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String tensions of $SU(N)$ gauge theories in $2+1$ dimensions

Barak Bringoltz[*] and Michael Teper
Rudolf Peierls Centre for Theoretical Physics, University of Oxford, OX1 3NP Oxford, UK
E-mail: `barak@thphys.ox.ac.uk`, `m.teper1@physics.ox.ac.uk`

We calculate the energy spectrum of closed strings in $SU(N)$ gauge theories with $N = 2,3,4,6,8$ in $2+1$ dimensions to a high accuracy. We attempt to control all systematic errors, and this allows us to perform a precise comparison with different theoretical predictions. When we study the dependence of the string mass on its length L we find that the Nambu-Goto prediction is a very good approximation down to relatively short lengths, where the Lüscher term alone is insufficient. We then isolate the corrections to the Lüscher term, and compare them to recent theoretical predictions, which indeed seem to be mildly preferred by the data. When we take these corrections into account and extract string tensions from the string masses, we find that their continuum limit is lower by $2\% - 1\%$ from the predictions of Karabli, Kim, and Nair. The discrepancy decreases with N, but when we extrapolate our results to $N = \infty$ we still find a discrepancy of 0.88% which is a 4.5σ effect.

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1. Introduction

There are many indications for a connection between QCD and string theory, which appear in a wide range of experimental phenomena and theoretical works [1]. Here we are interested on the QCD side and study the energy spectrum of the 'original' string - the flux tube of pure Yang-Mills - with an emphasis on its large-N limit. We focus on three Euclidean dimensions, and our motivation is two-fold. First, we wish to perform a precise test of the remarkable work of Karabali, Kim, and Nair [2]. This work yields the following prediction for the string tension σ

$$\frac{\sqrt{\sigma}}{g^2N} = \sqrt{\frac{1-1/N^2}{8\pi}}, \tag{1.1}$$

where g is the super-renormalised coupling[1]. A previous comparison of Eq. (1.1) to lattice results for $N = 2,3,4,6$ [3] showed that Eq. (1.1) is higher from the data by about $2\% - 1\%$, but also that this discrepancy decreases with N. This and the fact that [2] predicts no screening of zero N-ality charges suggests the possibility that the analysis in [2] may be exact at $N = \infty$. This would appear to be contradicted by the $N \to \infty$ extrapolation of [3], but the results presented there included several unestimated systematic errors which, while small, may be significant at the 1% level. Two of these lead to an underestimate of the lattice string tensions, and we check whether by controlling them, the lattice results become consistent with Eq. (1.1) at $N = \infty$.

The first error rises in the process of extracting the string tension from the string mass, where it is typical to neglect corrections that are sub-leading to the Lüscher term. Indeed, as the string length L becomes larger, these corrections decrease as $1/L^4$, but we wish to estimate them on a quantitative level. The way the string mass changes with L is also interesting theoretically (and this constitutes our second motivation for this study). It gives us information on the effective string theory of the flux tube, that can be compared with older [4] and more recent [5] predictions. The second systematic error that we remove is the neglect of $O(g^4)$ terms in the continuum extrapolation of the string tensions.

Here we present an analysis of these systematic errors, and in the case of the $O(1/L^4)$ error, we compare to the predictions of [5]. After removing both systematics we compare the resulting string tensions to Eq. (1.1). The results presented here were obtained in a preliminary analysis of the data. Publications with a more extensive analysis, that will also include the raw data, are forthcoming [8].

2. Methodology

There exists a plethora of lattice works studying properties of flux-tubes by looking at open or closed strings (for example see the review in [6]). To avoid 'contamination' from perturbative effects, and to compare directly with the spectrum of the effective string theories, we choose to measure the large distance exponential fall-off of correlation functions of strings closed around the spatial torus. Our analysis proceeds in two stages.

[1]Recall that in $2+1$ dimensions the coupling g^2 has dimensions of mass.

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2.1 Stage I - the string mass dependence on the string length

We first work with a fixed lattice spacing (by fixing the lattice coupling) and study the dependence of the string mass m on its length L. Fitting these results we are able to test the theoretical predictions in [4, 5]. In addition, these fits provide us with a practical tool to extract string tensions from strings which are shorter then usual, and with a quantitative estimate of the systematic error induced by using the Lüscher term alone.

2.2 Stage II - continuum extrapolation of string tensions

Here we perform measurements of string masses with a fixed physical length L and different lattice spacings. We choose $L\sqrt{\sigma} \gtrsim 3-3.5$ and use the fits we obtain from stage I to extract string tensions. We then extrapolate these to the continuum limit and compare with Eq. (1.1) for all values of N as well as for the extrapolation to $N=\infty$.

2.3 Lattice construction

We define the gauge theory on a discretized periodic Euclidean three dimensional space-time with $N_0 \times N_1 \times N_2$ sites, and perform Monte-Carlo simulations of a simple Wilson action. We use the Kennedy-Pendelton heat bath algorithm for the link updates, followed by five over-relaxations of all the $SU(2)$ subgroups of $SU(N)$. We measure correlation functions of Polyskov loops that wind around the $\hat{0}$ direction so $L = aN_0$. The correlations are measured along direction $\hat{1}$ and we project to zero transverse momentum by averaging over direction $\hat{2}$. Although here we mainly present results for the ground state, we study all possible values of the N-ality $k = 1, 2, \ldots, [N/2]$, and their first few excited states. Using improved operators [7] we are able to obtain overlaps which are almost perfect for the ground state of $k=1$, but somewhat lower for the excited states. To avoid finite volume effects we increase the lattice in the $\hat{1}$ and $\hat{2}$ directions for the shorter strings [7]. Stage-I is studied for $N = 3, 4, 6, 8$, and $1.3-1.6 \lesssim L\sqrt{\sigma} \lesssim 3-6.2$, while in stage-II we study $SU(2)$ as well and restrict to $L\sqrt{\sigma} \gtrsim 3-3.5$ with $0.1 \lesssim a\sqrt{\sigma} \lesssim 0.75$. The raw data will be presented in [8].

3. Results - dependence of the string mass on its length

We first check the universality class of the string by looking at the behaviour of an effective central charge defined as $\frac{m(L)}{L} \equiv \sigma - \frac{\pi C_{\rm eff}(L)}{6L^2}$. Here $C_{\rm eff}(L)$ should be 1 at $L \gg 1/\sqrt{\sigma}$ for the bosonic string, and is obtained from our data by performing fits to successive pairs of adjacent points. The results are shown in Fig. 1 where we see that for $\sqrt{\sigma}L \gtrsim 3$ the charge $C_{\rm eff}$ becomes consistent with 1.

Next we fit our data to the general form[2]

$$\frac{m(L)}{L} = \sigma - \frac{B}{L^2} - \frac{C}{(\sigma L^2)L^2} - \frac{D}{(\sigma L^2)^{3/2}L^2} - \frac{E}{(\sigma L^2)^2 L^2}. \tag{3.1}$$

Here we focus on the following fits. First we fix the $O(1/L^2)$ to be the Lüscher term with $B = \pi/6$ and (i) let C be a free fit parameter but fix $D = E = 0$ (ii) follow the theoretical predictions [5] and

[2] The $1/L^3$ term is missing since it was unacceptable fit for our data, and was suggested by Lüscher and Wiesz to be disfavoured theoretically [5].

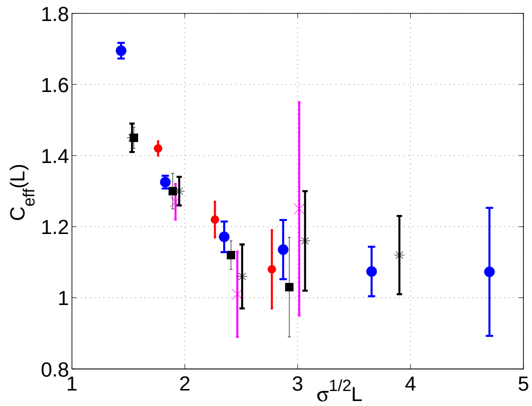

Figure 1: Effective central charge as a function of the string length, for $SU(3), \beta = 14.7172$ (blue circles), $SU(4), \beta = 28.00$ (red dots), $SU(6), \beta = 59.40, 90.00$ (black stars and squares), and $SU(8), \beta = 108.00$ (pink crosses).

constrain $C = \pi^2/72$ while fitting with $D \neq 0, E = 0$ or $D = 0, E \neq 0$. We also compare our fits to what one obtains when one uses the Lüscher term alone $m_{\text{Luscher}}(L)/L = \sigma - \frac{\pi}{6L^2}$, or the Nambu-Goto formula $m_{\text{NG}}(L)/L = \sigma\sqrt{1 - \frac{\pi}{3\sigma L^2}}$ as obtained by Arvis [4]. (Here σ is the string tension obtained from our fit) As a demonstration, we present results for $SU(6)$ and $\beta = 90.00$ in the left panel of Fig. 2, where one can see that the Lüscher term is a good approximation for $\sqrt{\sigma}L \gtrsim 3$ while the Nambu-Goto prediction works remarkably well down to almost the deconfinement length, but not exactly.

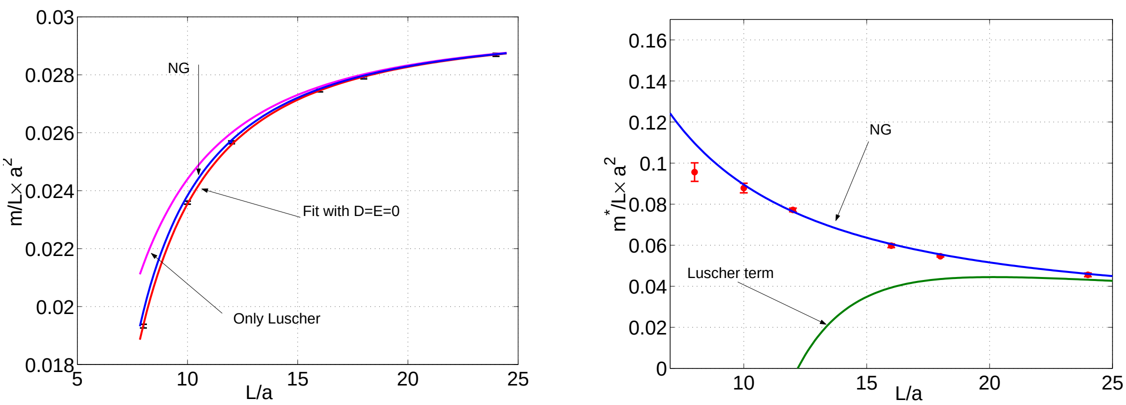

Figure 2: Left: $m(L)/L$ for the ground state closed string of $SU(6)$ at $\beta = 90.00$. The fit (red line) gives $a\sqrt{\sigma} = 0.172148(70)$. Substituting this in m_{Luscher} and m_{NG} gives the magenta and blue lines. Right: $m(L)/L$ for the first excited state of the $k = 1$ string. The lines are plots of m_{Luscher} and m_{NG} for a quantum $n = 1$ and σ as extracted from the ground state.

The results for all the other gauge groups are presented in Table 1, where one can see that the scaling and $1/N$ effects are relatively small. Our best fits in the case (i) and (ii) have comparable confidence levels of 43% – 90%, but a mild preference is seen toward option (ii). In apparent contrast to that, the coefficient C obtained in the fits of type (i) turns out to be about twice as large as predicted in [5]. Nonetheless, when we perform fits in which both C and either E are free (denoted as type (iii) in the last two columns in Table 1) we find that the data is almost consistent with $C \simeq \pi^2/72$ as [5] predicts.

Data set		fit (i), $D=E=0$		fit (ii), $C=\pi^2/72$		fit (iii), $D=0$	
N	β	$a\sqrt{\sigma}$	$\frac{C}{\pi^2/72}$	D	E	$\frac{C}{\pi^2/72}$	E
3	14.7172	0.261157(82)	1.96(8)	0.230(6)	0.299(8)	1.32(12)	0.23(3)
4	28.00	0.25198(16)	2.26(8)	0.23(4)	0.36(2)	1.36(32)	0.25(9)
6	59.40	0.27870(12)	1.91(6)	0.17(1)	0.25(2)	1.45(23)	0.12(6)
6	90.00	0.172148(70)	1.85(6)	0.16(1)	0.22(2)	1.21(24)	0.16(6)
8	108.00	0.27402(34)	1.58(2)	0.13(4)	0.19(6)	0.77(32)	0.28(9)

Table 1: The results of our fits of type i-iii (see text). The values of $a\sqrt{\sigma}$ for fits of type ii,iii were consistent mostly within 1.5σ with those for type i, which are presented in the table.

Finally we can use the value of σ obtained from the ground state of the string, and substitute it in the Lüscher and Nambu-Goto formulas, but with a quantum $n=1$ (see for example the work of Arvis [4]). We compare the result to the mass of the first excited state that we measure. The comparison is presented in the right panel of Fig. 2. We see that the Lüscher formula does not describe the data at all, but that, surprisingly, the full Nambu-Goto describes the data quite well. This seems to suggest that it is more natural to fit the data for m^2 rather than for m, since the former has the full Nambu-Goto expression as a zeroth approximation. An analysis in this form will be presented in a forthcoming publication [8].

To conclude this section we find that the systematic error induced by assuming $C=D=E=0$ for strings with $\sqrt{\sigma}L \simeq 3$ is to underestimate the string mass by about a third of a percent. In the next section we use this result and extract string tensions using the form Eq. (3.1) with $B=\pi/6$, $D=E=0$ and C as given by the fits to the different gauge groups (we assume that the scaling violations in C are small, as observed for $SU(6)$ - see Table 1).

4. Results - extrapolation of string tensions to the continuum

Equipped with the fitting formula for $m(L)$ we extract string tensions from a series of mass measurements where we keep the physical length of the string to be $L\sqrt{\sigma} \gtrsim 3-3.5$ and change the lattice spacing by changing β. We perform a continuum extrapolation with the ansatz

$$\beta_{MF}\frac{a\sqrt{\sigma}}{2N^2} = \left(\frac{\sqrt{\sigma}}{g^2N}\right)_{\text{continuum}} - \frac{b_0}{\beta_{MF}} - \frac{b_1}{\beta_{MF}^2}, \tag{4.1}$$

Here, β_{MF} is the mean field improved coupling $\beta_{MF} = \beta \cdot \langle u_p \rangle$, for which we measure the expectation value of the lattice plaquettes $\langle u_p \rangle$ at each β. Fitting with Eq. (4.1) is the way we remove the systematic error related to the $O(g^4)$ terms that we mention in the introduction. As a demonstration we present the results for $SU(4)$ and $SU(6)$ in the left panel of Fig. 3 where it can seen that $b_1 > 0$ and therefore that the removal of this systematic error increases the estimate of the continuum string tension. Nonetheless, as seen in the Figure, the extrapolation of the string tension are still lower compared to the value predicted by Karabali, Kim, and Nair (KKN) for these gauge groups.

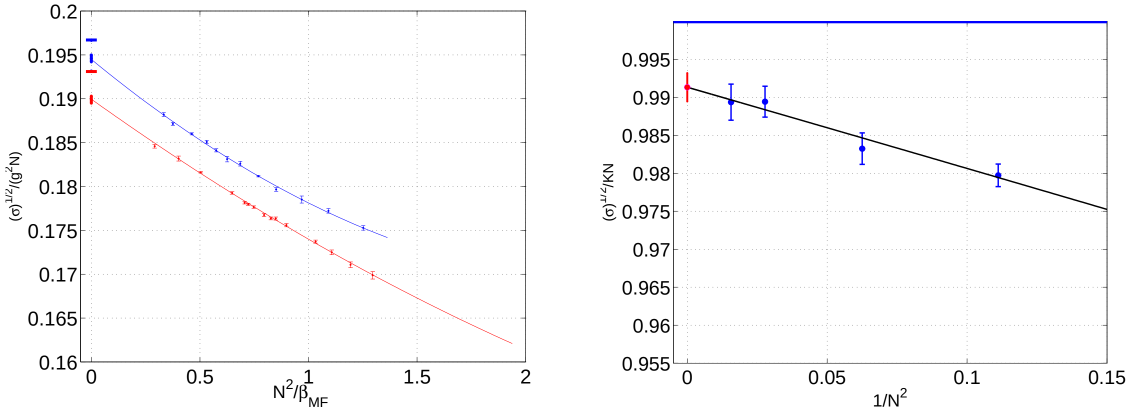

Figure 3: Left: The dimensionless quantity $\beta_{MF}\frac{a\sqrt{\sigma}}{2N^2}$ as a function of the improved coupling $1/\beta_{MF}$ for $SU(4)$ (lower plot, in red) and $SU(6)$ (higher plot, in blue). The error bars at $1/\beta_{MF}=0$ denote the result of the continuum extrapolation, while the horizontal bars denote the values predicted by Karabali, Kim, and Nair [2]. Right: The ratio between the prediction of [2] and our data, r, as a function of $1/N^2$, for $N=3,4,6,8$. The error bar at $1/N^2=0$ denotes the extrapolation to the $N=\infty$ limit, while the horizontal line at $r=1$ denotes where the prediction of [2] exactly matches the lattice result.

In the right panel of Fig. 3 we present the ratio $r\equiv\frac{(\sqrt{\sigma}/g^2N)_{\rm KKN}}{(\sqrt{\sigma}/g^2N)_{\rm Lattice}}$ between the Karabai-Kim-Nair prediction [2] and the lattice data (the numerical values are given in Table 2).

5. Summary

We measure string masses in $SU(N)$ gauge theories with $N=2,3,4,6,8$ in $2+1$ dimensions using lattice techniques. Here we present the results of two studies performed with these masses. In the first we investigate the behaviour of the closed string masses as a function of their length at a fixed lattice spacing. We find that for $\sqrt{\sigma}L\gtrsim 3$ our data is consistent with a Lüscher term of a unit central charge. For shorter strings the Lüscher term is insufficient and we find that the Nambu-Goto prediction works surprisingly well, but is not exact. We fit the deviations from the Lüscher term and compare the result to the recent theoretical predictions [5], which seem to be mildly supported by the data. We estimate that the systematic error induced by neglecting these sub-leading terms at $L\sqrt{\sigma}\simeq 3$ is to underestimate the string masses by about a third of a percent.

In the second study we calculate string tensions by restricting to string lengths of $\sqrt{\sigma}L\gtrsim 3-3.5$ and change the lattice spacing. Using the fit from the first study we extract the string tensions and extrapolate to the continuum limit. The results are given in Table 2, where we also give the Karabali-Kim-Nair (KKN) predictions. As indicated in the table, when we extrapolate to $N=\infty$ we find that the lattice result is lower by about 0.88% then the prediction of [2] which is a 4.5σ effect.

6. Future prospects

An additional systematic error is related to the contamination from excited states, which we take partially into account by fitting correlation functions with a single exponential at large t. Elim-

	$N=2$	$N=3$	$N=4$	$N=6$	$N=8$	$N=\infty$
KKN	0.17275	0.1881	0.1931	0.1967	0.19791	0.19947
Lattice	0.16678(42)	0.18425(28)	0.1898(4)	0.1946(4)	0.19580(47)	0.1977(4)

Table 2: $k=1$ string tensions for $SU(N)$ pure gauge theories in the continuum and the predictions of Karabali, Kim, and Nair (KKN) [2].

inating this effect will tend to give a lower string tension, which would *increase* the discrepancy with Eq. (1.1). Also, since in the case of k-strings our operators have typically a lower overlap than in the case of $k=1$, it is possible that one will obtain a significantly lower k-string tension there. This may bring the results numerically much closer to Casimir scaling, which may have interesting theoretical implications, and an initial analysis of this issue will be present in [8]. Other directions of research include a detailed spectrum calculation of excited string states and of glueballs [9] with a larger basis of operators that could be compared with recent theoretical predictions [10]

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