

Particle Physics from D-Branes at Gepner Points

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We discuss the building of Standard-like models from configurations of stacks of orientifold planes and D-branes on an internal space with the structure $(\text{Gepner model})^{c=3n} \times \mathbf{T}^{2(3-n)}/\mathbf{Z}_N$, for $n = 1, 2, 3$. In particular we concentrate in the, interesting, $n = 2$ case, which would correspond to orbifolds of a $K3$ six dimensional manifold times a two dimensional torus.

Such constructions are performed in two steps. In a first step, we consider $D = 6$ Type II B orientifolds on Gepner points, in the diagonal invariant case and for both, odd and even, affine levels. We build up the explicit expressions for B-type boundary states and crosscaps and obtain the amplitudes among them. From such amplitudes we read the corresponding spectra and the tadpole cancellation equations. In a second step we further compactify on a \mathbf{T}^2 torus, by simultaneously orbifolding the Gepner and the torus internal sectors. The embedding of the orbifold action in the brane sector breaks the original gauge groups and leads to $\mathcal{N} = 1$ supersymmetric chiral spectra. Whenever even orbifold action on the torus is considered, new branes, with world-volume transverse to torus coordinates, must be included. Interestingly enough, large transverse extra dimensions can be used to lower the string scale.

As an illustration we present a 3 generations Left-Right symmetric model that can be further broken to a MSSM model.

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1. Introduction

The quest of the Standard Model like vacua, from open string interacting conformal field theories, received considerable attention in last years. In particular, much progress has been achieved in the context of orientifolds of Type II string compactified on Gepner models [1, 2, 3, 4, 5, 6, 7] Gepner models [8] are special points of Calabi Yau manifolds, at string scale, that allow for a description in terms of an exactly solvable rational CFT. Preliminary works in heterotic string theories and Type II orientifolds on Gepner points were presented [9, 10].

The rules for computing spectra and the tadpole cancellation equations have been derived for generic situations. Nevertheless, even if concise and rather simple generic expressions can be obtained, the computation of spectra for specific models can become rather cumbersome due to the, generically, huge number of open string states involved. Only solving the tadpole consistency equations can represent a difficult task even for a fast computer. Therefore a systematics is needed in order to be able to extract any useful information. In this sense a remarkable computing search for models with Standard like model spectra was performed in [5] by restricting the scan to four stacks of SM branes, by following the ideas advanced in [11] in the context of intersecting brane models [13] on toroidal like manifolds. In fact, thousands of SM like models were found. It is worth mentioning that even the simplest of these models requires to introduce a huge number of projections and to solve several tadpole equations. In Ref. [6] a hybrid Type IIB orientifold construction was proposed where the internal sector is built up from a Gepner sector times a torus. By choosing a torus invariant under some of the known \mathbf{Z}_N phase symmetries of Gepner models, an orbifold by such symmetries was then performed. Thus, schematically, in $D = 4 + 2n$ the internal sector is given by $(\text{Gepner model})^{c_{int}=9-3n} \times \mathbf{T}^{2n}/\mathbf{Z}_N$ (where c_{int} is the internal central charge). The orbifold action is simultaneously embedded as a twist on Chan Paton factors on the open string sector resulting in a breaking of the starting D dimensional Gepner orientifold gauge groups. In particular, such constructions lead to $\mathcal{N} = 1$ $D = 4$ chiral models. Further elaboration on this idea was advanced in Ref. [7].

Hybrid Gepner-torus models have some interesting features. An important, practical, observation [6] is that the number of Gepner models (see [10]) involved, 3 in $D = 8$ or 16 in $D = 6$, is remarkably lower than the 168 models in $D = 4$ (without including moddings) and so it is the number of internal states. Also, many features can be studied analytically without the need of computers. From the phenomenological point of view, the possibility of having large extra dimensions, in the torus directions, could be an appealing feature allowing for some control over the string scale.

In this talk we discuss this hybrid models proposal, mainly based on the work of Ref. [7], where the internal sector is built up from $D = 6$ Gepner models times a two torus. From expressions for B-type boundary states and crosscaps we read the tadpole cancellation equations and the rules for reading the spectra. An explicit example (the 6620 model) is developed in detail. We further compactify on a \mathbf{T}^2 torus by simultaneously orbifolding the Gepner and the torus internal sectors and by embedding the orbifold action on the brane sector. Detailed rules for obtaining the $D = 4$ model spectra and tadpole equations are shown. We conclude with an illustration of how to obtain a 3 generations Left-Right symmetric model (which can be further broken into a MSSM model) from a \mathbf{Z}_4 orbifold of the, $D = 6$, 6620 diagonal Gepner model times a torus.

2. Type II orientifolds, crosscaps and boundary states

Essentially, an orientifold model is obtained by dividing out the orientation reversal symmetry of Type II string theory (see for instance [15, 3]). Schematically, Type IIB torus partition function is defined as

$$\mathcal{Z}_T(\tau, \bar{\tau}) = \sum_{a,b} \chi_a(\tau) \mathcal{N}^{ab} \bar{\chi}_b(\bar{\tau}) \quad (2.1)$$

where the characters $\chi_a(\tau) = \text{Tr}_{\mathcal{H}_a} q^{L_0 - \frac{c}{24}}$, with $q = e^{2i\pi\tau}$, span a representation of the modular group of the torus generated by $\mathbf{S}: \tau \rightarrow -\frac{1}{\tau}$ and $\mathbf{T}: \tau \rightarrow \tau + 1$ transformations. \mathcal{H}_a is the Hilbert space of a conformal field theory with central charge $c = 15$, generated from a conformal primary state ϕ_a (similarly for the right moving algebra). In particular $\chi_a(-\frac{1}{\tau}) = S_{aa'} \chi_{a'}(\tau)$ and modular invariance require $S\mathcal{N}S^{-1} = \mathcal{N}$ (for left-right symmetric theories $\mathcal{N}^{ab} = \mathcal{N}^{ba}$). Generically, the characters can be split into a spacetime piece, contributing with $c_{st} = \bar{c}_{st} = \frac{3}{2}D$ and an internal sector with $c_{int} = \bar{c}_{int} = \frac{3}{2}(10 - D)$.

Let Ω be the reversing order (orientifolding) operator permuting right and left movers. Modding by order reversal symmetry is then implemented by introducing the projection operator $\frac{1}{2}(1 + \Omega)$ into the torus partition function. The resulting vacuum amplitude reads

$$\mathcal{Z}_\Omega(\tau, \bar{\tau}) = \mathcal{Z}_T(\tau, \bar{\tau}) + \mathcal{Z}_K(\tau - \bar{\tau}). \quad (2.2)$$

The first term is just the symmetrization (or anti-symmetrization in case states anticommute) of left and right sector contributions indicating that two states differing in a left-right ordering must be counted only once. The second term is the Klein bottle contribution and takes into account states that are exactly the same in both sectors. In such case, the operator $e^{2i\pi\tau L_0} e^{-2i\pi\bar{\tau} L_0}$, when acting on the same states, becomes $e^{2i\pi 2it_K L_0}$ with $\tau - \bar{\tau} = 2it_K$ and thus

$$\mathcal{Z}_K(2it_K) = \frac{1}{2} \sum_a \mathcal{K}^a \chi_a(2it_K), \quad (2.3)$$

where $|\mathcal{K}^a| = \mathcal{N}^{aa}$. The Klein bottle amplitude in the *transverse channel* is obtained by performing an \mathbf{S} modular transformation such that

$$\tilde{\mathcal{Z}}_K(il) = \frac{1}{2} \sum_a O_a^2 \chi_a(il) \quad (2.4)$$

with $l = \frac{1}{2it_K}$ and

$$(O^a)^2 = 2^D \mathcal{K}^b S_{ba} \quad (2.5)$$

This notation for the closed channel coefficients highlights the fact that the Klein bottle transverse channel represents a closed string propagating between two crosscaps (orientifold planes) states. Namely, a quantum state $|C\rangle$, describing the crosscap can be found such that the KB amplitude can be expressed as

$$\tilde{\mathcal{Z}}_K(il) = \frac{1}{2} \langle C | q^{\frac{1}{2}H_{cl}} | C \rangle. \quad (2.6)$$

with $H_{cl} = L_0 - \tilde{L}_0 - \frac{c}{12}$.

Indeed, crosscap states can be formally expanded in terms of Ishibashi states [16] such that

$$|C\rangle = O_a|a\rangle\rangle_C \quad (2.7)$$

with

$${}_C\langle\langle b|q^{\frac{1}{2}H_{cl}}|a\rangle\rangle_C = \delta_{a,b}\chi^a(\tilde{q}), \quad (2.8)$$

and $\tilde{q} = e^{2i\pi l}$.

When integrated over the tube length, such amplitude leads, for massless states, to tadpole like divergences. In particular, for RR massless states, such tadpoles must be cancelled for the theory to be consistent. Notice that, for such fields, O_a represents the charge of the orientifold plane (crosscap) under them and, therefore, inclusion of an open string sector with D-branes carrying $-O_a$ RR charge provides a way for having a consistent theory [17] with net vanishing charge.

Therefore, we introduce stacks of boundary states $|\alpha\rangle$ (referred to as “brane- α ”)

$$|\alpha\rangle = \sum_a D_\alpha^a |a\rangle\rangle_B \quad (2.9)$$

such that the amplitude, describing propagation of strings between “intersecting” stacks α and β can be written as

$$\tilde{\mathcal{Z}}_{\beta,\alpha}(il) = \langle\beta|q^{\frac{1}{2}H_{cl}}|\alpha\rangle = \sum_a D_\alpha^a D_\beta^a \chi^a(l) = \sum_b C_{\beta,\alpha}^b \chi^b(l/2) \quad (2.10)$$

where in the last step we have performed an S modular transformation to direct channel.

Here

$$C_{\beta,\alpha}^b = \sum_a D_\beta^a D_\alpha^a S_{ab} \quad (2.11)$$

is the multiplicity of open string states contained in χ_a . Namely, it counts open string sector states of the form

$$|\Phi_a; \beta, \alpha\rangle \quad (2.12)$$

where Φ_a is a world sheet conformal field and α, β label the type of “branes” where the string endpoints must be attached to. $C_{\beta,\alpha}^a$ are positive integers (actually $C_{\beta,\alpha}^a = 0, 1, 2$ [3]) generated when the trace over open states $|\Phi_a; \beta, \alpha\rangle$ is computed.

The full cylinder partition function is obtained when summing over all possible stacks of n_α branes, namely

$$\tilde{\mathcal{Z}}_C(il) = \sum_a D_a^2 \chi^a(l) \quad (2.13)$$

with $D_a = \sum_\alpha n_\alpha D_\alpha^a$.

In a similar manner, strings propagating between branes and orbifold planes give rise to Möbius strip amplitude

$$\tilde{\mathcal{Z}}_M(il) = 2D_a O_a \hat{\chi}^a(l + \frac{1}{2}) \quad (2.14)$$

By modular transforming to direct channel we obtain multiplicities of open string states between a brane and its orientifold image

$$O_a(n_\alpha D_\alpha^b) P_{ba} = \mathcal{M}_a = M_\alpha^b n_\alpha \quad (2.15)$$

where we have used the fact that characters in the direct and transverse channels of the Möbius strip are related by the transformation [18] $P: it_M + \frac{1}{2} \rightarrow \frac{i}{4t_M} + \frac{1}{2}$ generated from the modular transformations S and T as $P = TST^2S$.

For indices a representing massless RR fields D_a is the D-brane RR charge. Therefore zero net RR charge requires the

$$O_a + n_\alpha D_\alpha^a = 0 \quad (2.16)$$

tadpole cancellation equations.

3. Orientifolds of $D = 6$ Gepner models

In Gepner models [8], in D space time dimensions, the internal sector is given by a tensor product of r copies of $N = 2$ superconformal minimal models with levels k_j , $j = 1, \dots, r$ and central charge

$$c_j = \frac{3k_j}{k_j + 2}, \quad k_j = 1, 2, \dots \quad (3.1)$$

such that internal central charge $c_{int} = \sum_{j=1}^r c_j^{int} = 12 - 3(D - 2)/2$.

Unitary representations of $N = 2$ minimal models are encoded in primary fields labelled by three integers (l, m, s) such that $l = 0, 1, \dots, k$; $l + m + s = 0 \pmod{2}$. These fields belong to the NS or R sector when $l + m$ is even or odd respectively. The primary field information of the complete theory can be conveniently collected in the vectors λ and μ [7]. Thus, the index a in the previous section amounts here for $a = (\lambda, \mu)$ in Gepner's case. Spacetime supersymmetry and modular invariance are implemented by keeping in the spectrum only states for which the total $U(1)$ charge is an odd integer.

In six dimensions $c_{int} = 6$ and 16 different possible Gepner models exist, which are associated to $K3$ surfaces [10, 19] i.e. $\bar{k} = (1, 1, 1, 1, 1, 1)$, $\bar{k} = (2, 2, 2, 2, 2)$, $\bar{k} = (0, 2, 6, 6)$, etc.

Notice that, in some cases, $k = 0$ blocks have been added. Even if such terms are irrelevant in a closed string theory (for instance the central charge remains invariant), they have been shown to have a non trivial (K-theory) effect when open string sector is included. In fact, an even (odd) number of internal minimal blocks is required (see for instance [4, 14]) in $D = 6$ ($D = 4$) for consistency. Actually, for the sake of simplicity we will consider the case where the internal sector is a tensor product of $r = 6$ conformal blocks. This will allow us to simultaneously consider cases with 3, 4, 5, and 6 conformal blocks such as $(6)^2(2)$, $(2)^4$, 1^4 or $(1)^6$ by adding, if necessary, conformal blocks with level $k = 0$.

The Klein bottle amplitude is determined from that of the torus up to signs representing different ways of “dressing” the world-sheet parity Ω [6, 2]). We denote the dressed orientifold operator as $\Omega_{\Delta, \omega_j}^B$.

From the Klein bottle amplitude in the transverse channel we can read the expression for the crosscap state up to signs that can be fixed from the Möbius strip amplitude. The result is that the crosscap state is given by¹

¹For explicit expressions for modular matrices $P_{\ell, l}$ and $S_{\ell, l}$ see [6, 2].

$$\begin{aligned}
|C\rangle_B^{NS} &= \frac{1}{\kappa_c} \sum_{\lambda', \mu'}^{ev} \sum_{v_0=0}^{\frac{K}{2}-1} \sum_{v_j, \tilde{v}_1=0}^1 \sum_{\varepsilon_j=0}^1 \left(\prod_{k<l} (-1)^{v_k v_l} \right) (-1)^{v_0} (-1)^{\sum_j v_j} \\
&(-1)^{\sum \omega_j \Lambda'_0 / 2} e^{2\pi i v_0 \sum \frac{\Delta_j}{k_j+2}} \delta_{\Lambda'_0, 2+2v_0+2\tilde{v}_1+2\sum v_j+2\sum \omega_j}^{(4)} \delta_{\Lambda'_1, 2v_0+2\tilde{v}_1}^{(4)} \\
&\prod_{j=1}^r \left(\sigma_{(l'_j, m'_j, s'_j)} \frac{P_{l'_j, \varepsilon_j k_j}}{\sqrt{S_{l'_j, 0}}} \delta_{m'_j, 2v_0+(1-\varepsilon_j+\omega_j)(k_j+2)}^{(2k_j+4)} \delta_{s'_j, 2v_0+2v_j+2(1-\varepsilon_j)}^{(4)} (-1)^{\varepsilon_j \frac{(m'_j+s'_j)}{2}} \right) |\lambda', \mu'\rangle_c |
\end{aligned} \tag{3.2}$$

The normalization is chosen so that the overlap of the crosscap with itself yields the transverse Klein amplitude 2.6. Here

$$\Delta = H \sum_{i=1}^r \frac{\Delta_i}{k_i+2}, \quad \omega = \sum_{i=1}^r \omega_i \tag{3.3}$$

with $H = \text{lcm}\{k_i+2\}$, $K = \text{lcm}(4, 2k_j+4)$ and Δ, ω_i denote the ‘‘dressings’’.

In order to cancel tadpole-like divergences, boundary states must be introduced. We consider the B-type RS-boundary states [20]

$$\begin{aligned}
|\alpha\rangle_B &= |S_0, \tilde{S}_1; (L_j, M_j, S_j)_{j=1}^r\rangle_B = \frac{1}{\kappa_B} \sum_{\lambda', \mu'}^{\beta, b} (-1)^{\frac{\Lambda_0^2}{2}} e^{-i\pi \Lambda'_0 \frac{S_0}{2}} e^{-i\pi \Lambda'_1 \frac{S_1}{2}} \\
&\prod_{j=1}^r \frac{S_{l'_j, L_j}}{\sqrt{S_{l'_j, 0}}} e^{i\pi \frac{m'_j M_j}{k_j+2}} e^{-i\pi \frac{s'_j S_j}{2}} |\lambda', \mu'\rangle
\end{aligned} \tag{3.4}$$

where ‘‘b’’ in the summatory indicates that $m_j = b \pmod{k_j+2}$. In fact, due to supersymmetry and field identifications these B-type boundary states only depend on $\mathbf{L} = (L_1, \dots, L_r)$ with $L_i \leq k_i/2$, $M = H \sum \frac{M_i}{k_i+2}$ and $S = \sum S_i$. However, whenever a label L_i reaches $k_i/2$, extra copies of the gauge field may appear propagating on the brane world-volume. In this case, it is necessary to resolve the branes into elementary branes such that only a single gauge field is propagating on the world-volume. The details depend on the values of $|\mathbf{S}|$ counting the number of i such that $L_i = k_i/2$. It can be shown [4] that when $|\mathbf{S}|$ is an odd integer the elementary branes are given by

$$|\alpha_{ele}\rangle_B = \frac{1}{2^{\frac{|\mathbf{S}|-1}{2}}} |\alpha\rangle_B \tag{3.5}$$

Instead, if $|\mathbf{S}|$ is even there is an extra \mathbf{Z}_2 -valued label ψ taking values \pm so that the elementary boundary states are now labelled by $\mathbf{L}, \mathbf{M}, \mathbf{S}, \psi$. The original boundary states can be written in terms of the elementary ones as

$$|\alpha\rangle_B = \frac{1}{2^{|\mathbf{S}|}} \{ |\alpha+\rangle_B + |\alpha-\rangle_B \} \tag{3.6}$$

where α stands for all labels different from ψ . The two boundary states $|\alpha\pm\rangle_B$ contain new states from the twisted (c, c) RR sector. Explicit expressions for such states are given in [7].

The massless fields in the 6D spacetime theory are the vector field $(2, 0)(0, 0, 0)^6$ and the hypermultiplets $(0, 0) \prod_j (l_j, l_j, 0)$ with $\sum_j \frac{l_j}{k_j+2} = 1$. They are contained in the cylinder and Möbius

amplitude which we present next. The bosonic and massless part of the cylinder amplitude between two D-branes $|\mathbf{L}, M\rangle$ and $|\mathbf{L}', M'\rangle$ is generally given by

$$\frac{1}{2} \frac{1}{N_{\mathbf{S}}} \frac{1}{N_{\mathbf{S}'}} \sum_{\lambda, \mu} \sum_{\varepsilon_1, \dots, \varepsilon_r=0}^{sr} \left(\prod_{i=1}^r N_{L_j, L'_j}^{|\varepsilon_j k_j - l_j|} \right) \delta_{\sum_i \varepsilon_i = 1 + \frac{s_0}{2}}^{(2)} \chi_{\mu}^{\lambda} \quad (3.7)$$

where N_{L_1, L_2}^l are the $SU(2)$ fusion coefficients ([21, 22]) and

$$N_{\mathbf{S}} = \begin{cases} 2^{|\mathbf{S}|/2} & \text{if } |\mathbf{S}| \text{ even} \\ 2^{[|\mathbf{S}|-1]/2} & \text{if } |\mathbf{S}| \text{ odd} \end{cases}$$

eliminates any extra counting when some of the D-branes are short. Besides, we have defined an extra label $s_0 = \Lambda_0 + \Lambda_1 \pmod{4}$ (see Appendix A) taking the values 0, 2 whenever the fields are in the scalar and vector representations, respectively (Note that in six dimensional spacetime $s_0 = 0, 2$ also for the spinor representations, so this definition strictly makes sense when we restrict ourselves to bosonic representations). When the amplitude between two short-orbit branes $|\mathbf{L}, \psi\rangle$ and $|\mathbf{L}', \psi'\rangle$ such that $\mathbf{S} = \mathbf{S}'$ and $|\mathbf{S}| \in 2\mathbf{Z}^+$ is considered, an additional projection must be taken into account, due to the ψ labels, leading to

$$\frac{1}{2^{|\mathbf{S}|}} \sum_{\lambda, \mu} \sum_{\varepsilon_1, \dots, \varepsilon_r=0}^{sr} \left(\prod_{i=1}^r N_{L_j, L'_j}^{|\varepsilon_j k_j - l_j|} \right) \delta_{\sum_{i \in \mathbf{S}} \varepsilon_i = \frac{1}{2}(1 - \psi \psi')}^{(2)} \delta_{\sum_i \varepsilon_i = 1 + \frac{s_0}{2}}^{(2)} \chi_{\mu}^{\lambda} \quad (3.8)$$

On the other hand, the massless states in the bosonic Möbius strip amplitude are given by

$$-\frac{1}{2N_{\mathbf{S}}} \prod_{k < l} (-1)^{\rho_k \rho_l} \delta_{\sum \rho_j + 1 + \frac{s_0}{2}, \sum \omega_j}^{(2)} (-1)^{\sum \omega_j \frac{s_0}{2}} e^{i \frac{\pi}{2} \sum \omega_j (m_j - 2L_j + \varepsilon_j (k_j + 2))} (-1)^{\varepsilon_j} N_{L_j, L'_j}^{|\varepsilon_j k_j - l_j|} \chi_{\mu}^{\lambda} \quad (3.9)$$

where $\rho_j = \frac{s_0}{2} + 1 + \varepsilon_j + \sum \omega_j$.

In particular, we see the vector ($s_0 = 2$) has the sign

$$-\frac{1}{N_{\mathbf{S}}} (-1)^{\sum \omega_j} (-1)^{\sum \omega_j L_j} \sum_{\varepsilon_1, \dots, \varepsilon_r=0}^1 \prod_{k < l} (-1)^{\varepsilon_k \varepsilon_l} \prod (-1)^{\omega_j \varepsilon_j \frac{k_j + 2}{2}} \delta_{\varepsilon_j L_j, \varepsilon_j \frac{k_j}{2}} \delta_{\sum \varepsilon_j, \sum \omega_j} \quad (3.10)$$

A plus (minus) sign indicates a symplectic (orthogonal) gauge group while a zero leads to a unitary gauge group. In a similar manner, the gauge group representations in which matter states transform, can be identified (an example is given in next section).

The tadpole cancellation conditions can be easily read from the expressions for the crosscap (3.2) and boundary states (3.4). They take the general form $\text{Tad}_D(\lambda, \mu) - 8\text{Tad}_O(\lambda, \mu) = 0$. For the massless fields $(2, 0)(0, 0, 0)^6$ and $(0, 0) \prod_j (l_j, l_j, 0)$ the NS-NS tadpoles of the orientifold plane read

$$\text{Tad}_O(\lambda, \mu)_B = \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\varepsilon_1, \dots, \varepsilon_r=0}^1 \left(\prod_{k < l} (-1)^{\varepsilon_k \varepsilon_l} \right) (-1)^{\nu_0 \sum \varepsilon_j} \delta_{s_0/2, 1 + \sum \varepsilon_j + \sum \omega_j}^{(2)} (-1)^{\sum \omega_j (s_0/2)} (-1)^{\Delta_j (1 - \varepsilon_j)} \prod_{j=1}^r \left(\sin \left[\frac{1}{2} (l_j, \varepsilon_j k_j) \right] \delta_{l_j + (1 - \varepsilon_j) k_j, 0}^{(2)} \delta_{m'_j, 2\nu_0 + (1 - \varepsilon_j + \omega_j)(k_j + 2)}^{(2k_j + 4)} (-1)^{\varepsilon_j \frac{m_j}{2}} \right). \quad (3.11)$$

Also, collecting all terms from the boundary states and their $\Omega_{\Delta_j, \omega, \omega_\alpha}$ images, we obtain, for their massless tadpoles

$$\text{Tad}_D(\lambda, \mu) = \sum_{a=1}^N \frac{N_a}{N_S} \cos \left[\pi \sum_j \frac{m_j(M_j^a - \Delta_j)}{k_j + 2} \right] \prod_j \sin(l_j, L_j^a). \quad (3.12)$$

These expressions are valid up to common phases. We have also renormalized the tadpole equations by introducing the factor N_S so that the Chan-Paton factors N_a truly represent the multiplicity of elementary D-branes.

3.1 $(6)^2(2)(0^3)$ model

We exemplify the construction presented in the preceding section for the specific Gepner model $(6)^2(2)(0^3)$. We will later consider this example to discuss model building in four dimensions.

The gauge groups and massless spectrum is obtained from Eqs. (3.8) and (3.9). For instance, we see that brane $\mathbf{L}_{10} = 3310^3$ is short, with $|\mathbf{S}| = 6$. Thus, for the vector ($s_0 = 2$), a non vanishing contribution in (3.8) implies $\delta_{\sum_i \varepsilon_i \frac{1}{2}(1-\psi\psi')}^{(2)} \delta_{\sum_i \varepsilon_i = 0}^{(2)} \neq 0$, namely $\psi = \psi'$. Moreover, for such choice of ε'_i 's we see that (3.9) vanishes thus leading to the unitary group. In a similar way, for the scalars ($s_0 = 0$) the states propagating between a boundary state and its orientifold image are selected, $\psi = -\psi'$. Möbius amplitude (3.9) is non vanishing in this case and produces a minus sign thus leading to antisymmetric representations.

The tadpole equations (3.11,3.12) for this set of branes reads

$$\begin{aligned} N_2 + 2N_3 + N_4 + N_5 + 2N_6 + N_7 + 2N_8 + N_9 + 2N_{10} + N_{11} + 3N_{12} &= 16 \\ N_1 + 2N_2 + 2N_3 + 2N_4 + N_5 + 3N_6 + 2N_8 + 2N_9 + 2N_{10} + 2N_{11} + 4N_{12} &= 24 \end{aligned} \quad (3.13)$$

States propagating between branes can be easily computed from (3.7) and (3.8). Two tensor multiplets are found in the internal sector (see for instance [3]). It can be checked that all gauge and gravitational anomalies cancel.

At this point it may be instructive and useful for our subsequent calculations to illustrate this in a detailed example containing only $(\mathbf{L}_1, \mathbf{L}_6, \mathbf{L}_{10})$ states.

The tadpole equations for the reduced set of D-branes lead to

$$N_{10} = 8 - N_6 = N_1 \quad (3.14)$$

The gauge group is $Sp(N_1) \times Sp(N_6) \times U(N_{10})$ with matter hypermultiplets in

$$\begin{aligned} &3[(1, 1, \square) + (1, 1, \bar{\square})] + 7(1, \square, 1) + (1, \square, 1) \\ &+ (N_1, N_6, 1) + (N_1, 1, N_{10}) + (N_1, 1, \bar{N}_{10}) + 3[(1, N_6, N_{10}) + (1, N_6, \bar{N}_{10})] \end{aligned} \quad (3.15)$$

It is easy to check that this spectrum (plus a closed sector containing two tensor multiplets and nineteen hypermultiplets) leads to vanishing of gauge and gravitational anomalies if tadpole equations (3.14) are satisfied.

4. Orbifolding $(\text{Gepner model})^{c=6} \times \mathbf{T}^2$

Orbifolds of Gepner models are also easily implemented in the language of boundary and crosscap states. The internal sector described by $(\text{Gepner model})^{c=6} \times \mathbf{T}^2$ has a discrete symmetry acting on fields in the following way

$$g : Z \rightarrow e^{2\pi i v} Z \quad g : \Phi_{m_i s_i}^i \rightarrow e^{2\pi i \frac{m_i \gamma_i}{k_i + 2}} \Phi_{m_i s_i}^i \quad i = 1, \dots, r \quad (4.1)$$

where $Z = X^4 + iX^5$ denotes the complex coordinate on \mathbf{T}^2 and $(v; \gamma_i)$ are labels for the generator $\hat{g} \in G$. For a torus with symmetry \mathbf{Z}_N we have $Nv \in \mathbf{Z}$. The labels $(v; \gamma_i)$ are conveniently encoded in terms of a simple current vector j

$$j = (0, v; 2\gamma_1, \dots, 2\gamma_r; 0, \dots, 0). \quad (4.2)$$

which satisfies $2\beta_0 \bullet j \in \mathbf{Z}$ or in components

$$-\frac{v}{2} + \sum \frac{\gamma_i}{k_i + 2} = 0 \quad \text{mod } \mathbf{Z}. \quad (4.3)$$

As it is well known, twisted sectors must be included in order to ensure the modular invariance of the torus partition function. As a consequence, new tadpoles are expected to appear, in the transverse channel, due to RR fields propagating in the twisted sectors. Thus, the boundary states required to cancel the tadpoles include the RR fields in the twisted sector of the theory.

Interestingly enough, it is possible to rewrite the projection by simple currents in such a way that its relation to the usual orbifolds of toroidal manifolds is much more evident. To see this we recall that open string states formally read

$$|\Phi_k; i, j\rangle \lambda_{ji}^k \quad (4.4)$$

where λ^k encodes the gauge group representation into which the state Φ_k transforms. For instance, if the state Φ_0 corresponds to gauge bosons, λ^0 represents gauge group G generators².

Let us assume that such Chan-Paton factors have been determined already and that we further act on string states with a generator θ of a Z_N symmetry group. Such action which manifests as a phase $e^{2\pi i \frac{\gamma_i m_i}{k_i + 2}}$ on world sheet field Φ_k should, in principle, be accompanied by corresponding representation of group action such that

$$\hat{\theta} |\Phi_k; i, j\rangle \lambda_{ji} = \gamma_{i'j'} |\hat{\theta} \Phi_k; i', j'\rangle \gamma_{j'i} \lambda_{ji} = e^{2\pi i \frac{\gamma_i m_i}{k_i + 2}} (\gamma^{-1} \lambda \gamma)_{j'i'} |\Phi_k; i', j'\rangle$$

Therefore, invariance under such action requires

$$e^{2\pi i \frac{\gamma_i m_i}{k_i + 2}} \gamma^{-1} \lambda^k \gamma = \lambda^k \quad (4.5)$$

By following the same steps as in Ref. [23], we can represent Z_N Chan-Paton twist in terms of Cartan generators as $\gamma = e^{2\pi i V H}$ where V is a "shift" eigenvalues vector of the generic form

$$V = \frac{1}{N} (0^{N_0}, 1^{N_1}, \dots, (N-1)^{N_{N-1}}) \quad (4.6)$$

²Which generically will be a product of unitary, orthogonal and symplectic groups.

(ensuring $\gamma^N = 1$) and Cartan generators are represented by 2×2 σ_3 submatrices.

On this basis, projection equation (4.5) reduces to the simple condition

$$\rho_k V = \frac{\gamma_i m_i}{k_i + 2} + \frac{v s_{-1}}{2} \quad (4.7)$$

where ρ_k is the weight vector associated to the corresponding λ^k representation.

Recall that in the orientifold theory we must introduce a boundary state and its image under Ω .

Tadpole conditions can be generalized for orbifolded hybrid models $T^2 \times \text{Gepner}$ in the following way [6]

$$D_\mu^\lambda \left(\text{Tr } \gamma_{N,2x} + \sqrt{f} \text{Tr } \gamma_{D,2x,I} \right) + O_\mu^\lambda \cos \pi x v = 0 \quad (4.8)$$

$$D_\mu^\lambda \left(\text{Tr } \gamma_{N,2x+1} + \sqrt{f} \text{Tr } \gamma_{D,2x+1,I} \right) = 0 \quad (4.9)$$

for all states (λ, μ) such that $\chi_{\mu+2x\Gamma}^\lambda$ is massless.

Here O_μ^λ is the orientifold charge we have in six dimensions for the state (λ, μ) while the factor $f = 4 \sin^2 \pi x v$ is a non-trivial contribution from the fixed points in the complex torus T^2 in the NN sector. The labels N and D are used to distinguish between D-branes with Neumann and Dirichlet boundary conditions in the torus T^2 .

5. A MSSM example

As an illustration of the general techniques discussed above we concentrate here on a \mathbf{Z}_4 modding of the $[(6)^2(2)(0)]^{c=6} \times T^2$ model³. Let us notice that inspection of allowed internal states indicates that only 3 massless chiral ($l_i = m_i, s_i = 0$) states, namely those such $(m_1, m_2, m_3, m_4) = (3, 3, 1, 0), (5, 1, 1, 0), (1, 5, 1, 0)$, do propagate between brane \mathbf{L}_{10} (with a $U(N_{10})$ gauge group living on its worldvolume) and \mathbf{L}_6 (with an $Sp(N_6)$ gauge group). Therefore, an internal modding of the form $\Gamma = (0, 0, 1, 0)$ acting on the Gepner model will allow such states to remain in the spectrum and, by appropriately embedding it as a twist γ^{l_0}, γ^6 on the D-brane sector, the original $U(N_{10}) \times Sp(N_6)$ gauge group could be broken into a Standard-like model with 3 generations. Moreover, in order to have $\mathcal{N} = 1$ supersymmetry in four dimensions, we must accompany this modding with a \mathbf{Z}_2 modding on T^2 , namely $v_3 = 1/2$, so as to satisfy Eq.(4.3). Thus, our starting point is

$$\Gamma = (0, 0, 1, 0) \left(\frac{1}{2} \right) \otimes \gamma^a \quad (5.1)$$

Note that the actual internal modding (see (4.1)) is $\gamma_i / (k_i + 2)$ so it represents a \mathbf{Z}_4 action.

As we stressed in the previous section, performing a \mathbf{Z}_2 modding on the torus will require the introduction of a new set of branes having Dirichlet boundary conditions on the open string ends living on T^2 . We quote them with an index D while introducing an index N to label the original branes with Neumann conditions on the third complex coordinate Z . We will refer to them as D_Z and N_Z branes respectively. Besides the untwisted tadpole equations 3.13, several new equations must now be solved (see [7] due to the twisting, according to Eq.(4.8,4.9).

³We will write the internal sector in terms of four theories in what follows.

As mentioned, \mathbf{L}_{10} and \mathbf{L}_6 constitute the basic branes which, after splitting under modding action, will give rise to our model. It is interesting to remark that, both boundary states can be placed on the same NN sector, or either \mathbf{L}_{10} in NN sector and \mathbf{L}_6 in the DD sector (or viceversa) or both in the DD sector. The basic features, discussed below, will be independent of the sector choice. However phenomenological details will be different, mainly due to the extra branes that must be added to satisfy RR tadpole cancellation.

We choose two (minimal) $N_{10} = N_6 = 4$ stacks of \mathbf{L}_6 and \mathbf{L}_{10} branes to start with a $U(4) \times Sp(4)$ gauge group. The modding Γ in (5.1) is embedded as twists γ_6 and γ_{10} , on each respective stack, as

$$\gamma_6 \rightarrow V_6 = \frac{1}{4}(0, 2) \quad (5.2)$$

$$\gamma_{10} \rightarrow V_{10} = \frac{1}{4}(1, 1, 1, 3) \quad (5.3)$$

For the vector $\Gamma \cdot \mu = 0$ and therefore from (4.7) we find

$$U(4) \rightarrow SU(3) \times U(1)^2 \quad (5.4)$$

$$Sp(4) \rightarrow SU(2) \times SU(2)$$

where $Sp(2) \equiv SU(2)$. Thus, a LR symmetric-like model group is obtained.

Moreover, the correct LR spectrum with 3 massless generations is found. Namely, massless chiral states propagating in between $\mathbf{L}_{10} - \mathbf{L}_6$ (these are $(0; 3310; 0)$, $(0; 15100)$, $(0; 5110; 0)$) satisfy $\Gamma \mu = \frac{1}{4}$ and therefore we find the spectrum representations under $SU(3) \times SU(2)_L \times SU(2)_R \times Q_{B-L}$ to be

$$3[(3, 2, 1)_{\frac{1}{3}} + (\bar{3}, 1, 2)_{-\frac{1}{3}} + (1, 1, 2)_1 + (1, 2, 1)_{-1}] \quad (5.5)$$

where the subindex indicates the charge eigenvalue of

$$Q_{B-L} = \frac{1}{3}Q_3 + Q \quad (5.6)$$

Q_3 being the generator of the $U(1)$ in $U(3)$ and Q the other $U(1)$ generator in (5.5).

Actually, it is possible to establish a correspondence with an intersecting brane model picture in toroidal manifolds (see for instance [11]). Namely, under the action of γ_{10} and γ_6 , boundary states \mathbf{L}_{10} and \mathbf{L}_6 intersecting at a six dimensional manifold, split into four stacks of boundary states as

$$\mathbf{L}_{10}[U(4)] \rightarrow \mathbf{L}_{10}^a[U(3)] + \mathbf{L}_{10}^d[U(1)] \quad (5.7)$$

$$\mathbf{L}_6[Sp(4)] \rightarrow \mathbf{L}_6^b[Sp(2)] + \mathbf{L}_6^c[Sp(2)] \quad (5.8)$$

where we have indicated in brackets the gauge group living on the corresponding brane. Thus, boundary states $\mathbf{L}_{10}^a, \mathbf{L}_{10}^d, \mathbf{L}_6^b, \mathbf{L}_6^c$ do match with the basic branes a, b, c, d arising in intersecting brane models on toroidal constructions [11, 24].

Thus, drawing boundary states as lines and interpreting multiplicities as intersection numbers we are lead to a graphic representation as the one given in left side of Figure 1.

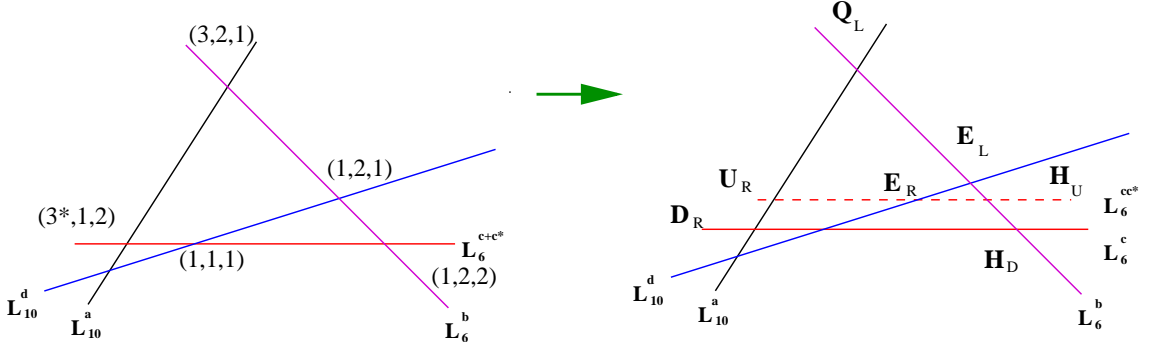


Figure 1: By separating $\mathbf{L}_6^{c+c^*} \rightarrow \mathbf{L}_6^c + \mathbf{L}_6^{c^*}$ away from the orientifold point breaking from LR to MSSM with 3 righthanded neutrini is achieved

Besides states propagating between different branes we must consider states along the same type of branes. They lead to vector like matter.

Interestingly enough, massless states (4020) , (0420) and (2220) do propagate in $\mathbf{L}_6 - \mathbf{L}_6$ sector. They satisfy $\Gamma \cdot \mu = \frac{1}{2}$ and thus, together with $(1)(0000)$, descending from the six dimensional vector, lead to $9(1, 2, 2)_0$ candidates to LR Higgses.

Branes c and its image c^* are placed here on top of each other and on top of an orientifold point (leading to $Sp(2)$). Since such branes are parallel in the torus, following similar steps as discussed [12], we can think into separating them away from the orientifold point in the torus. Thus, $SU(2)_R \rightarrow U(1)_c$ where $U(1)_c$ charges are given by T_R^3 eigenvalues. Therefore, by introducing the weak hypercharge $Y = -T_R^3 + \frac{1}{2}Q_{B-L}$ we find that the original LR symmetric model breaks down to $SU(3) \times SU(2)_L \times U(1)_Y$ MSSM with three chiral generations

$$3[(3, 2, 1)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{2}{3}} + (\bar{3}, 1)_{\frac{1}{3}} + (1, 1)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}} + (1, 1)_0] \quad (5.9)$$

including three right handed neutrini. Moreover, LR chiral states $(1, 2, 2)_0$ decompose into $(1, 2)_{-1/2} + (1, 2)_{1/2}$ with the correct MSSM Higgs charges. It can be checked that, non-anomalous, hypercharge remains massless in the model under consideration [7]. A pictorial representation is presented in Figure 1.

Besides these basic boundary states leading to the MSSM structure, additional stacks of branes must be added in order to satisfy tadpole cancellation equations. Different choices are possible and each of them will give rise to particular phenomenological features. A simple explicit example can be found Ref. [7].

Some general remarks about Yukawa couplings can be advanced. As a general observation notice that a Yukawa coupling will have the form

$$Y_{ijk} \Phi_{ba}^i \Phi_{ac}^j \Phi_{cb}^k \quad (5.10)$$

where Φ_{ab} is the chiral superfield insertion connecting boundaries a and b and i, j, k refers to internal CFT labels. Such a term should be a singlet of the gauge group and invariant under Γ modding. Moreover, it must be allowed by the fusion rules of the internal conformal field theory [25, 21]. Namely, $Y_{ijk} \propto \langle ijk \rangle \propto \mathcal{N}_{ij}^k$

For instance, couplings like

$$[(\underline{5110})(3,2,1)_{1/3}^{ab} \times [(3310)](\bar{3},1,2)_{-1/3}^{ac} [(2220)](1,2,2)_0^{bc}] \quad (5.11)$$

(where we have indicated the internal charges in brackets) are non vanishing and lead to degenerate masses for two quark generations. Fusion rules forbid masses for the first quark generation. A similar result is obtained for lepton masses since the same internal states are involved for leptonic Yukawa couplings. The general pattern is very similar (the number of Higgses is different) to the one found in Ref. [26] in the context of branes at singularities.

Actually, (see i.e. [26], the full picture of mass structures becomes more complicated due, for instance, to the presence of Yukawa couplings of quarks with colored triplets present at other intersections. Notice also that, from the 9 candidates to be interpreted as Higgs particles coming from $\mathbf{L}_6 - \mathbf{L}_6$ sector, only those with CFT quantum numbers (2220) are allowed in Yukawa couplings. For all of them, on the other hand, mass term like couplings are allowed. Thus, we can imagine a scenario where some of the $(1,2,2)$ multiplets become very massive.

In the example discussed above the basic branes \mathbf{L}_{10} , containing strong group, and \mathbf{L}_6 , where $SU(2)_L \times SU(2)_R$ lives, were placed in the same NN sector. However, other schemes, which might be useful future phenomenological applications, are also possible. For instance, consistent models where part of the spectrum containing the $SU(2)_L \times SU(2)_R$ gauge theory is placed on the branes in the DD sector can be built.

An interesting feature of the hybrid construction is that lowering of the string scale [27] could be achieved by considering large extra dimensions in the T^2 torus, transverse to the whole configuration of intersecting boundary states.

Indeed, in the present examples of the type $Gepner \times T^2/\mathbf{Z}_N$, boundary states would correspond to branes wrapping cycles on K_3 and stuck at a \mathbf{C}/\mathbf{Z}_N singularity. Thus, if we denote by V_4 the volume of the Gepner piece, which should be of the order of the string scale $V_4 \propto 1/M_s^4$, and by V_2 that of two dimensional manifold. Then we expect the Planck scale, after dimensional reduction to four dimensions, to be

$$M_{Planck} = \frac{2}{\lambda} M_s^4 \sqrt{V_4 V_2} \quad (5.12)$$

where λ is the string coupling. Therefore the string scale M_s can be lowered by choosing the volume V_2 ($V_2 = \frac{M_{Planck}^2 \lambda^2}{4M_s^4}$) sufficiently large. Recall that the models constructed here are fully supersymmetric and though lowering the scale could be phenomenologically attractive in some cases it is not as compelling as in non supersymmetric models.

Presumably, having these large extra dimension could allow for the introduction of dilute fluxes in a supergravity limit of some of these hybrid construction [28].

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