PROCEEDINGS OF SCIENCE

Non-Abelian Asymmetry

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Given a symmetry: before looking for its possible extensions one should first search for its corresponding asymmetry. This means to find out the opportunities that such symmetry offers. Then our effort here is to study an asymmetry possibility by introducing different potential fields rotating under a same single group.

In this work asymmetric classical properties are studied for the non-abelian case. The presence of asymmetry is first derived through new strength field tensors and collective aspects. Then field equations, Bianchi identities, Noether theorem and conserved currents are obtained.

Diversity and connectivity are the main results from asymmetry. Diversity can be seen through the various quanta carrying different masses, spin and coupling constants obtained through equations of motion and their corresponding conserved currents. Connectivity through new inductive relationships derived from new Bianchi identities and coupled equations. It is a gauge theory that emphasizes the meaning of entanglement.

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1. Introduction

Symmetry alone does not solve physics. In order to do physics one needs asymmetry. Thus the concept of asymmetry has always being walking on the side of physics. It goes beyond the symmetry perfection and understands diversity. The spontaneous breaking symmetry case is its most well known example [1]. However there are another situations to be understood.

By asymmetry one defines the opportunities that a given symmetry can offer. A possibility is to consider the introduction of a set of fields transforming under a common gauge group as

$$A_{\mu I}(x) \longrightarrow A'_{\mu I}(x) = UA_{\mu I}U^{-1} + \frac{i}{g}\partial_\mu U \cdot U^{-1} \tag{1.1}$$

where $I = 1,...,N$. Eq. 1.1 can be assumed as a established mathematical expression. Different origins based on Kaluza-Klein, supersymmetry, fibre bundle, σ-model have already been studied and proved its existence [2].

Thus one understands that by asymmetry does not mean symmetry violation but the possibility of including differences inside symmetry. For showing more explicitly this asymmetry presence one should rewrite the field basis. According to Borscher's theorem [3], physics must be independent under fields reparametrizations, and so, one can rewrite the basis $\{A_{\mu I}\}$ in terms of $\{D_\mu, X_{\mu i}\}$ defined as

$$\begin{aligned} D_\mu &= \sum_I A_{\mu I} \\ X_{\mu i} &= A_{\mu(I+1)} - A_{\mu I} \end{aligned} \tag{1.2}$$

$\{D, X_i\}$ is called the constructor basis due to the fact that, under this field-referential, the gauge invariance origin is more immediate where D_μ is the genuine gauge field, while $X_{\mu i}$ correspond to the opportunities that symmetry offers.

Thus the field D_μ works as the usual gauge field and the fields X_μ^i as a kind of vector-matter fields transforming in the adjoint representation. It gives

$$D_\mu \longrightarrow D'_\mu = UD_\mu U^{-1} + \frac{i}{g}\left(\partial_\mu U\right)U^{-1} \tag{1.3}$$

$$X_\mu^i \longrightarrow X_\mu'^{\,i} = U X_\mu^i U^{-1} \tag{1.4}$$

where X_μ^i fields express the meaning of asymmetry. The indice i varying from 2 to N is showing its diversity richness [4]. Geometrically, the potential fields X_μ^i comes from the torsion tensor of the higher-dimensional manifold that spontaneously compactify to M^4xB^4,where $B^4$is the Minkowski space-time and B^k some k-dimensional internal space. Thus the origin of the potential fields can be treated back to the vielbein, spin-connection and potential fields of higher-dimensional gravity-matter coupled theory spontaneously compactified for an internal space with torsion [2].

2. Lagrangian

The general expression for this non-abelian asymmetric Lagrangian is

$$\mathscr{L}(D,X_i) = \mathscr{L}_A + \mathscr{L}_S + \mathscr{L}_{GF} - \frac{1}{2} m_{ij}^2 X_\mu^i, X^{\mu j} \tag{2.1}$$

$$\begin{aligned}\mathscr{L}_A &= \lambda_1 Z_{[\mu\nu]} Z^{[\mu\nu]} + \lambda_2 z_{[\mu\nu]} z^{[\mu\nu]} + \lambda_3 Z_{[\mu\nu]} z^{[\mu\nu]} \\ \mathscr{L}_S &= \xi_1 Z_{(\mu\nu)} Z^{(\mu\nu)} + \xi_2 z_{(\mu\nu)} z^{(\mu\nu)} + \xi_3 Z_{(\mu\nu)} z^{(\mu\nu)} \\ \mathscr{L}_{GF} &= -\frac{1}{\alpha}\left[\partial_\mu\left(D^\mu + p_i X^{\mu i}\right)\right]^2\end{aligned}$$

where the anti-symmetric field strength is

$$Z_{[\mu\nu]} = d D_{\mu\nu} + \alpha_i X_{[\mu\nu]}^i \tag{2.2}$$

with

$$\begin{aligned}D_{\mu\nu} &= \partial_\mu D_\nu - \partial_\nu D_\mu + ig[D_\mu, D_\nu] \\ X_{[\mu\nu]}^i &= \partial_\mu X_\nu^i - \partial_\nu X_\mu^i + ig([D_\mu, X_\nu^i] - [D_\nu, X_\mu^i])\end{aligned}$$

and the symmetric field strength is

$$Z_{(\mu\nu)} = \beta_i X_{(\mu\nu)}^i + \rho_i g_{\mu\nu} X_\alpha^{\alpha i} \tag{2.3}$$

where

$$X_{(\mu\nu)}^i = \partial_\mu X_\nu^i + \partial_\nu X_\mu^i + ig([D_\mu, X_\nu^i] + [D_\nu, X_\mu^i]) \tag{2.4}$$

Another type of field strength is $z_{\mu\nu}$. It is a collective field which does not depend on derivatives

$$z_{\mu\nu} = z_{[\mu\nu]} + z_{(\mu\nu)} \tag{2.5}$$

where

$$z_{[\mu\nu]} = a_{(ij)}[X_\mu^i, X_\nu^j] + b_{[ij]}\{X_\mu^i, X_\nu^j\} + \gamma_{[ij]} X_\mu^i X_\nu^j \tag{2.6}$$

and

$$z_{(\mu\nu)} = a_{[ij]}[X_\mu^i, X_\nu^j] + u_{[ij]} g_{\mu\nu}[X_\alpha^i, X^{\alpha j}] + b_{(ij)}\{X_\mu^i, X_\nu^j\} + v_{(ij)} g_{\mu\nu}\{X_\alpha^i, X^{\alpha j}\} \tag{2.7}$$

It is understood the notation $A_\mu \equiv A_\mu^a\, t_a$, where t_a are the matrices which satisfy the Lie algebra for SU(N). Observe that $Z_{\mu\nu}$ is not Lie algebra valued as it is $F_{\mu\nu}$ in the usual QCD. However in order to explore the abundance of gauge scalars that such extended model offers one should also consider other group-valued structures in the non-irreducible sector contribution. Notice that coefficients d, α_i, β_i, ρ_i, m_{ij}, $a_{(ij)}$ and so on are identified as the free coefficients of the theory because they can take any value without the symmetry be broken.

3. Classical Equations

The equation of motion associated to D_μ field is

$$
\begin{gathered}
4\lambda_1\left(d\,\partial_\nu Z^{[\mu\nu]}\,t_a + ig\left(d\,D_\nu^b + \alpha_i X_\nu^{ib}\right) Z^{[\mu\nu]}\,[t_a,t_b]\right) + \\
+4\xi_1\, ig\left(\beta_i X_\nu^{ib} Z^{(\mu\nu)} + \rho_i X^{\mu ib} Z_{(\nu}{}^{\nu)}\right)[t_a,t_b] + \\
+2\lambda_3\left(d\,\partial_\nu z^{[\mu\nu]}\,t_a + i\frac{g}{N}\left(d\,D_\nu^b + \alpha_i X_\nu^{ib}\right) z^{[\mu\nu]}\,[t_a,t_b]\right) + \\
+2\xi_3\, ig\left(\beta_i X_\nu^{ib} z^{(\mu\nu)} + \rho_i X^{\mu ib} z_{(\nu}{}^{\nu)}\right)[t_a,t_b] + \frac{1}{\alpha} g^{\mu\nu}\left(2\,\partial_\alpha D^\alpha + p_i\,\partial_\alpha X^{\alpha i}\right) = 0
\end{gathered}
\tag{3.1}
$$

Considering the asymmetry presence, one also derives the following $(N-1)$ equations of motion for the $X_{\mu i}$ fields:

$$
\begin{gathered}
4\lambda_1\,\alpha_i\left(\partial_\nu Z^{[\mu\nu]}\,t_a + ig D_\nu^b Z^{[\mu\nu]}\,[t_a,t_b]\right) + \\
-4\xi_1\left(\beta_i\,\partial_\nu Z^{(\mu\nu)}\,t_a + \rho_i\,\partial^\mu Z_{(\nu}{}^{\nu)}\,t_a + ig\left(\beta_i D_\nu^b Z^{(\mu\nu)} + \rho_i D^{\mu b} Z_{(\nu}{}^{\nu)}\right)[t_a,t_b]\right) + \\
+2\lambda_2\left(2\,a_{(ij)} X_\nu^{jb} z^{[\mu\nu]}\,[t_a,t_b] + \left(2\,b_{[ij]} + \gamma_{[ij]}\right) X_\nu^{jb} z^{[\mu\nu]}\{t_a,t_b\}\right) + \\
+4\xi_2\left(\left(a_{[ij]} X_\nu^{jb} z^{(\mu\nu)} + u_{[ij]} X^{\mu\, jb} z_{(\nu}{}^{\nu)}\right)[t_a,t_b] + \left(b_{(ij)} X_\nu^{jb} z^{(\mu\nu)} + v_{(ij)} X^{\mu\, jb} z_{(\nu}{}^{\nu)}\right)\{t_a,t_b\}\right) + \\
+\lambda_3\left(\begin{array}{c} 2\,\alpha_i\left(\partial_\nu z^{[\mu\nu]} t_a + ig D_\nu^b\, z^{[\mu\nu]}\,[t_a,t_b]\right) + \\ +2\,a_{(ij)} X_\nu^{jb} Z^{[\mu\nu]}\,[t_a,t_b] + \left(2b_{[ij]} + \gamma_{[ij]}\right) X_\nu^{jb} Z^{[\mu\nu]}\{t_a,t_b\} \end{array}\right) + \\
+2\xi_3\left(\begin{array}{c} -\beta_i\,\partial_\nu z^{(\mu\nu)}\,t_a - \rho_i\,\partial^\mu z_{(\nu}{}^{\nu)}\,t_a - ig\left(\beta_i D_\nu^b z^{(\mu\nu)} + \rho_i D^{\mu b} z_{(\nu}{}^{\nu)}\right)[t_a,t_b] + \\ + \left(a_{[ij]} X_\nu^{jb} Z^{(\mu\nu)} + u_{[ij]} X^{\mu\, jb} Z_{(\nu}{}^{\nu)}\right)[t_a,t_b] + \left(b_{(ij)} X_\nu^{jb} Z^{(\mu\nu)} + v_{(ij)} X^{\mu\, jb} Z_{(\nu}{}^{\nu)}\right)\{t_a,t_b\} \end{array}\right) + \\
+\frac{1}{\alpha} g^{\mu\nu}\left(\partial_\alpha D^\alpha + 2\,p_i\,p_j\,\partial_\alpha X^{\alpha j}\right) - m_{ij}^2 X^{\mu j} = 0
\end{gathered}
\tag{3.2}
$$

Field equations are complemented by identities as Bianchi identities and Noether theorem. Considering the minimal covariant derivative $\mathscr{D}_\mu = \partial_\mu\,. + ig\left[D_\mu,\,.\right]$, $\mathscr{D}'_\mu = \partial_\mu + ig\left[D_\mu,\,.\right] + ig\left[X_\mu^i,\,.\right]$ and the collective fields $x_{\mu\nu}^{ij} = \left[X_\mu^i, X_\nu^j\right]$, one gets the following Bianchi identities:

$$
\begin{gathered}
\mathscr{D}_\mu D_{\nu\rho} + \mathscr{D}_\nu D_{\rho\mu} + \mathscr{D}_\rho D_{\mu\nu} = 0 \\
\mathscr{D}_\mu X^i_{[\nu\rho]} + \mathscr{D}_\nu X^i_{[\rho\mu]} + \mathscr{D}_\rho X^i_{[\mu\nu]} + ig\left(\left[X_\mu^i, D_{\nu\rho}\right] + \left[X_\nu^i, D_{\rho\mu}\right] + \left[X_\nu^i, D_{\mu\nu}\right]\right) = 0 \\
\mathscr{D}_\mu X^i_{(\nu\rho)} + \mathscr{D}_\nu X^i_{[\rho\mu]} - \mathscr{D}_\rho X^i_{(\mu\nu)} + ig\left(\left[X_\mu^i, D_{\nu\rho}\right] - \left[X_\nu^i, D_{\rho\mu}\right] + \left[X_\nu^i, D_{\mu\nu}\right]\right) = 0 \\
\mathscr{D}_\mu x^{ij}_{\nu\rho} + \mathscr{D}_\nu x^{ij}_{\rho\mu} + \mathscr{D}_\rho x^{ij}_{\mu\nu} + \left[X_\nu^j, X^i_{[\rho\mu]}\right] + \left[X_\mu^j, X^i_{[\nu\rho]}\right] + \left[X_\rho^j, X^i_{[\mu\nu]}\right] = 0
\end{gathered}
\tag{3.3}
$$

Then, defining $z'_{[\mu\nu]} = a_{(ij)}\, x^{ij}_{\mu\nu}$ and $z'_{(\mu\nu)} = a_{[ij]}\, x^{ij}_{\mu\nu}$, one also derives the following expressions

$$
\begin{aligned}
&\mathscr{D}_\mu z'_{[\nu\rho]} + \mathscr{D}_\nu z'_{[\rho\mu]} + \mathscr{D}_\rho z'_{[\mu\nu]} + a_{(ij)} \left(\left[X^{\,i}_\nu, X^{\,j}_{[\rho\mu]}\right] + \left[X^{\,i}_\mu, X^{\,j}_{[\nu\rho]}\right] + \left[X^{\,i}_\rho, X^{\,j}_{[\mu\nu]}\right]\right) = 0 \\
&\mathscr{D}_\mu z'_{(\nu\rho)} + \mathscr{D}_\nu z'_{(\rho\mu)} + \mathscr{D}_\rho z'_{(\mu\nu)} - a_{[ij]} \left(\left[X^{\,i}_\mu, X^{\,j}_{(\nu\rho)}\right] + \left[X^{\,i}_\nu, X^{\,j}_{(\rho\mu)}\right] + \left[X^{\,i}_\rho, X^{\,j}_{(\mu\nu)}\right]\right) = 0 \\
&\mathscr{D}_\mu z'_{[\nu\rho]} - \mathscr{D}_\nu z'_{[\rho\mu]} + \mathscr{D}_\rho z'_{[\mu\nu]} - a_{(ij)} \left(\left[X^{\,i}_\mu, X^{\,j}_{(\nu\rho)}\right] - \left[X^{\,i}_\nu, X^{\,j}_{[\rho\mu]}\right] - \left[X^{\,i}_\rho, X^{\,j}_{(\mu\nu)}\right]\right) = 0 \\
&\mathscr{D}_\mu z'_{(\nu\rho)} - \mathscr{D}_\nu z'_{(\rho\mu)} + \mathscr{D}_\rho z'_{(\mu\nu)} + a_{[ij]} \left(\left[X^{\,i}_\mu, X^{\,j}_{[\nu\rho]}\right] - \left[X^{\,i}_\nu, X^{\,j}_{(\rho\mu)}\right] - \left[X^{\,i}_\rho, X^{\,j}_{[\mu\nu]}\right]\right) = 0
\end{aligned}
\tag{3.4}
$$

The last identity to be worked out is the local Noether theorem. It provides three relationships:

$$
\partial_\mu \left\{ \left(\left[\frac{\partial \mathscr{L}}{\partial\left(\partial_\mu D_\nu\right)}, D_\nu \right] + \left[\frac{\partial \mathscr{L}}{\partial\left(\partial_\mu X_{\nu i}\right)}, X_{\nu i} \right] \right) \right\}^a \alpha_a = 0 \tag{3.5}
$$

$$
\left\{ \left[\frac{\partial \mathscr{L}}{\partial\left(\partial_\mu D_\nu\right)}, D_\nu \right] + \left[\frac{\partial \mathscr{L}}{\partial\left(\partial_\mu X_{\nu i}\right)}, X_{\nu i} \right] - \frac{n}{g} \frac{\delta \mathscr{L}}{\delta\left(\partial_\mu D_\nu\right)} \right\}^a \partial_\mu \alpha_a = 0 \tag{3.6}
$$

$$
\left[\frac{\delta \mathscr{L}}{\delta\left(\partial_\mu D_\nu\right)} \right]^a \partial_\mu \partial_\nu \alpha_a = 0 \tag{3.7}
$$

which yields,

$$
\partial_\mu \left\{ \left(\begin{array}{c}
2\lambda_1 \left[Z^{[\mu\nu]}, d\,D_\nu + \alpha_i X_{\nu i}\right] + \lambda_3 d \left[z^{[\mu\nu]}, D_\nu\right] + \\
+2\xi_1 \left(\beta_i \left[Z^{(\mu\nu)}, X_{\nu i}\right] + \rho_i g^{\mu\nu} \left[Z_{(\alpha}^{\ \ \alpha)}, X_{\nu i}\right]\right) + \\
+\lambda_3 \alpha_i \left[z^{[\mu\nu]}, X_{\nu i}\right] + \xi_3 \left(\beta_i \left[z^{(\mu\nu)}, X_{\nu i}\right] + \rho_i g^{\mu\nu} \left[z_{(\alpha}^{\ \ \alpha)}, X_{\nu i}\right]\right)
\end{array} \right) \right\}^a \alpha_a = 0 \tag{3.8}
$$

$$
\left\{ \begin{array}{c}
2\lambda_1 \left[Z^{[\mu\nu]}, d\,D_\nu + \alpha_i X_{\nu i}\right] + \lambda_3 d \left[z^{[\mu\nu]}, D_\nu\right] + \\
+2\xi_1 \left(\beta_i \left[Z^{(\mu\nu)}, X_{\nu i}\right] + \rho_i g^{\mu\nu} \left[Z_{(\alpha}^{\ \ \alpha)}, X_{\nu i}\right]\right) + \\
+\lambda_3 \alpha_i \left[z^{[\mu\nu]}, X_{\nu i}\right] + \xi_3 \left(\beta_i \left[z^{(\mu\nu)}, X_{\nu i}\right] + \rho_i g^{\mu\nu} \left[z_{(\alpha}^{\ \ \alpha)}, X_{\nu i}\right]\right) + \\
-2\frac{n}{g} d \left(\lambda_1 Z^{[\mu\nu]} + \lambda_3 z^{[\mu\nu]}\right)
\end{array} \right\}^a \partial_\mu \alpha_a = 0 \tag{3.9}
$$

$$
\left[4\,d\,\lambda_1 Z^{[\mu\nu]} + 2\,d\,\lambda_3\, z^{[\mu\nu]}\right]^a \partial_\mu \partial_\nu \alpha_a = 0 \tag{3.10}
$$

Finally, adding to eq. 3.8, one gets N-conserved currents which can be derived from the equations of motion. They are showing the asymmetry diversity on the couplings.

4. Global vector equations

Eq. 1.1 is producing new elements. Thus in order to understand the new entities to be measured we should rewrite the classical equations in terms of gauge invariant objects. Defining

$\phi \equiv D_0$, $\vec{D} \equiv -D_i$, $\phi^I \equiv X_0{}^I$, $\vec{X}^I \equiv -X_i{}^I$, where here notation is changed for $I = 2,...,N$ and $i = 1,2,3$. It yields, the following invariants

$$\vec{E} \equiv D_{0i}\ ,\ \vec{B} \equiv \frac{1}{2}\varepsilon_{imn}D_{nm}\ ,\ \vec{E}^I \equiv X_{[0i]}{}^I\ ,\ \vec{B}^I \equiv \frac{1}{2}\varepsilon_{ijk}X_{[kj]}{}^I$$
$$\vec{e} \equiv z_{[0i]}\ ,\ \vec{b} \equiv \frac{1}{2}\varepsilon_{ijk}z_{[kj]}\ ,\ \vec{e}\,' \equiv z'_{[0i]}\ ,\ \vec{b}\,' \equiv \frac{1}{2}\varepsilon_{ijk}z'_{[kj]} \tag{4.1}$$

The longitudinal sector also develops new invariants

$$\sigma \equiv \beta_I X_{(00)}{}^I,\quad \vec{\sigma} \equiv \beta_I X_{(0i)}{}^I,\quad \sigma_{ij} \equiv \beta_I X_{(ij)}{}^I,\quad \theta \equiv \rho_I\, g_{00} X_{(\alpha}{}^{\alpha)i},\ \theta_{ij} \equiv \rho_I\, g_{ij} X_{(\alpha}{}^{\alpha)i}$$
$$\vec{\Lambda} \equiv z_{(0i)},\ \Gamma_{ij} \equiv z_{(ij)},\ \tau \equiv z_{(\mu}{}^{\mu)},\quad \tau + \Gamma_{ii} \equiv z_{(00)},\quad \vec{\Lambda}\,' \equiv z'_{(0i)},\ \Gamma'_{ij} \equiv z'_{(ij)} \tag{4.2}$$

Rewriting eq. 3.1 in terms of these new measurable objects:

$$\begin{gathered}
2\lambda_1 \left(-d\vec{\nabla}\cdot\left(d\vec{E}^a + \alpha_J \vec{E}^{Ja}\right) + g\ f_{abc}\left(d\vec{D}^b + \alpha_I \vec{X}^{Ib}\right)\cdot\left(d\vec{E}^c + \alpha_J \vec{E}^{Jc}\right)\right) + \\
+2\xi_1 \begin{pmatrix} g\beta_I\ f_{abc}\vec{X}^{Ib}\cdot\vec{\sigma}^e - g\beta_I\ f_{abc}\phi^{Ib}\left(\sigma^c + \theta^c\right) + \\ +g\rho_I\ f_{abc}\phi^{Ib}\left(\sigma_j{}^{jc} + \theta_{(j}{}^{j)c}\right) - g\rho_I\ f_{abc}\phi^{Ib}\left(\sigma^c + \theta^c\right) \end{pmatrix} + \\
+\lambda_3\left(-d\vec{\nabla}\cdot\vec{e}^a + g\ f_{abc}\left(d\vec{D}^b + \alpha_I \vec{X}^{Ib}\right)\cdot\vec{e}^c +\right) + \\
+\xi_3\left(g\beta_I\ f_{abc}\vec{X}^{Ib}\cdot\vec{\Lambda}^e - g\beta_I\ f_{abc}\phi^{Ib}\left(\tau^c + \Gamma^{iic}\right) - g\rho_I\ f_{abc}\phi^{Ib}\tau^c\right) = 0\,,
\end{gathered}$$

$$\begin{gathered}
2\lambda_1 \begin{pmatrix} d\frac{\partial}{\partial t}\left(d\vec{E}^a + \alpha_J \vec{E}^{Ja}\right) - d\vec{\nabla}\times\left(d\vec{B}^a + \alpha_J \vec{B}^{Ja}\right) + \\ +g f_{abc}\left(d\vec{D}^b + \vec{X}^{Ib}\right)\times\left(d\vec{B}^c + \alpha_J \vec{B}^{Jc}\right) - g f_{abc}\left(d\phi^b + \alpha_I\phi^{Ib}\right)\left(d\vec{B}^c + \alpha_J \vec{B}^{Jc}\right) \end{pmatrix} + \\
+2\xi_1 \begin{pmatrix} g\beta_I\ f_{abc}\phi^{Ib}\vec{\sigma}^e - g\beta_I\ f_{abc}\vec{X}^{Ib}\left(\sigma^{ijc} + \theta^{ijc}\right) + \\ +g\rho_I\ f_{abc}\vec{X}^{Ib}\left(\sigma^c + \theta^c\right) - g\rho_I\ f_{abc}\vec{X}^{Ib}\left(\sigma_j{}^{jc} + \theta^c\right) \end{pmatrix} + \\
+\lambda_3\left(d\frac{\partial}{\partial t}\vec{e}^a - d\vec{\nabla}\times\vec{b}^a + g f_{abc}\left(d\vec{D}^b + \vec{X}^{Ib}\right)\times\vec{b}^c - g f_{abc}\left(d\phi^b + \alpha_I\phi^{Ib}\right)\vec{e}^c\right) + \\
+\xi_3\left(g\beta_I f_{abc}\phi^{Ib}\vec{\Lambda}^c - g\beta_I f_{abc}\vec{X}^{Ib}\Gamma^{ijc} + g\rho_I\ f_{abc}\vec{X}^{Ib}\tau^c\right) = 0
\end{gathered} \tag{4.3}$$

for the genuine gauge field.

$$
\begin{aligned}
&2\,\lambda_1\left(-\alpha_I\,\vec{\nabla}\cdot\left(d\,\vec{E}^{a}+\alpha_J\,\vec{E}^{Ja}\right)+g\,\alpha_I\;f_{abc}\,\vec{D}^{b}\cdot\left(d\,\vec{E}^{c}+\alpha_J\,\vec{E}^{Jc}\right)\right)+\\
&+2\,\xi_1\left(\begin{array}{c}
-\beta_I\,\dfrac{\partial}{\partial t}\left(\sigma^{a}+\theta^{a}\right)+\beta_I\,\vec{\nabla}\cdot\vec{\sigma}^{a}+\\
+g\,\beta_I\,f_{abc}\,\phi^{b}\left(\sigma^{c}+\theta^{c}\right)-g\,\beta_I\;f_{abc}\,\vec{D}^{b}\cdot\vec{\sigma}^{e}+\\
-\rho_I\,\dfrac{\partial}{\partial t}\left(\sigma^{a}+\theta^{a}\right)+\rho_I\,\dfrac{\partial}{\partial t}\left(\sigma_j^{\;ja}+\theta_j^{\;ja}\right)+\\
+g\,\rho_I\;f_{abc}\,\phi^{b}\left(\sigma^{c}+\theta^{c}\right)-g\,\rho_I\,f_{abc}\,\phi^{b}\left(\sigma_j^{\;jc}+\theta_j^{\;jc}\right)
\end{array}\right)+\\
&-2\,\lambda_2\left(i\,a_{(IJ)}\,f_{\,abc}+b_{[IJ]}\,d_{abc}+\frac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\vec{X}^{b}\cdot\vec{e}^{c}+\\
&+2\,\xi_2\left(\begin{array}{c}
-\left(i\,a_{[IJ]}\;f_{abc}+b_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\cdot\vec{\Lambda}^{c}+\\
+\left(i\,a_{[IJ]}\;f_{abc}+b_{(IJ)}\,d_{abc}\right)\phi^{Jb}\,\Gamma^{iie}+\\
+\left(i\,a_{[IJ]}\;f_{abc}+i\,u_{[IJ]}\;f_{abc}+b_{(IJ)}\,d_{abc}+v_{(IJ)}\,d_{abc}\right)\phi^{Jb}\,\tau^{c}
\end{array}\right)+\\
&+\lambda_3\left(\begin{array}{c}
-\alpha_I\,\vec{\nabla}\cdot\vec{e}^{a}+g\,\alpha_I\;f_{abc}\,\vec{D}^{b}\cdot\vec{e}^{c}\\
-\left(i\,a_{(IJ)}\;f_{abc}+b_{[IJ]}\,d_{abc}+\dfrac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\vec{X}^{Jb}\cdot\left(d\,\vec{E}^{c}+\alpha_K\,\vec{E}^{Kc}\right)
\end{array}\right)+\\
&+\xi_3\left(\begin{array}{c}
-\beta_I\,\dfrac{\partial}{\partial t}\left(\tau^{c}+\Gamma^{iia}\right)+\beta_I\,\vec{\nabla}\cdot\vec{\Lambda}^{a}+g\,\beta_I\,f_{abc}\left(\tau^{c}+\Gamma^{iic}\right)+\\
-g\,\beta_I\;f_{abc}\,\vec{D}^{b}\cdot\vec{\Lambda}^{c}+\rho_I\,\dfrac{\partial}{\partial t}\,\tau^{a}+g\,\rho_I\;f_{abc}\,\phi^{b}\,\tau^{c}+\\
-\left(i\,a_{[IJ]}\;f_{abc}+b_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\cdot\vec{\sigma}^{c}+\\
-\left(i\,u_{[IJ]}\;f_{abc}+v_{(IJ)}\,d_{abc}\right)\phi^{Jb}\left(\sigma_j^{\;jc}+\theta_j^{\;jc}\right)+\\
+\left(i\,a_{[IJ]}\;f_{abc}+i\,u_{[IJ]}\;f_{abc}+b_{(IJ)}\,d_{abc}+v_{(IJ)}\,d_{abc}\right)\phi^{Jb}\left(\sigma^{c}+\theta^{c}\right)
\end{array}\right)=0
\end{aligned}
$$

$$
\begin{aligned}
&2\,\lambda_1 \begin{pmatrix} \alpha_I \,\dfrac{\partial}{\partial t}\left(d\,\vec{E}^a + \alpha_J\,\vec{E}^{Ja}\right) - \alpha_I\,\vec{\nabla}\times\left(d\,\vec{B}^a + \alpha_J\,\vec{B}^{Ja}\right) + \\ -g\,\alpha_I\, f_{abc}\,\phi^b\left(d\,\vec{E}^c + \alpha_J\,\vec{E}^{Jc}\right) + g\,\alpha_I\, f_{abc}\,\vec{D}^b\times\left(d\,\vec{B}^c + \alpha_J\,\vec{B}^{Jc}\right) \end{pmatrix} + \\
&+2\,\xi_1 \begin{pmatrix} \beta_I\,\dfrac{\partial}{\partial t}\,\vec{\sigma}^a - 2\,\beta_I\,\vec{\nabla}\left(\sigma^{ija} + \theta^{ija}\right) - g\,\beta_I\, f_{abc}\,\phi^b\,\vec{\sigma}^c + g\,\beta_I\, f_{abc}\,\vec{D}^b\left(\sigma^{ijc} + \theta^{ijc}\right) + \\ +\rho_I\,\vec{\nabla}\left(\sigma^a + \theta^a\right) - \rho_I\,\vec{\nabla}\left({\sigma_j}^{ja} + {\theta_j}^{ja}\right) - g\,\rho_I\, f_{abc}\,\vec{D}^b\left(\sigma^c + \theta^c\right) - g\,\rho_I\, f_{abc}\,\vec{D}^b\left({\sigma_j}^{jc} + {\theta_j}^{jc}\right) \end{pmatrix} + \\
&+2\,\lambda_2 \begin{pmatrix} \left(i\,a_{(IJ)}\, f_{abc} + b_{[IJ]}\,d_{abc} + \frac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\phi^{Jb}\,\vec{e}^c + \\ -\left(i\,a_{(IJ)}\, f_{abc} + b_{[IJ]}\,d_{abc} + \frac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\vec{X}^{Jb}\times\vec{b}^c \end{pmatrix} + \\
&+2\,\xi_2 \begin{pmatrix} \left(i\,a_{[IJ]}\, f_{abc} + b_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\,\Gamma^{ijc} - \left(i\,a_{[IJ]}\, f_{abc} + b_{(IJ)}\,d_{abc}\right)\phi^{Jb}\,\vec{\Lambda}^c + \\ -\left(i\,u_{[IJ]}\, f_{abc} + v_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\,\tau^c \end{pmatrix} + \\
&+\lambda_3 \begin{pmatrix} \alpha_I\,\dfrac{\partial}{\partial t}\,\vec{e}^a - \alpha_I\,\vec{\nabla}\times\vec{b}^a - g\,\alpha_I\, f_{abc}\,\phi^b\,\vec{e}^c + g\,\alpha_I\, f_{abc}\,\vec{D}^b\times\vec{b}^c + \\ +\left(i\,a_{(IJ)}\, f_{abc} + b_{[IJ]}\,d_{abc} + \frac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\phi^{Jb}\left(d\,\vec{E}^c + \alpha_K\,\vec{E}^{Kc}\right) + \\ -\left(i\,a_{(IJ)}\, f_{abc} + b_{[IJ]}\,d_{abc} + \frac{1}{2}\,\gamma_{[IJ]}\,d_{abc}\right)\vec{X}^{Jb}\times\left(d\,\vec{B}^c + \alpha_K\,\vec{B}^{Kc}\right) \end{pmatrix} + \\
&+\xi_3 \begin{pmatrix} \beta_I\,\dfrac{\partial}{\partial t}\,\vec{\Lambda}^a - \beta_I\,\vec{\nabla}\,\Gamma^{ija} - g\,\beta_I\, f_{abc}\,\phi^b\,\vec{\Lambda}^c + g\,\beta_I\, f_{abc}\,\vec{D}^b\,\Gamma^{ijc} + \rho_I\,\vec{\nabla}\,\tau^a - g\,\rho_I\, f_{abc}\,\vec{D}^b\,\tau^c + \\ +\left(i\,a_{[IJ]}\, f_{abc} + b_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\left(\sigma^{ijc} + \theta^{ijc}\right) - \left(i\,a_{[IJ]}\, f_{abc} + b_{(IJ)}\,d_{abc}\right)\phi^{Jb}\,\vec{\sigma}^c + \\ -\left(i\,u_{[IJ]}\, f_{abc} + v_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\left(\sigma^c + \theta^c\right) + \left(i\,u_{[IJ]}\, f_{abc} + v_{(IJ)}\,d_{abc}\right)\vec{X}^{Jb}\left({\sigma_j}^{jc} + {\theta_j}^{jc}\right) \end{pmatrix} = 0
\end{aligned} \tag{4.4}
$$

The Bianchi identities are rewritten as

$$
\begin{aligned}
\overrightarrow{\mathscr{D}}'\times\overrightarrow{E} &= -\frac{\mathscr{D}'}{\mathscr{D}'t}\,\overrightarrow{B} \\
\overrightarrow{\mathscr{D}}'\cdot\overrightarrow{B} &= 0
\end{aligned} \tag{4.5}
$$

$$
\begin{aligned}
\overrightarrow{\mathscr{D}}'\times\overrightarrow{E}^{\,I} + \frac{\mathscr{D}'}{\mathscr{D}'t}\,\overrightarrow{B}^{\,I} &= g\, f_{abc}\,\overrightarrow{X}^{\,Ia}\times\overrightarrow{E}^{\,b}\,t^c + g\, f_{abc}\,\phi^{Ia}\,\overrightarrow{B}^{\,b}\,t^c \\
\overrightarrow{\mathscr{D}}'\cdot\overrightarrow{B}^{\,I} &= g\, f_{abc}\,\overrightarrow{X}^{\,Ia}\cdot\overrightarrow{B}^{\,b}\,t^c
\end{aligned} \tag{4.6}
$$

$$
\begin{aligned}
\overrightarrow{\mathscr{D}}'\times\overrightarrow{E}^{\,I} - 2\,g\, f_{abc}\,\phi^{Ia}\,\overrightarrow{B}^{\,b}\,t^c &= \beta_I\,\varepsilon_{ijk}\,\frac{\mathscr{D}'}{\mathscr{D}'t}\,\sigma^{ija}\,t_a - \beta_I\,\varepsilon_{ijk}\,\mathscr{D}'_i\,\sigma^a\,t_a \\
\overrightarrow{\mathscr{D}}'\cdot\overrightarrow{B}^{\,I} &= g\, f_{abc}\,\overrightarrow{X}^{\,Ia}\cdot\overrightarrow{B}^{\,b}\,t^c
\end{aligned} \tag{4.7}
$$

$$
\begin{aligned}
\overrightarrow{\mathscr{D}}'\times\overrightarrow{e}^{\,\prime a}\,t_a + \frac{\mathscr{D}'}{\mathscr{D}'t}\,\overrightarrow{b}^{\,\prime a}\,t_a + i\, f_{abc}\,a_{(IJ)}\,\overrightarrow{X}^{\,Ia}\times\overrightarrow{E}^{\,Jb}\,t^c + i\, f_{abc}\,a_{(IJ)}\,\phi^{Ia}\,\overrightarrow{B}^{\,Jb}\,t^c &= 0 \\
\overrightarrow{\mathscr{D}}'\cdot\overrightarrow{b}^{\,\prime a}\,t_a + i\, f_{abc}\,a_{(IJ)}\,\overrightarrow{X}^{\,Ia}\cdot\overrightarrow{B}^{\,Jb}\,t^c &= 0
\end{aligned} \tag{4.8}
$$

$$
\begin{aligned}
\varepsilon_{ijk}\,\frac{\mathscr{D}'}{\mathscr{D}'t}\,\Gamma'^{ija}\,t_a &= i\, f_{abc}\,\varepsilon_{ijk}\,a_{[IJ]}\,\beta^J\,\phi^{Ia}\,\sigma^{ijb}\,t^c \\
\varepsilon_{ijk}\,\mathscr{D}'^i\,\Gamma'^{jka}\,t_a &= i\, f_{abc}\,\varepsilon_{ijk}\,a_{[IJ]}\,\beta^J\,X^{iIa}\,\sigma^{jkb}\,t^c
\end{aligned} \tag{4.9}
$$

$$2\,\frac{\mathscr{D}'}{\mathscr{D}'t}\,\vec{b}^{\,\prime a}\,t_a + i\,f_{abc}\,a_{(IJ)}\,\vec{X}^{\,Ia}\times\vec{E}^{\,Jb}\,t^c + i\,f_{abc}\,a_{(IJ)}\,\beta^J\,\vec{X}^{\,Ia}\times\vec{\sigma}^{\,b}\,t^c + \\ + i\,f_{abc}\,\varepsilon_{ijk}\,a_{(IJ)}\,\beta^J\,\phi^{Ia}\,\sigma^{ijb}\,t^c = 0$$

$$\vec{\mathscr{D}}^{\,\prime}\cdot\vec{b}^{\,\prime a}\,t_a + i\,f_{abc}\,a_{(IJ)}\,\vec{X}^{\,Ia}\cdot\vec{B}^{\,Jb}\,t^c = 0 \tag{4.10}$$

$$i\,f_{abc}\,a_{[IJ]}\,\vec{X}^{\,Ia}\times\vec{E}^{\,Jb}\,t^c - 2\,i\,f_{abc}\,a_{[IJ]}\,\phi^{Ia}\,\vec{B}^{\,Jb}\,t^c + \\ + \varepsilon_{ijk}\,\frac{\mathscr{D}'}{\mathscr{D}'t}\,\Gamma'^{ija}\,t_a = 2\,\vec{\mathscr{D}}^{\,\prime}\times\vec{\Lambda}^{\,\prime a}\,t_a + i\,f_{abc}\,a_{[IJ]}\,\beta^J\,\vec{X}^{\,Ia}\times\vec{\sigma}^{\,b}\,t^c$$

$$3\,\varepsilon_{ijk}\,\mathscr{D}'_i\,\Gamma'^{jka}\,t_a + i\,f_{abc}\,\varepsilon_{ijk}\,a_{[IJ]}\,\beta^J\,X^{iIa}\,\sigma^{jkb}\,t^c = 0 \tag{4.11}$$

5. Conclusion

Physics should not be only made by searching for new symmetries, but also for looking for the corresponding asymmetries. While group theory defines by symmetry a number of gauge fields equal to the number of generators corresponding to this group [5], by asymmetry here one defines a number of potential fields rotating under a same group different from the number of generators and so, this new aspect is explored in this work through non-abelian asymmetry.

Thus the effort here is to show how opportunities as diversity and connectivity can appear from SU(N). On diversity, it shows a variety of fields carrying different masses without requiring Higgs and associated to different coupling constants. On connectivity, it presents different Bianchi identities between its various fields as equations (3.3) and (3.4) are showing.

We would also relate that although this work has worked with unitary groups, we would say that it can also be extended to other classical groups including the exceptional Lie groups.

Consequently, it emerges a possibility where nature evolution is a consequence from opportunities taken from a given symmetry. Physical models containing asymmetry are another possibility to be understood through LHC.

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