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Analysis of the String Vortex Potential in a Softly Broken Supersymmetric Scenario

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The main propose of this work is to pursue an investigation of the potential for a supersymmetric scenario of a global vortex. In our formulation, the duality relation between the vortex configuration and a 2-form gauge field is the key-element. The Lorentz-breaking background is also suitably accomodated in a superfield, and the duality between the vortex and the Kalb-Ramond field is duely formulated in the N=1-superspace. We then find that the embedding of the string vortices in the supersymmetric model dictates the introduction of terms that softly break supersymmetry. The global string vortex may be modelling the superfluid vortex if there is a Lorentz-breaking background.

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1. Introduction

Cosmic strings [1, 2, 6], most likely produced during fase transitions [7], appear in some Grand-Unified Gauge Theories and carry a large energy density [2]. These strings are associated with the breaking of local symmetries. In this way, together with other structures, they may provide a possible origin for the seed density perturbations which became the large scale structure of the Universe observed today [8, 9]. However, local strings are important by their contribution to the gravitational radiation background[10]. There are other types of strings that present similar cosmological implications, in particular those formed when a global symmetry is broken. Instead of radiating gravitationally, the dominant radiation mechanism for these strings is the emission of massless Nambu-Goldstone bosons [5]. In recent publications, it has been shown that, in the low-energy regime, the effective action that, presents the Kalb-Ramond fields, provides an accurate description of the dynamics of global global strings[13]. The Kalb-Ramond fields[14] are a anti-symmetric tensor. This tensor intercting with a massive Higgs fields give us a source. The system have a applications to superfluid helium and axion cosmology. The global vortex behavior as a superfluid if the Kalb-Ramond field breaks Lorentz symmetry[17, 15] in the background. In view of the possibility that Supersymmetry (SUSY) was realized in the early Universe, and it was broken approximately at the same time as cosmic strings were formed, many recent works investigate cosmic strings by adopting a supersymmetric framework [18, 19, 16].

In this work, we present the preliminary results for the supersymmetric version to these vortex-superfluids. The outline of this paper is as follows: in Section 2, we start by presenting a simple supersymmetric model with the global symmetry breaking induced by an explicit SUSY breaking term. In Section 3, we study the bosonic configurations that are relevant to the superfluid description. Section 4 is devoted to the study of the global vortex configurations, the potential breaking and stability, the superfiels identifications and the component action with the Kalb-Ramond fields. In Section 5, we analyse Lorentz-symmetry breaking and superfluid stability, finally, in Section 6, we draw our preliminary Conclusions.

2. Review of the non-supersymmetric vortex-superfluid model

In this section, we review the model of the global strings that may can model vortices in a superfluid. The global strings appear when a U(1) global symmetry is spontaneously broken. The lagrangian that describes this model reads as follows:

$$L = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi) \tag{2.1}$$

The potential is given by

$$V(\phi) = \frac{\lambda}{4}\left[|\phi|^2 - \eta^2\right]^2. \tag{2.2}$$

The vortex ansatz is

$$\phi = f(r)e^{i\alpha}, \tag{2.3}$$

where the boundary conditions are given by

$$f(r) = 0 \quad \text{tor} = 0$$
$$f(r) = \eta \quad \text{tor} \to \infty; \tag{2.4}$$

with this configuration, we have the effective Lagrangian.

$$L = \partial_\mu f \partial^\mu f + f^2 \partial_\mu \alpha \partial^\mu \alpha - V(\phi) \tag{2.5}$$

This vortex is does not stable, i.e. a global string behaves like a vortex in a superfluid only in the presence of a special background field. In this work we adopt the fact that the vortex superfluid is immersed in a Lorentez-breaking fluid. To understand this problem, it is more convenient to rewrite the model in terms of a 2-form field, $B_{\mu\nu}$, with the duality parametrization, far from of the vortex, given by

$$\eta \partial_\mu \alpha \equiv \varepsilon_{\mu\nu\rho\lambda} \partial^\nu B^{\rho\lambda}. \tag{2.6}$$

The interaction of the vortex with the classical Goldstone boson field is described by

$$L = \frac{1}{6} G_{\mu\nu\lambda} G^{\mu\nu\lambda} + B_{\mu\nu} J^{\mu\nu}, \tag{2.7}$$

where $G_{\mu\nu\lambda}$ is the Kalb-Ramond field-strength:

$$G_{\alpha\mu\nu} = \partial_\alpha B_{\mu\nu} + \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu}. \tag{2.8}$$

The relativistic force law for the response of a vortex to the local field $G^{\mu\nu\rho}$, analogous to the Lorentz force law in Electrodynamics, is given ao below:

$$F^\lambda = j_{\mu\nu} G^{\mu\nu\lambda}, \tag{2.9}$$

where

$$\partial^\mu G_{\mu\nu\lambda} = j^{\nu\lambda} \tag{2.10}$$

It is easily verified that, for a straight string at rest with no background, this force vanishes. To solve this problem, we better split $G_{\mu\nu\lambda)}$ into two parts, $G^{\mu\nu\lambda} = G^{\mu\nu\lambda}_{(self)} + G^{\mu\nu\lambda}_{(ext)}$. The external Lorentz breaking bacground is given by

$$G^{\mu\nu\lambda}_{(ext)} = \sqrt{\rho}\varepsilon^{0\mu\nu\lambda} = \sqrt{\rho}\varepsilon^{ijk}. \tag{2.11}$$

The fact that the superfluid vortex is immersed in a Lorentz-noninvariant fluid suggests that the correct model for superfluid vortex involves choosing a special background. In this case, $F^\lambda \neq 0$.

3. The supersymmetric model

The ingredient superfields of the model: a chiral scalar supermultiplet, $\Phi(\phi,\chi,F)$, that contains a complex scalar field, ϕ, a spinor, χ_a, and an auxiliary complex scalar field, F; a chiral supermultiplet, $S(s,\zeta,H)$, that contains a complex scalar field, s, a spinor, ζ_α, and an auxiliary scalar, H. The kinetic piece of the Lagrangian of the global vortex in superfields is written as below:

$$L_K = \Phi\Phi^\dagger|_{\theta\theta\bar{\theta}\bar{\theta}} + SS^\dagger|_{\theta\theta\bar{\theta}\bar{\theta}}. \tag{3.1}$$

The chiral scalar supermultiplets Φ and S are θ-expandes according to the following expressions:

$$\Phi = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}[\phi(x) + \theta^a\chi_a(x) + \theta^2 F(x)], \tag{3.2}$$

$$S = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}[s(x) + \theta^a\zeta_a(x) + \theta^2 H(x)]. \tag{3.3}$$

These superfields satisfy a chirality constraint, given by the condition $\bar{D}_{\dot{a}}\Phi = 0$. This scalar superfield contains the scalar field, ϕ, the fermionic field, χ, and the auxiliary field, F, needed to close the supermultiplet degree, of freedom.

The superpotential is the supersymmetric quantity that yields the potential of the theory. In this case, to give us a global stable vortex without flat directions, no D-term should be present (and this is a difficulty), since we do not wish a local symmetry. Then:

$$W = gS\Phi^2|_{\theta\theta} + h.c. \tag{3.4}$$

The superpotential given by eq.(3.4) is not be able to yield alone the global symmetry breaking; normally, it arises from a D-term. Here, it is not possible. So, the global symmetry breaking may be accomplished by an explicit SUSY breaking term induced by the breaking of SUSY in the hidden sector

$$L_{SUSY-B} = -\mu\theta^2\bar{\theta}^2\Phi\Phi^\dagger|_{\theta\theta\bar{\theta}\bar{\theta}}. \tag{3.5}$$

The parameter μ is related with the supersyymmetry breaking scale, that we can associate to some physical experiment.

4. The bosonic component fields in the supersymmetric model

The bosonic Lagrangian that results from this supersymmetric model and that is relevant to the superfluid can be written as:

$$L_B = \partial_\mu\phi\partial^\mu\phi^\dagger + \partial_\mu s\partial^\mu s^\dagger - V. \tag{4.1}$$

The potential V, that comes from the eq.(3.4), is given by

$$V' = g^2|\phi|^4 + 4g^2|s|^2|\phi|^2. \tag{4.2}$$

The (soft) SUSY breaking mass term given by eq.(3.5) reads as follows:

$$L_{SUSY-B} = -\mu^2|\phi|^2. \tag{4.3}$$

Then, the full potential is

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$$V = g^2|\phi|^4 - (\mu^2 - 4g^2|s|^2)|\phi|^2. \tag{4.4}$$

Spontaneous global symmetry breaking takes place whenever

$$(\mu^2 - 4g^2|s|^2) > 0. \tag{4.5}$$

A solution to the vortex configuration exists without flat directions. The global vortex presents a minimum roll and the a central maximum characterise the Mexican - hat potential.

5. The vortex and the Kalb-Ramond superfields

The global string is a time-independent vortex solution to the equations of motion of a spontaneously broken global U(1) Higgs model; in our case, the potential that breaks the symmetry is given by eq.(4.4), as we have analysed. We can be split the scalar field ϕ into a massive (real) component, φ, and a massless (real periodic) Goldstone boson, α, according to the expression:

$$\phi = \varphi(r)e^{i\alpha}. \tag{5.1}$$

However, we can used the well-known duality between a massless scalar field and a two-index antisymmetric tensor gauge field, $B_{\mu\nu}$, to replace the Goldstone boson by using in superfield the relation:

$$\bar{\Phi}\Phi \sim \mathscr{G}. \tag{5.2}$$

The component-field expansion for the superfield-strength of the Kalb Ramond, $\mathscr{G}$, is given by

$$\begin{aligned} \mathscr{G} = & -\tfrac{1}{2}M + \tfrac{i}{4}\theta\xi_a - \tfrac{i}{4}\bar{\theta}_{\dot{a}}\bar{\xi}^{\dot{a}} + \tfrac{1}{2}\theta\sigma^{\mu}_{a\dot{a}}\bar{\theta}^{\dot{a}}\tilde{G}_{\mu} + \\ & \tfrac{1}{8}\theta^a\sigma^{\mu}_{a\dot{a}}\bar{\theta}^2\partial_{\mu}\bar{\xi}^{\dot{a}} - \tfrac{1}{8}\theta^2\sigma^{\mu}_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_{\mu}\xi^a + \\ & -\tfrac{1}{8}\theta^2\bar{\theta}^2\partial_{\mu}\partial^{\mu}M; \end{aligned} \tag{5.3}$$

$\tilde{G}_{\mu}$ is the dual of the 3-form field strength, $G_{\mu\nu k}$, related to the 2-form Kalb-Ramond (KR) field, $B_{\mu\nu}$:

$$\tilde{G}_{\mu} = \frac{1}{3!}\varepsilon_{\mu\nu\alpha\beta}G^{\nu\alpha\beta}. \tag{5.4}$$

The $B_{\mu\nu}$-field shows up only through its field-strength, located in the chiral scalar superfield $\mathscr{G}$. As for the χ, λ and ξ, these are the fermionic partners of the scalar matter, the photon and the KR gauge potential, respectively.
The identification (5.2) gives us vanishing fermionic background:

$$\varphi^2 = 2\eta M \tag{5.5}$$

$$\varphi^2\partial_{\mu}\alpha = \frac{1}{2}\eta\varepsilon_{\mu\nu\lambda\rho}\partial^{\nu}B^{\lambda\rho}. \tag{5.6}$$

The identification given by (5.6) is the usual are for the vortex configuration. To analyse the Lorentz-breaking background, we adopt the same ansatz (2.11) for the bosonic sector.

6. Preliminary results

In this work, we have shown that it is possible to construct a string vortex by modelling the vortex superfluid in a supersymmetric context. We have analysed the potential that gives us the correct string vortex configuration; it presents a soft supersymmetry breaking induced by the hidden sector. We have analysed the bosonic aspects of the duality representation of the vortex. The next step is study the fermionic implication of the the Lorentz-breaking Kalb-Ramond background. Another aspect of relevance is the origin of the Kalb-Ramond background [20] and its relation with the hidden soft breaking of global SUSY.

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