. These effects are not necessarily unrelated, as e.g. instantons can contribute to the condensate $\langle \alpha_s F_{\mu\nu}^2 \rangle$ [4], the Faddeev-Popov operator has zero modes in an instanton background [5], etc.

Although propagators are not gauge invariant quantities, they are, to some extent, the simplest Green functions which can be studied from the analytic point of view. As far as four-dimensional Yang-Mills theory is considered, this task appears to be very difficult for more complicated Green functions.

During the last decade, there has been an intensive activity from the lattice community in the study of the gluon and ghost propagators. We can thus compare, at least qualitatively, our theoretical predictions with the available lattice data.

So far, a certain number of gauges have been considered extensively from the theoretical as well as from the lattice point of view. This is the case for the Landau, Coulomb and maximal Abelian gauges. Throughout this paper, we shall mainly be interested in the Landau gauge

$$\partial^\mu A_\mu^a = 0 \,. \tag{1.5}$$

In particular, we shall focus on the effect of the dimension two condensate $\left\langle A^2_{\min} \right\rangle$ on the infrared behaviour of the gluon propagator, in combination with the effects arising from a treatment of the Gribov problem.

2. The dimension two condensate

The possible existence and relevance of a gauge condensate of dimension two has been advocated a few years ago by [6, 7]. The idea was put forward that the gauge invariant, dimension two operator $A^2_{\min}$, obtained by minimizing A^2 along its gauge orbit,

$$
\begin{aligned}
A^2_{\min} &\equiv \min_{U \in SU(N)} \frac{1}{VT} \int d^4x \left(A^U_\mu \right)^2 , \\
A^U_\mu &= U A_\mu U^\dagger + U \partial_\mu U^\dagger ,
\end{aligned}
\tag{2.1}
$$

might condense, i.e.

$$
\left\langle A^2_{\min} \right\rangle \neq 0
\tag{2.2}
$$

This nonvanishing condensate might be important for several reasons:

- $\left\langle A^2_{\min} \right\rangle$ could serve as an order parameter for the condensation of monopoles, relevant for the dual Meissner effect in the dual superconductivity picture of color confinement [6, 7]

- it could account for certain power corrections in $\frac{1}{Q^2}$ which have been reported in the study of two and three point correlation functions, see e.g. [8, 9].

- it could give rise to a dynamical gluon mass m_g via the OPE [10], which could be relevant for the dual Meissner effect [11] as well as for obtaining analytic estimates of glueball spectra [12].

3. A closer look at $A^2_{\min}$

The relevance of the operator $A^2_{\min}$ for Yang-Mills gauge theories is known since many years. It plays a key role in the study of the Gribov copies and of the geometrical and topological properties of the space of gauge orbits [13, 14, 15]. It can be expressed as an infinite series of nonlocal terms, see [16], namely

$$
A^2_{\min} = \int d^4x \left[A^a_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A^a_\nu - g f^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a \right) \left(\frac{1}{\partial^2} \partial A^b \right) A^c_\nu \right] + O(A^4) .
\tag{3.1}
$$

From this expression, one can also check explicitly order by order in the coupling constant that $A^2_{\min}$ is gauge invariant. However, it is immediately clear that $A^2_{\min}$ is a highly nonlocal expression, a fact making it very difficult to handle. In general, it also falls beyond the applicability of the OPE, which refers to local operators.

So far, the only possibility to study the operator $A^2_{\min}$ by analytical tools relied on choosing the Landau gauge (1.5). Due to the transversality condition, all nonlocal terms drop out, so that $A^2_{\min}$ reduces to the local operator A^2,

$$
A^2_{\min} \equiv A^2 \text{ in the Landau gauge}
\tag{3.2}
$$

3.1 Effective potential for the local composite operator

The operator A^2 is an example of a local composite operator (LCO). In [17], a method was developed to construct a sensible effective potential for such operators. This so-called LCO method was later applied to the case of A^2 in the Landau gauge [18]. We couple the operator A^2 to the Yang-Mills action by means of a source J,

$$S_J = S_{YM} + \int d^4x \left(\frac{1}{2} J A_\mu^a A_\mu^a - \frac{1}{2} \zeta J^2 \right). \tag{3.3}$$

The last term, quadratic in the source J, is necessary to kill the divergences in vacuum correlators like $\langle A^2(x) A^2(y) \rangle$ for $x \to y$, or equivalently in the generating functional $W(J)$, defined as

$$e^{-W(J)} = \int [\text{fields}] e^{-\int d^4x S_J}. \tag{3.4}$$

The presence of the LCO parameter ζ ensures a homogenous renormalization group equation for $W(J)$. Its arbitrariness can be overcome by making it a function $\zeta(g^2)$ of the strong coupling constant g^2, allowing one to fix $\zeta(g^2)$ order by order in perturbation theory in accordance with the renormalization group equation [17, 18].

In order to recover an energy interpretation, the term $\propto J^2$ can be removed by employing a Hubbard-Stratonovich transformation

$$1 = \int \sigma e^{-\frac{1}{2\zeta} \left(\frac{\sigma}{g} + \frac{1}{2} A^2 - \zeta J \right)^2}, \tag{3.5}$$

leading to the action

$$\begin{aligned} S &= S_{YM} + S_\sigma, \\ S_\sigma &= \int d^4x \left(\frac{\sigma^2}{2g^2\zeta} + \frac{1}{2g^2\zeta} g\sigma A^2 + \frac{1}{8\zeta} (A^2)^2 \right). \end{aligned} \tag{3.6}$$

A key ingredient for the LCO method is the renormalizability of the operator A^2. It was proven in [19] that A^2 is renormalizable to all orders of perturbation theory, making use of the Ward identities in the presence of the operator A^2. In addition, an interesting identity was proven concerning the anomalous dimension γ_{A^2} of the operator A^2, first noticed in [20]. It can be shown that γ_{A^2} can be expressed as a linear combination of the gauge beta function β and of the anomalous dimension γ_A of the gauge field A, according to the relationship [19]

$$\gamma_{A^2}(a) = -\left(\frac{\beta(a)}{a} + \gamma_A(a) \right), \qquad a \equiv \frac{g^2}{16\pi^2}. \tag{3.7}$$

Starting from (3.6) it is possible to calculate the effective potential $V(\sigma)$. The correspondence $\langle \sigma \rangle = -g \langle A^2 \rangle$ consequently provides evidence for a nonvanishing dimension two gluon condensate using an effective potential approach, if $\langle \sigma \rangle \neq 0$. It is clear from (3.6) that $\langle \sigma \rangle \neq 0$ also induces an effective gluon mass. $V(\sigma)$ was calculated to two loop order in [18, 21], and a nonvanishing condensate is favoured as it lowers the vacuum energy. The ensuing effective gluon mass was found to be a few hundred MeV.

3.2 Restriction to the Gribov horizon

In the Landau gauge, it is necessary to restrict the domain of integration in the Feynman path integral at least to the so-called Gribov region Ω, whose boundary $\partial\Omega$ is the first Gribov horizon, where the first vanishing eigenvalue of the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu \left(\partial_\mu \delta^{ab} + g f^{acb} A_\mu^c \right) \,, \tag{3.8}$$

appears. This restriction is necessary due to the existence of the Gribov copies, which implies that the Landau condition (1.5) does not uniquely fix the gauge [3].

It has been discussed in [22, 23] how this restriction can be accomplished at the Lagrangian level. More precisely, the starting Yang-Mills measure in the Landau gauge is given by

$$d\mu_\gamma = DA\delta(\partial_\mu A_\mu^a)\det(\mathcal{M})e^{-\left(S_{\text{YM}}+\gamma^4 H\right)} \,, \tag{3.9}$$

where

$$H = \int d^4 x h(x) = g^2 \int d^4 x f^{abc} A_\mu^b \left(\mathcal{M}^{-1} \right)^{ad} f^{dec} A_\mu^e \,, \tag{3.10}$$

is the so-called horizon function, which implements the restriction to the Gribov region Ω. Notice that H is nonlocal. The parameter γ, known as the Gribov parameter, has the dimension of a mass and is not free, being determined by the horizon condition

$$\langle h(x) \rangle = 4 \left(N^2 - 1 \right) \,, \tag{3.11}$$

where the expectation value $\langle h(x) \rangle$ has to be evaluated with the measure $d\mu_\gamma$. To the first order, the horizon condition (3.11) reads, in d dimensions,

$$1 = \frac{N(d-1)}{4} g^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^4 + 2Ng^2\gamma^4} \,. \tag{3.12}$$

This equation coincides with the original gap equation derived by Gribov for the parameter γ [3].

Albeit nonlocal, the horizon function H can be localized through the introduction of a suitable set of additional fields. The final action reads

$$S = S_0 - \gamma^2 g \int d^d x \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} - d(N^2 - 1)\gamma^4 \right) \,,$$

$$S_0 = S_{\text{YM}} + \int d^d x \left(b^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu \left(D_\mu c \right)^a \right)$$

$$+ \int d^d x \left(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) \right.$$

$$\left. -g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} (D_\nu c)^b \varphi_\mu^{mc} \right) \,. \tag{3.13}$$

The fields $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac} \right)$ are a pair of complex conjugate bosonic fields. Similarly, the fields $\left(\overline{\omega}_\mu^{ac}, \omega_\mu^{ac} \right)$ are anticommuting. The horizon condition is equivalent with the demand that the quantum effective action Γ obeys

$$\frac{\partial\Gamma}{\partial\gamma^2} = 0 \tag{3.14}$$

Gauge	LOperator
linear covariant	$\frac{1}{2}A^a_\mu A^a_\mu$
Curci-Ferrari	$\frac{1}{2}A^a_\mu A^a_\mu + \alpha \bar{c}^a c^a$
maximal Abelian	$\frac{1}{2}A^\beta_\mu A^\beta_\mu + \alpha \bar{c}^\beta c^\beta$

Table 1: Gauges and their renormalizable dimension two operator

As shown in [22, 23, 24], the resulting local action turns out to be renormalizable to all orders of perturbation theory. Remarkably, we have been able to prove that this feature is preserved when the local operator A^2_μ is coupled to the Zwanziger action [26]. This allows for a simultaneous study of the effects of the Gribov parameter and condensate $\langle A^2 \rangle$.

A main consequence of the restriction of the domain of integration to the Gribov region is the fact that the ghost propagator gets enhanced in the infrared region. Using the gap equation arising from the horizon condition, one finds that [3, 22, 23, 25, 26, 27]

$$\left\langle c^a \bar{c}^b \right\rangle_p \sim \frac{\delta^{ab}}{p^4} \text{ for } p^2 \to 0 \tag{3.15}$$

This enhancement remains valid in the presence of $\langle A^2 \rangle$ [25, 26]. The infrared enhancement of the ghost propagator in the Landau gauge has also been observed from lattice simulations [29, 28] or solutions of the Schwinger-Dyson equations [30, 31, 32].

The Gribov restriction and $\langle A^2 \rangle$ also affect the gluon propagator in a nontrivial fashion, more precisely one finds [25, 26]

$$\left\langle A^a_\mu A^b_\nu \right\rangle_p = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{p^2}{p^4 + m^2 p^2 + 2g^2 N \gamma^4} \tag{3.16}$$

It is worth noticing that a propagator like (3.16) is not new, as it has been considered already some time ago by Stingl in order to solve the Schwinger-Dyson equations [33]. It induces an infrared suppressed gluon propagator, a fact in qualitative agreement with lattice [29] and Schwinger-Dyson results [30, 31, 32]. Let us also mention that (3.16) violates spectral positivity, giving an indication that the gauge bosons are unphysical particles [26]

4. Beyond the Landau gauge

It is unclear what the role of $A^2_{\min}$ might be in other gauges. This is a very difficult question, without any answer at the moment. Due to the severe nonlocality in the expression (3.1), it seems that explicit calculations outside the Landau gauge are almost prohibitive, as a localization procedure looks quite hopeless.

Nevertheless, in several other gauges, we have shown that other dimension two, renormalizable, local operators exist. We generalized the LCO method and showed that these operators condense and give rise to a dynamical gluon mass, see Table 1 and [34, 35, 36]. In the maximal Abelian gauge, it was found that only the off-diagonal gluons A^β_μ acquire a dynamical mass, a fact qualitatively consistent with the lattice results from [37, 38].

We have been able to make some connection between the various gauges and their dimension two operators by constructing renormalizable interpolating gauges and operators [34, 39]. However, these operators are explicitly gauge dependent, hence there does not seem to exist a clear relation with the gauge invariant operator $A^2_{\min}$.

Recently, the issue of Gribov copies has also been addressed in the maximal Abelian gauge [40, 41].

5. Another gauge invariant dimension two operator

Recently, we have considered the fact that, perhaps, another gauge invariant dimension two operator might be of some relevance. We took a look at

$$\mathscr{O} \equiv \frac{1}{VT} \int d^4x F^a_{\mu\nu} \left[\left(D^2 \right)^{-1} \right]^{ab} F^b_{\mu\nu} \,. \tag{5.1}$$

This operator was already introduced by Jackiw and Pi during their analysis of a dynamical mass generation in 3-dimensional gauge theories [42].

We can add the operator (5.1) to the Yang-Mills action as a mass term via

$$S_{YM} + S_{\mathscr{O}} \,, \tag{5.2}$$

with

$$S_{\mathscr{O}} = -\frac{m^2}{4} \int d^4x F^a_{\mu\nu} \left[\left(D^2 \right)^{-1} \right]^{ab} F^b_{\mu\nu} \,. \tag{5.3}$$

As we have discussed in [43], the action (5.2) can be localized by introducing a pair of complex bosonic antisymmetric tensor fields, $(B^a_{\mu\nu}, \overline{B}^a_{\mu\nu})$, and a pair of complex anticommuting antisymmetric tensor fields, $(\overline{G}^a_{\mu\nu}, G^a_{\mu\nu})$, belonging to the adjoint representation, according to which

$$
\begin{aligned}
e^{-S_{\mathscr{O}}} &= \int D\overline{B}DBD\overline{G}DG \exp\left[-\left(\frac{1}{4} \int d^4x \overline{B}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} \right. \right. \\
&\quad \left. \left. - \frac{1}{4} \int d^4x \overline{G}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu} + \frac{im}{4} \int d^4x \left(B - \overline{B} \right)^a_{\mu\nu} F^a_{\mu\nu} \right) \right] \,.
\end{aligned}
\tag{5.4}
$$

Doing so, we obtain a classical local action which reads

$$S_{YM} + S_{BG} + S_m \,, \tag{5.5}$$

where

$$
\begin{aligned}
S_{BG} &= \frac{1}{4} \int d^4x \left(\overline{B}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu} \right) \,, \\
S_m &= \frac{im}{4} \int d^4x \left(B - \overline{B} \right)^a_{\mu\nu} F^a_{\mu\nu} \,,
\end{aligned}
\tag{5.6}
$$

which is left invariant by the gauge transformations

$$
\begin{aligned}
\delta A^a_\mu &= -D^{ab}_\mu \omega^b \,, \\
\delta B^a_{\mu\nu} &= g f^{abc} \omega^b B^c_{\mu\nu} \,, \quad \delta \overline{B}^a_{\mu\nu} = g f^{abc} \omega^b \overline{B}^c_{\mu\nu} \,, \\
\delta G^a_{\mu\nu} &= g f^{abc} \omega^b G^c_{\mu\nu} \,, \quad \delta \overline{G}^a_{\mu\nu} = g f^{abc} \omega^b \overline{G}^c_{\mu\nu} \,.
\end{aligned}
\tag{5.7}
$$

In order to discuss the renormalizability of (5.5), we relied on the method introduced by Zwanziger in [22, 23] to discuss the renormalizability of the nonlocal horizon function (3.10). Instead of using (5.5) with m coupled to the composite operators $B^a_{\mu\nu}F^a_{\mu\nu}$ and $B^a_{\mu\nu}F^a_{\mu\nu}$, we introduce 2 suitable external sources $V_{\rho\sigma\mu\nu}$ and $\overline{V}_{\rho\sigma\mu\nu}$ and replace S_m by

$$\frac{1}{4}\int d^4x \left(V_{\sigma\rho\mu\nu}\overline{B}^a_{\sigma\rho}F^a_{\mu\nu} - \overline{V}_{\sigma\rho\mu\nu}B^a_{\sigma\rho}F^a_{\mu\nu}\right) . \tag{5.8}$$

At the end, the sources $V_{\sigma\rho\mu\nu}(x)$, $\overline{V}_{\sigma\rho\mu\nu}(x)$ are required to attain their physical value, namely

$$\overline{V}_{\sigma\rho\mu\nu}\Big|_{\text{phys}} = V_{\sigma\rho\mu\nu}\Big|_{\text{phys}} = -\frac{im}{2}\left(\delta_{\sigma\mu}\delta_{\rho\nu} - \delta_{\sigma\nu}\delta_{\rho\mu}\right) , \tag{5.9}$$

so that (5.8) reduces to S_m in the physical limit.

We assume the linear covariant gauge fixing, implemented through

$$S_{gf} = \int d^4x \left(\frac{\alpha}{2}b^a b^a + b^a \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b\right) , \tag{5.10}$$

In [43], we wrote down a list of symmetries enjoyed by the action

$$S_{YM} + S_{BG} + S_{gf} , \tag{5.11}$$

i.e. in absence of the sources. Let us only mention here the BRST symmetry, generated by the nilpotent transformation s given by

$$\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b , \quad sc^a = \frac{g}{2}f^{abc}c^a c^b , \\
sB^a_{\mu\nu} &= gf^{abc}c^b B^c_{\mu\nu} + G^a_{\mu\nu} , \quad s\overline{B}^a_{\mu\nu} = gf^{abc}c^b \overline{B}^c_{\mu\nu} , \\
sG^a_{\mu\nu} &= gf^{abc}c^b G^c_{\mu\nu} , \quad s\overline{G}^a_{\mu\nu} = gf^{abc}c^b \overline{G}^c_{\mu\nu} + \overline{B}^a_{\mu\nu} , \\
s\overline{c}^a &= b^a , \quad sb^a = 0 , \quad s^2 = 0 .
\end{aligned} \tag{5.12}$$

It turns out that one can introduce all the necessary external sources in a way consistent with the starting symmetries. This allows to write down several Ward identities by which the most general counterterm is restricted using the algebraic renormalization formalism [43]. After a very cumbersome analysis, it turns out that the action (5.5) must be modified to

$$S_{phys} = S_{cl} + S_{gf} , \tag{5.13}$$

with

$$\begin{aligned}
S_{cl} = \int d^4x \Bigg[&\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{im}{4}(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{4}\left(\overline{B}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu}\right) \\
&- \frac{3}{8}m^2\lambda_1\left(\overline{B}^a_{\mu\nu}B^a_{\mu\nu} - \overline{G}^a_{\mu\nu}G^a_{\mu\nu}\right) + m^2\frac{\lambda_3}{32}\left(\overline{B}^a_{\mu\nu} - B^a_{\mu\nu}\right)^2 \\
&+ \frac{\lambda^{abcd}}{16}\left(\overline{B}^a_{\mu\nu}B^b_{\mu\nu} - \overline{G}^a_{\mu\nu}G^b_{\mu\nu}\right)\left(\overline{B}^c_{\rho\sigma}B^d_{\rho\sigma} - \overline{G}^c_{\rho\sigma}G^d_{\rho\sigma}\right) \Bigg] ,
\end{aligned} \tag{5.14}$$

in order to have renormalizability to all orders of perturbation theory. We notice that we had to introduce a new invariant quartic tensor coupling λ^{abcd}, subject to the generalized Jacobi identity

$$f^{man}\lambda^{mbcd} + f^{mbn}\lambda^{amcd} + f^{mcn}\lambda^{abmd} + f^{mdn}\lambda^{abcm} = 0, \tag{5.15}$$

and the symmetry constraints

$$\lambda^{abcd} = \lambda^{cdab} \,,$$
$$\lambda^{abcd} = \lambda^{bacd} \,, \tag{5.16}$$

as well as two new mass couplings λ_1 and λ_3. Without the new couplings, i.e. when $\lambda_1 \equiv 0$, $\lambda_3 \equiv 0$, $\lambda^{abcd} \equiv 0$, the previous action would not be renormalizable. We refer to [43, 44] for all the details. We also notice that the novel fields $B_{\mu\nu}^a$, $\overline{B}_{\mu\nu}^a$, $G_{\mu\nu}^a$ and $\overline{G}_{\mu\nu}^a$ are no longer appearing at most quadratically. As it should be expected, the classical action S_{cl} is still gauge invariant w.r.t. (5.7).

The BRST transformation (5.12) no longer generates a symmetry of the gauge fixed action S_{phys}. However, we are able to define a natural generalization of the usual BRST symmetry that does constitute an invariance of the gauge fixed action (5.13). Indeed, after inspection, one shall find that

$$\widetilde{s}S_{phys} = 0 \,,$$
$$\widetilde{s}^2 = 0 \,, \tag{5.17}$$

with

$$\widetilde{s}A_\mu^a = -D_\mu^{ab}c^b \,, \quad \widetilde{s}c^a = \frac{g}{2}f^{abc}c^ac^b \,,$$
$$\widetilde{s}B_{\mu\nu}^a = gf^{abc}c^bB_{\mu\nu}^c \,, \quad \widetilde{s}\overline{B}_{\mu\nu}^a = gf^{abc}c^b\overline{B}_{\mu\nu}^c \,,$$
$$\widetilde{s}G_{\mu\nu}^a = gf^{abc}c^bG_{\mu\nu}^c \,, \quad \widetilde{s}\overline{G}_{\mu\nu}^a = gf^{abc}c^b\overline{G}_{\mu\nu}^c \,,$$
$$\widetilde{s}\overline{c}^a = b^a \,, \quad \widetilde{s}b^a = 0 \,. \tag{5.18}$$

In [43, 44], we also calculated explicitly various renormalization group equations to two loop order, confirming the renormalizability at the practical level. Various consistency checks are at our disposal in order to establish the reliability of these results, e.g. the gauge parameter independence of the anomalous dimension of gauge invariant quantities or the equality of others, in accordance with the output of the Ward identities in [43]. Furthermore, we proved in [44] the equivalence of the model (5.13) with the ordinary Yang-Mills theory in the case that $m \equiv 0$, making use of the nilpotent transformation

$$\delta_s B_{\mu\nu}^a = G_{\mu\nu}^a \,, \qquad \delta_s G_{\mu\nu}^a = 0 \,,$$
$$\delta_s \overline{G}_{\mu\nu}^a = \overline{B}_{\mu\nu}^a \,, \qquad \delta_s \overline{B}_{\mu\nu}^a = 0 \,,$$
$$\delta_s(\text{rest}) = 0 \,, \tag{5.19}$$

which generates a "supersymmetry" of the action $S_{phys}^{m\equiv0}$.

Acknowledgments

References

[1] D. J. Gross and F. Wilczek, *Ultraviolet behaviour of non-Abelian gauge theories*, Phys. Rev. Lett. **30** (1973) 1343.

[2] H. D. Politzer, *Reliable perturbative results for strong interactions?*, Phys. Rev. Lett. **30** (1973) 1346.

[3] V. N. Gribov, *Quantization Of Nonabelian Gauge Theories*, Nucl. Phys. B **139** (1978) 1.

[4] T. Schafer and E. V. Shuryak, *Instantons in QCD*, Rev. Mod. Phys. **70** (1998) 323 [arXiv:hep-ph/9610451].

[5] A. Maas, *On the spectrum of the Faddeev-Popov operator in topological background fields*, Eur. Phys. J. C **48** (2006) 179 [arXiv:hep-th/0511307].

[6] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, *On the significance of the vector potential squared*, Phys. Rev. Lett. **86** (2001) 2220 [arXiv:hep-ph/0010057].

[7] F. V. Gubarev and V. I. Zakharov, *On the emerging phenomenology of $\langle A_{\min}^2 \rangle$*, Phys. Lett. B **501** (2001) 28 [arXiv:hep-ph/0010096].

[8] Ph. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, *Testing Landau gauge OPE on the lattice with a $\langle A^2 \rangle$ condensate*, Phys. Rev. D **63** (2001) 114003 [arXiv:hep-ph/0101302].

[9] E. Ruiz Arriola, P. O. Bowman and W. Broniowski, *Landau-gauge condensates from the quark propagator on the lattice*, Phys. Rev. D **70** (2004) 097505 [arXiv:hep-ph/0408309].

[10] K. I. Kondo, *Vacuum condensate of mass dimension 2 as the origin of mass gap and quark confinement*, Phys. Lett. B **514** (2001) 335 [arXiv:hep-th/0105299].

[11] T. Suzuki, K. Ishiguro, Y. Mori and T. Sekido, *The dual Meissner effect and Abelian magnetic displacement currents*, Phys. Rev. Lett. **94** (2005) 132001 [arXiv:hep-lat/0410001].

[12] J. M. Cornwall, *Dynamical Mass Generation In Continuum QCD*, Phys. Rev. D **26** (1982) 1453.

[13] G. Dell'Antonio and D. Zwanziger, *Ellipsoidal bound on the Gribov horizon contradicts the perturbative renormalization group*, Nucl. Phys. B **326** (1989) 333.

[14] G. Dell'Antonio and D. Zwanziger, *Every gauge orbit passes inside the Gribov horizon*, Commun. Math. Phys. **138** (1991) 291.

[15] Semenov-Tyan-Shanskii and V.A. Franke, Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V.A. Steklov AN SSSR, Vol. **120** (1982) 159. English translation: New York: Plenum Press 1986.

[16] M. Lavelle and D. McMullan, *Constituent quarks from QCD*, Phys. Rept. **279** (1997) 1 [arXiv:hep-ph/9509344].

[17] H. Verschelde, *Perturbative Calculation Of Nonperturbative Effects In Quantum Field Theory*, Phys. Lett. B **351** (1995) 242.

[18] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, *The non-perturbative groundstate of QCD and the local composite operator A_μ^2*, Phys. Lett. B **516** (2001) 307 [arXiv:hep-th/0105018].

[19] D. Dudal, H. Verschelde and S. P. Sorella, *The anomalous dimension of the composite operator A^2 in the Landau gauge*, Phys. Lett. B **555** (2003) 126 [arXiv:hep-th/0212182].

[20] J. A. Gracey, *Three loop MS-bar renormalization of the Curci-Ferrari model and the dimension two BRST invariant composite operator in QCD*, Phys. Lett. B **552** (2003) 101 [arXiv:hep-th/0211144].

[21] R. E. Browne and J. A. Gracey, *Two loop effective potential for $\langle A_\mu^2 \rangle$ in the Landau gauge in quantum chromodynamics*, JHEP **0311** (2003) 029 [arXiv:hep-th/0306200].

[22] D. Zwanziger, *Local and renormalizable action from the Gribov horizon*, Nucl. Phys. B **323** (1989) 513.

[23] D. Zwanziger, *Renormalizability of the critical limit of lattice gauge theory by BRS invariance*, Nucl. Phys. B **399** (1993) 477.

[24] N. Maggiore and M. Schaden, *Landau Gauge Within The Gribov Horizon*, Phys. Rev. D **50** (1994) 6616 [arXiv:hep-th/9310111].

[25] R. F. Sobreiro, S. P. Sorella, D. Dudal and H. Verschelde, *Gribov horizon in the presence of dynamical mass generation in Euclidean Yang-Mills theories in the Landau gauge*, Phys. Lett. B **590** (2004) 265 [arXiv:hep-th/0403135].

[26] D. Dudal, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *The Gribov parameter and the dimension two gluon condensate in Euclidean Yang-Mills theories in the Landau gauge*, Phys. Rev. D **72** (2005) 014016 [arXiv:hep-th/0502183].

[27] J. A. Gracey, *Two loop correction to the Gribov mass gap equation in Landau gauge QCD*, Phys. Lett. B **632** (2006) 282 [arXiv:hep-ph/0510151].

[28] S. Furui and H. Nakajima, *Effects of the quark field on the ghost propagator of lattice Landau gauge QCD*, Phys. Rev. D **73** (2006) 094506 [arXiv:hep-lat/0602027].

[29] A. Maas, A. Cucchieri and T. Mendes, *On the infrared behavior of Green's functions in Yang-Mills theory* [arXiv:hep-lat/0610006].

[30] P. Watson and R. Alkofer, *Verifying the Kugo-Ojima confinement criterion in Landau gauge QCD*, Phys. Rev. Lett. **86** (2001) 5239 [arXiv:hep-ph/0102332].

[31] C. Lerche and L. von Smekal, *On the infrared exponent for gluon and ghost propagation in Landau gauge QCD*, Phys. Rev. D **65** (2002) 125006 [arXiv:hep-ph/0202194].

[32] D. Zwanziger, *Non-perturbative Landau gauge and infrared critical exponents in QCD*, Phys. Rev. D **65** (2002) 094039 [arXiv:hep-th/0109224].

[33] M. Stingl, *Propagation Properties And Condensate Formation Of The Confined Yang-Mills Field*, Phys. Rev. D **34** (1986) 3863 [Erratum-ibid. D **36** (1987) 651].

[34] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *An analytic study of the off-diagonal mass generation for Yang-Mills theories in the maximal Abelian gauge*, Phys. Rev. D **70** (2004) 114038 [arXiv:hep-th/0406132].

[35] D. Dudal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, *Dynamical gluon mass generation from $\langle A_\mu^2 \rangle$ in linear covariant gauges*, JHEP **0401** (2004) 044 [arXiv:hep-th/0311194].

[36] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, *Gluon-ghost condensate of mass dimension 2 in the Curci-Ferrari gauge*, Annals Phys. **308** (2003) 62 [arXiv:hep-th/0302168].

[37] K. Amemiya and H. Suganuma, *Effective mass generation of off-diagonal gluons as the origin of infrared Abelian dominance in the maximally Abelian gauge in QCD*, Phys. Rev. D **60** (1999) 114509 [arXiv:hep-lat/9811035].

[38] V. G. Bornyakov, M. N. Chernodub, F. V. Gubarev, S. M. Morozov and M. I. Polikarpov, *Abelian dominance and gluon propagators in the maximally Abelian gauge of SU(2) lattice gauge theory*, Phys. Lett. B **559** (2003) 214 [arXiv:hep-lat/0302002].

[39] D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, R. Thibes and H. Verschelde, *Remarks on a class of renormalizable interpolating gauges*, JHEP **0507** (2005) 059 [arXiv:hep-th/0505037].

[40] M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and R. Thibes, *The influence of the Gribov copies on the gluon and ghost propagators in Euclidean Yang-Mills theory in the maximal Abelian gauge*, Phys. Rev. D **72**, 085021 (2005) [arXiv:hep-th/0507052].

[41] M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and R. Thibes, *A study of the maximal Abelian gauge in SU(2) Euclidean Yang-Mills theory in the presence of the Gribov horizon* [arXiv:hep-th/0609212].

[42] R. Jackiw and S. Y. Pi, *Threshhold Singularities and the Magnetic Mass in Hot QCD*, Phys. Lett. B **368** (1996) 131 [arXiv:hep-th/9511051].

[43] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *A study of the gauge invariant, nonlocal mass operator* $Tr \int d^4x F_{\mu\nu} \frac{1}{D^2} F_{\mu\nu}$, Phys. Rev. D **72** (2005) 105016 [arXiv:hep-th/0510240].

[44] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *Quantum properties of a non-Abelian gauge invariant action with a mass parameter*," Phys. Rev. D **74** (2006) 045008 [arXiv:hep-th/0605288].