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String–Localized Quantum Fields, Modular Localization, and Gauge Theories

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The concept of modular localization intoduced by Brunetti, Guido and Longo, and Schroer, can be used to construct quantum fields. It combines Wigner's particle concept with the Tomita-Takesaki modular theory of operator algebras. I report on the construction of free fields which are localized in semi-infinite strings extending to spacelike infinity (joint work with B. Schroer and J. Yngvason). Particular applications are: The first local (in the above sense) construction of fields for Wigner's massless "infinite spin" particles; String-localized vector/tensor potentials for Photons and Gravitons, respectively; Massive vector bosons. Some speculative ideas will be presented concerning the perturbative construction of gauge theories (and quantum gravity) completely within a Hilbert space, trading gauge dependence with dependence on the direction of the localization string.

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1. The Notion of String-Localized Quantum Fields.

The principle of locality demands that observables be measurable in bounded regions of space-time, and that observables localized in space-like separated regions be compatible. This principle is usually implemented by (point-like localized) quantum fields which commute for space-like separated arguments. In addition to the observables there may be, however, unobservable charge–carrying fields. In models, these are constructed first and then the observables are constructed from them, usually selected by a global gauge principle. (For example, the observables in the case of a charged scalar field $\varphi(x)$ are generated by the currents $j_\mu(x)$.)

The unobservable fields need, in general, not be localized in bounded regions. In some cases, the fundamental fields even *cannot be localized* in bounded regions: For example, if they carry a so-called "gauge charge" [8], that is a charge which can be determined at space-like infinity by a version of Gauss' law. Another instance are fields whose basic excitations are certain "exotic" particle types, namely Anyons [31] in $2+1$ dimensions and Wigner's so-called massless "infinite spin" particles [30]. The former correspond to irreducible massive positive-energy representations of the Poincaré group whose spin is not integer or half-integer (which is admitted in two space dimensions). The work of Doplicher *et. al.* [10] implies that the corresponding fields cannot be compactly localized. The latter correspond to irreducible massless representations with infinitely many polarization degrees of freedom, corresponding to a faithful representation of the little group $E(2)$. J. Yngvason has shown [32] that fields with such excitations cannot be point-like localized in the sense of Wightman fields.

On the other hand, the charge carrying fields do have to satisfy *some* localization properties since they must generate local observables. If the theory is purely massive, then it has been shown [9] that the charged fields are localized[1] in *space-like cones*. A space-like cone is a salient cone in space-time which extends to space-like infinity. Important structural results have been shown for theories with such localization, like the construction of scattering states [9], analiticity of the S-matrix [3], the analysis of the superselection charge structure [11], and the Bisognano-Wichmann and PCT theorems [19]. Similarly, Brunetti *et al.* [7] have shown the existence of a free field algebra localized in space-like cones for all (bosonic) particle types, including the massless "infinite spin" particles.

Steinmann [25], inspired by the ideas of Mandelstam [17], introduced the notion of quantum fields localized on *space-like strings*, idealizing a cone. Such "string" is a ray extending from a point in Minkowski space to infinity in some space-like direction. More precisley, if x is a point in $\mathbb{R}^4$ and e is a point in the manifold of space-like directions,

$$H := \{e \in \mathbb{R}^4,\ e\cdot e = -1\}, \tag{1.1}$$

then the string $S_{x,e}$ emanating from x in direction e is given by

$$S_{x,e} := x + \mathbb{R}_0^+ e. \tag{1.2}$$

[1]In the case of charged fields, localization means that space-like separated fields have vanishing commutators or anti-commutators or, in $2+1$ dimensions, satisfy more general (braided) commutation relations.

Steinmann proved the Jost-Schroer theorem for fields with such localization [25]. In [22, 23], we have elaborated on this concept, introducing the notion of a covariant string–localized quantum field, as follows.

Definition 1. *A covariant* string–localized quantum field *is an operator valued distribution* $\varphi(x,e)$ *on* $\mathbb{R}^4 \times H$ *satisfying*
i*) String–locality: If* $S(x,e)$ *and* $S(x',e')$ *are space-like separated, then*

$$[\varphi(x,e),\varphi(x',e')] = 0; \tag{1.3}$$

ii*) Covariance: There is a representation* U *of the Poincaré group* $\mathscr{P}_+^\uparrow$ *such that*

$$U(a,\Lambda)\varphi(x,e)U(a,\Lambda)^{-1} = \varphi(\Lambda x + a, \Lambda e) \tag{1.4}$$

holds for all $(a,\Lambda) \in \mathscr{P}_+^\uparrow$.

(We also consider the case where the fields have, in addition, tensor or spinor indices, *cf.* below.) Our original aim, motivating the introduction of this concept, was an explicit construction of free fields for the massless infinite spin particles, improving the existence result of Brunetti *et al.* [7]. This goal has been reached [22, 23], and the by-products (on other particle types) turned out to be more interesting than the original object, as I shall try to motivate in section 4.

The next section is meant to sketch the role of modular localization in our construction of free fields. Section 3 summarizes our results on free fields, and Section 4 gives a speculative outlook on the construction of interacting string-localized fields.

2. Modular Localization and the Construction of Free String-Localized Fields.

In the point-localized case, covariance of free fields essentially already implies locality. In the string-localized case, this is not so and there are no guidelines from the usual field theory methods in implementing locality (1.3). Hence independent ideas are warranted. In the context of algebraic QFT, there is an appropriate concept, the so-called *modular localization*. This concept has been introduced by Brunetti *et al.* [7] and by Schroer [14]. It is based on the Bisognano-Wichmann theorem [2, 19], which asserts that for a large class of models, certain algebraic invariants of the field algebra are fixed by the (ray) representation of the Poincaré group under which the field transforms, and the S-matrix. These algebraic invariants are the Tomita operators $S(\mathscr{O})$ of the algebra of fields localized in $\mathscr{O}$, for each space-time region $\mathscr{O}$. The principle of modular localizaton consists in inverting the argument: Namely, these invariants can be consistently constructed from the representation of the Poincaré group (and the S-matrix). This has been done by the author for Anyons in $d = 2+1$ at the single particle level [20]. Moreover, (in the non-Anyon case) the family of Tomita operators allows for the construction of free local fields. This concept of modular localizaton has been used by Brunetti *et al.* in the mentioned existence proof of free fields localized in space-like cones for every bosonic particle type, including the massless infinite spin particles [7].

As mentioned, the aim of [22, 23] was to achieve more, namely an explicit construction, and idealization of the space-like cones to strings $S_{x,e}$. By the Jost-Schroer theorem for string-localized fields [25], it suffices to solve the problem on the level of the single particle space, consisting of L^2

functions on the mass shell H_m^+ with values in the "little Hilbert space" (corresponding to the spin degrees of freedom). The field $\varphi(x,e)$ must create from the vacuum Ω a single particle state of the form

$$\langle p \,|\, \varphi(x,e)\,\Omega \rangle = e^{ipx}\, u(e,p), \quad p \in H_m^+, \tag{2.1}$$

where $u(e,p)$ is a distribution in $e \in H$ and a function in $p \in H_m^+$ with values in the "little Hilbert space" satisfying certain properties which encode locality and covariance: Firstly, our requirement of string-locality (1.3) implies that $\varphi(x,e)\Omega$ is (after smearing) in the domain of definition of each Tomita operator pertaining to any space-like cone containing the string $S_{x,e}$. The concept of modular localization then implies that for fixed p, $u(e,p)$ is the boundary value of an analytic function on the tuboid $\mathscr{T}_+$ in the natural complexification of H consisting of those complex e whose imaginary part is in the open forward light cone. This function is of moderate growth near the real "boundary" H, in the sense of [4], thereby defining $u(e,p)$ as a distribution on H. Secondly, covariance (1.4) implies that u satisfies the *intertwiner property*

$$D(R(\Lambda,p))\, u(\Lambda^{-1}e, \Lambda^{-1}p) = u(e,p) \tag{2.2}$$

for $(e,p) \in \mathscr{T}_+ \times H_m^+$ and $\Lambda \in \mathscr{L}_+^\uparrow$. Here, $R(\Lambda,p)$ is the Wigner rotation and D is the repesentation of the little group which induces the irreducible (ray) representation of the Poincaré group corresponding to the particle type at hand. With the intertwiner function $u(e,p)$ one associates a hermitean field acting in the Fock space over the corresponding one particle space, via

$$\varphi(x,e) = \int_{H_m^+} d\mu(p)\, \Big\{ e^{ip\cdot x}\, u(e,p) \circ a^*(p) \;+ e^{-ip\cdot x}\, \overline{u(e,p)} \circ a(p) \Big\}. \tag{2.3}$$

Here, $d\mu(p)$ is the Lorentz invariant measure on H_m^+, and the circle $\circ$ denotes (sloppily speaking) summation/integration over the spin degrees of freedom. (In Equ. (2.3) it has been assumed that $u(e,p)$ satisfies a certain self-conjugacy property (which can be achieved for all particle types), yielding a hermitean field.) This field is then in fact covariant in the sense of (1.4) under the second quantization U of the corresponding irreducible representation, and string-localized in the sense of Equ. (1.3).

3. Results on Free String-Localized Fields.

Along the indicated lines, we have constructed free covariant string-localized fields for all bosonic particle types, including the massless infinite spin particles in [23]. (String-localized fermions can also be constructed; they need an additional spinor index.) Our fields satisfy the Reeh-Schlieder and Bisognano-Wichmann properties.

We have also found the following uniqueness result. Every covariant string-localized field is of the form (2.3), and the intertwiner function u is unique up to multiplication with a function of $e \cdot p$ which is meromorphic on the upper complex half plane. (That is to say, if $\hat{u}$ is another intertwiner function, then $u(e,p) = F(e\cdot p)\, \hat{u}(e,p)$, where F is a numerical function, meromorphic on the complex upper half plane.)

Due to the worse localization, our fields have a better short distance behaviour than their point-like localized counterparts. For example, the Fourier transforms of their propagators[2] behave like

[2]By propagator, we mean time-ordered two-point function.

$|p|^{-2}$ for large p for any of these fields, independent of the spin (corresponding to a $|p|^0$ behaviour of the on-shell two-point functions). This includes massless particles with helicity ± 1 and ± 2, corresponding to photons and gravitons, *cf.* Equ.s (3.5) and (3.8) below. Note that this behaviour is a prerequisite for any non-trivial interaction. It is to be contrasted with the point-like case where the propagators for spin/helicity s behave at best like $|p|^{2s-2}$, and $|p|^{-2}$ can only be achieved in the setting of gauge theory, at the price of an indefinite metric space or loss of covariance.

Some special cases are worth mentioning in more detail.

Massless infinite spin particles. These correspond to representations of the Poincaré group where the inducing representation of the little group $E(2)$, corresponding to the spin degrees of freedom, is faithful. Such inducing representation D_κ is infinite dimensional, characterized by a parameter $\kappa > 0$, and acts on $L^2(\mathbb{R}^2, \delta(k^2 - \kappa^2) d^2k)$ as

$$(D_\kappa(c,R)\psi)(k) := e^{ic\cdot k}\,\psi(R^{-1}k).$$

For these particles, we have found intertwiner functions u^α characterized by a real parameter $\alpha < 0$:

$$u^\alpha(e,p)(k) = e^{-i\pi\alpha/2} \int d^2c\, e^{ikc}\,(B_p\Lambda_c\xi \cdot e)^\alpha, \tag{3.1}$$

where B_p is a boost which maps a fixed base point $(1,0,0,1) \in H_0^+$ to p, Λ_c is the Lorentz transformation corresponding to a c–translation in the stability group $E(2)$ of $(1,0,0,1)$, and ξ is a lightlike vector invariant under the rotation subgroup of the $E(2)$. This intertwiner function gives rise, via Equ. (2.3), to a quantum field which satisfies all requirements from Definition 1.

The problem which has thus been solved has already been posed by Wigner [30] and has resisted considerable efforts of several generations of elementary particle physicists [1, 16, 30, 32]. (In the mentioned articles, covariant fields have been constructed, but the issue of localization has not been solved.)

Vector potentials for photons. For massless particles with finite helicity (*ie.* finite-dimensional representation of the little group) the fields must carry, in addition to the string direction e, a vector index. For photons, we constructed a string-localized vector boson $A_\mu(x,e)$, acting in the physical photon Hilbert space (the second quantization of the direct sum of irreducible representation spaces for helicity 1 and -1). It transforms as

$$U(a,\Lambda)A_\mu(x,e)U(a,\Lambda)^{-1} = A_\nu(a+\Lambda x, \Lambda e)\,\Lambda^\nu{}_\mu \tag{3.2}$$

and satisfies string-locality in the sense of Eq. (1.3). It is indeed a vector potential for the field strength $F_{\mu\nu}$ (the unique free Wightman field corresponding to the electromagnetic field strength and acting in the mentioned Hilbert space) in the sense that its exterior derivative dA coincides with F, *ie.* $F_{\mu\nu}(x) = \partial_\mu A_\nu(x,e) - \partial_\nu A_\mu(x,e)$. It also satisfies the Lorentz and axial "gauge" conditions

$$\partial^\mu A_\mu(x,e) = 0, \quad e^\mu A_\mu(x,e) = 0. \tag{3.3}$$

However, these conditions are satisfied by *every* free vector field $A_\mu(x,e)$ for photons acting in the physical Hilbert space and transforming as in Eq. (3.2); hence they cannot be regarded as additional gauge conditions in this context. Our vector potential is completely fixed by the requirements of string-locality (1.3), covariance (3.2) and that its exterior derivative is independent of e [23, Prop. 5.1]. (The latter requirement is analogous to gauge independence in the usual formulation. It implies that dA coincides with the electromagnetic field strength [23, Proof of Prop. 5.1].) It is buildt as in Equ. (2.3) from the following intertwiner function:

$$u(e,p)_{\pm,\mu} = \lim_{\varepsilon\to 0} \frac{\hat{e}_\pm(p)\cdot e}{e\cdot p + i\varepsilon}\, p_\mu - \hat{e}_\pm(p)_\mu. \tag{3.4}$$

Here, $\hat{e}_\pm(p)$ are the polarization vecors $\hat{e}_\pm(p) := B_p\,\hat{e}_\pm$ where $\hat{e}_\pm := 2^{-1/2}\,(0,1,\mp i,0)$ and B_p is the mentioned boost which maps a $(1,0,0,1)$ to p. The ε-prescription in Equ. (3.4) refers to the fact that $u(e,p)_{\pm,\mu}$ is a distribution in e for fixed p: First integrate over a test function in e, then take the limit. Since $e\cdot p$ has positive imaginary part for $p\in H_0^+$ and e in the mentioned tuboid $\mathscr{T}_+$, this prescription correponds precisely to the one indicated above, before Equ. (2.2).

The two-point function of the corresponding field is given by

$$\big(\Omega, A_\mu(x,e)A_\nu(x',e')\Omega\big) = i\int_{H_0^+} d\mu(p)\, e^{ip\cdot(x'-x)}\, M_{\mu\nu}(p;e,e'), \tag{3.5}$$

$$M_{\mu\nu}(p;e,e') \doteq -g_{\mu\nu} + \frac{p_\mu p_\nu}{(e\cdot p - i\varepsilon)(e'\cdot p + i\varepsilon')} + \frac{e_\nu p_\mu}{e\cdot p - i\varepsilon} + \frac{e'_\mu p_\nu}{e'\cdot p + i\varepsilon'}.$$

Recall that in the quantization of the *point-like* localized vector potential, one has the freedom of a choice of gauge, with the following two alternatives: A covariant gauge only exists in an indefinite metric space [26]. In a Hilbert space representation, there are only non-covariant gauges, among them the axial gauge $e^\mu A_\mu(x,e) = 0$ where e is a *fixed* direction. In this gauge, the two-point function has the same form as in Equ. (3.5) (with $e = e'$), with two significant disadvantages compared with our string-localized fields: Firstly, it is not Poincaré invariant since e is fixed; and secondly, there is no convincing preferred regularization of the singularities $e\cdot p$ [29]. (In our approach, the factors $(e\cdot p + i\varepsilon)^{-1}$ are regular after smearing with a test function in e, and this regularization is fixed by the same requirements as the field $A_\mu(x,e)$ itself.)

Massive vector bosons. There is also a string-localized field for massive vector bosons with spin one [21]. As in the above massless case, it is fixed by the requirements of covariance (3.2), string-locality and that $dA(x,e)$ be independent of e. It has the same two-point function as the massless (photon) counterpart, *cf.* Equ. (3.5), except that it is concentrated on the positive mass shell H_m^+. This interesting fact might allow for a treatment of the infrared problem (adiabatic limit) in perturbative QED by starting from massive QED and letting $m\to 0$. The massive analogue of Equ. (3.5) implies that the propagator of our string-localized massive vector boson behaves like $|p|^{-2}$ for large momenta. This is worthwile comparing with the point-like localized counterpart, whose propagator contains a term $\sim |p|^0$, indicating that it admits no interesting interactions (unless one adds ghost degrees of freedom and uses an indefinite metric representation).

Tensor potentials for linearized gravitons. For massless particles with helicity ± 2, there is a string-localized tensor field $h_{\mu\nu}(x,e)$ transforming as a "string-tensor", similar to Equ. (3.2) [21]. It is a "potential" for the quantized (point-localized) free, *ie.* linearized, Riemann tensor $R_{\mu\nu\alpha\beta}$ [24], in the sense that the classical relation between the linearized Riemann tensor and the perturbation of the metric holds:

$$R_{\mu\nu\alpha\beta}(x) = \tfrac{1}{2}\big\{\partial_\mu\partial_\alpha h_{\nu\beta}(x,e) + \partial_\nu\partial_\beta h_{\mu\alpha}(x,e) \tag{3.6}$$

$$- \partial_\nu\partial_\alpha h_{\mu\beta}(x,e) - \partial_\mu\partial_\beta h_{\nu\alpha}(x,e)\big\}. \tag{3.7}$$

It is well-known that for point-like fields these conditions cannot be satisfied in a Hilbert space representation with positive energy [27]. The two-point function of our $h_{\mu\nu}$ is given by [21]

$$\big(\Omega, h_{\mu\alpha}(x,e)h_{\mu'\alpha'}(x',e')\Omega\big) = i\int d^4p\; e^{ip\cdot(x'-x)}\, M_{\mu\alpha,\mu'\alpha'}(p;e,e'), \tag{3.8}$$

$$M_{\mu\alpha,\mu'\alpha'}(p;e,e') \doteq \frac{1}{(e\cdot p - i\varepsilon)^2(e'\cdot p + i\varepsilon')^2}\, M^R_{\mu\nu\alpha\beta,\mu'\nu'\alpha'\beta'}(p)\; e^\nu e^\beta (e')^{\nu'}(e')^{\beta'}.$$

Here $M^R_{\mu\nu\alpha\beta,\mu'\nu'\alpha'\beta'}(p)$ is the on-shell two-point function of the free Riemann tensor, which is known to be a homogenous polynomial in p of degree four, *cf.* [24]. Consequently, the Fourier transform of the propagator goes like $|p|^{-2}$ for large p. (It is also regular for finite p since, as mentioned above, it is being considered as a distribution in e, e' so that the factors $(e\cdot p \pm i0)^{-1}$ do not cause singularities).

4. Outlook: Interacting String-Localized Fields.

The specific properties of our string-localized free fields raise the hope that they should be a good starting point for a perturbative construction of interacting string-localized fields. In contrast to the case of point-localized fields, the various construction schemes are not equivalent in the case at hand. For example, the Yang-Feldman approach does not seem to work for string-localized fields, for reasons similar to the ones found already in the 70's in the context of "non-local" interactions [18]: The (string-) localization is lost in higher orders. But there is one perturbative scheme which seems to work for string-localized fields: The so-called causal construction of Epstein and Glaser [13], based on ideas of Stueckelberg and Bogoliubov.

We shall briefly sketch this approach (see [5, 24] for a detailed account). One starts with free fields acting in a Hilbert space, and an "interaction Lagrangean" $\mathscr{L}_I$. This is a Wick polynomial in the free fields, interpreted as the first order of the S-matrix. (However, a Lagrangean formulation of the theory is not necessary [28]). The interaction Lagrangean determines a specific class of Wick polynomials, namely its derivatives w.r.t. basic fields. For Wick polynomials W_i in this class, one defines time-ordered products $TW_1(x_1)\cdots W_n(x_n)$ recursively, requiring that

$$TW_1(x_1)\cdots W_n(x_n) = TW_1(x_1)\cdots W_k(x_k)\; TW_{k+1}(x_{k+1})\cdots W_n(x_n) \tag{4.1}$$

if all of the events $x_1,\ldots,x_k$ are later than the events $x_{k+1},\ldots,x_n$ in some reference frame. Together with (translational) covariance, this fixes the time-ordered products up to the point $x_1 = \cdots = x_n = 0$. (Re-) normalization then consists in the extension into this point. Having constructed the time-ordered products, one defines Bogoliubov's S-Matrix, depending on a test function of compact support g and a Wick polynomial W in the mentioned class, as the formal series

$$S(gW) := \sum_{n=0}^{\infty}\frac{i^n}{n!}\int dx_1\cdots dx_n\, g(x_1)\cdots g(x_n)\; TW(x_1)\cdots W(x_n). \tag{4.2}$$

The interpretation of $S(g\mathscr{L}_I)$ is that it formally constitutes the S-matrix for the Hamiltonian $H_I(t) \doteq -\int d^3\mathbf{x}\,\mathscr{L}_I(t,\mathbf{x})\,g(t,\mathbf{x})$ in the interaction picture. (The infrared problem consists in the so-called adiabatic limit, $g \to$ const.) One then defines for every free field φ an interacting field φ_I via Bogoliubov's formula:

$$\varphi_I(f) := \frac{1}{i}\frac{d}{d\lambda} S(g\mathscr{L}_I)^{-1}\, S\big(g\mathscr{L}_I + \lambda\,\varphi(f)\big)\,|_{\lambda=0}. \tag{4.3}$$

Due to the time-ordering prescription (4.1), the S-matrix satisfies the so-called causal factorization property which in turn implies locality of the interacting fields.

This scheme might be transferred to the string-localized case as follows. The time-ordering prescription of string-localized Wick products $W(x,e)$ must take the strings $S_{x,e}$ into account: Equ. (4.1), with (x_i,e_i) instead of x_i, must hold if all strings $S_{x_1,e_1},\dots,S_{x_k,e_k}$ are later than the strings $S_{x_{k+1},e_{k+1}},\dots,S_{x_n,e_n}$ in some reference frame. Bogoliubov's S-matrix then depends on test functions $g(x,e)$ living on $\mathbb{R}^4\times H$, and the multiple integral in Equ. (4.2) extends also over $H^{\times n}$. $S(g\mathscr{L}_I)$ is then the formal S-matrix for the interaction Hamiltonian $H_I(t) \doteq -\int_{x^0=t} d^3\mathbf{x} \int_H d\sigma(e)\,\mathscr{L}_I(x,e)\,g(x,e)$ in the interaction picture, where $d\sigma(e)$ denotes the Lorentz invariant measure on H. The interacting fields are defined as in Equ. (4.3), with f a test function on $\mathbb{R}^4\times H$. As in the point-like case, the time-ordering prescription implies a causal factorization property of the S-matrix which in turn implies *string-locality* of the interacting fields in the sense of Equ. (1.3). It is at the moment unclear to what extent the time-ordered products are fixed by the mentioned prescription, and which normalization degrees of freedom one has as compared to the corresponding point-like case.

An indespensible requirement for the programme is that it admits the construction of *local* (*ie.*, compactly localized) observables. A possible mechanism achieving this is to imitate gauge theories, with "gauge dependence" being replaced by "dependence on the string e". Consider, for example, the massless or massive vector boson $A_\mu(x,e)$. Since the exterior derivative is independent of e, $A_\mu(x,e)$ and $A_\mu(x,e')$ differ by the derivative of a field $\Phi(x,e,e')$. (In constrast to the gauge theory case, this field is in the algebra of the A_μ's, and needs no new degrees of freedom.) Therefore, if we take the interaction Lagrangean $\mathscr{L}_I(x,e) = :j^\mu(x)A_\mu(x,e):$, where j^μ is the conserved current of a charged field, we have

$$\mathscr{L}_I(x,e') = \mathscr{L}_I(x,e) + i\partial_\mu W^\mu(x;e,e'). \tag{4.4}$$

This implies that Bogoliubov's S-matrix is independent of the string e at first order, in the adiabatic limit. (Independence of e means that $S(g\otimes h\mathscr{L}_I)$, $g\otimes h\in\mathscr{D}(\mathbb{R}^4\times H)$, factorizes as $\int_H d\sigma(e)h(e)$ times an operator $S(g,\mathscr{L}_I)$ which is independent of h.) Independence at higher orders then ammounts to a (re-) normalization condition on the time-ordered products analogous to the "perturbative gauge invariance" [24]. If this e-independence of the S-matrix in the adiabatic limit can be implemented, then the interacting counterpart $\varphi_I(x)$ of any field $\varphi(x)$ which does not depend on e also does not depend on e and is point-like localized. This holds in particular for the fields $F_I^{\mu\nu}(x)$, where $F = dA$, and $j_I^\mu(x)$. These fields will then generate an observable algebra with point-like localization.

We conclude with some rather speculative remarks on possible applications of this construction.

As indiated, this construction should be attempted to carry through for QED, and for massive vector bosons. A more speculative possible application is the perturbative construction of quantum gravity along rather conservative lines. Such construction would start from a family of string-localized free tensor potentials $h_{\mu\nu}(x,e)$ as described above, one for each background metric within a certain class of space-times. Here, the string $S_{x,e}$ might be defined as the semi-infinite geodesic curve starting from x in the direction $e \in T_xM$. Each $h_{\mu\nu}$ would describe the quantum fluctuations around the given classical background. As interaction Lagrangean $\mathscr{L}_I$ one would take the corresponding part of the Einstein-Hilbert Lagrangean. The family of resulting interacting fields for every background should be constructed in such a way that a change of background metric ammounts to a symmetry of the theory, in the sense explained by Brunetti and Fredenhagen in [6]. As explained there, this would implement independence of the gravitational background.

One might also speculate that the proposed scheme allows for the perturbative construction of non-Abelian gauge theory analogues, and that it might even admit (renormalizable) interactions which are not admitted in the gauge theory setting. For example, why should there not be a string-localized model with self-interacting vector bosons without a Higgs particle? (In the point-like case, such model would either violate unitarity or renormalizability [15].) In view of the renewed search for the Higgs particle, such model would be of great interest. Apart from possible new models, there is an esthetic motivation for these constructions, namely: In the gauge theoretic approach, the construction detours through a huge realm of unphysical quantities (ghosts and an indefinite metric space), which one would like to avoid, following Ockham's razor. Our approach, on the other hand, works completely in Hilbert space and does not need ghosts (this is in accord with the well-known fact that in the axial gauge the ghosts decouple).

If these constructions work, it would be interesting to discuss the following question. The work of Scharf *et al.* [12] show that the principles of gauge invariance (of the observable quantities) and renormalizability fix, to a great extent, the possible interactions for a given set of particle types. The question is if the same holds in our approach, where gauge invariance is replaced by independence of the string directions e. Since this indendence is equivalent with point-like localization, this would ultimatively mean the following: The principles of *locality* and renormalizability fix the possible interactions. This would be very satisfying, since these principle are, in contrast to the gauge principle, intrinsic to quantum field theory.

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