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The Z(2) gauge model revisited: as a possible testbed for the confinement and chiral symmetry phase transition of SU(2) lattice gauge theory

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Adopting the cooling technique to smooth the discontinuous $Z(2)$ lattice gauge field, we found that on $SU(2)$ gauge configurations obtained by this smoothing there exists clear discontinuity of the topological property at almost the same point as the confinement-deconfinement phase transition of the original $Z(2)$ gauge theory. This observation suggests the possibility that $Z(2)$ gauge model might be a testbed for analyzing the relation between the confinement and the chiral phase transition in which the topological objects are believed to play crucial roles.

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1. Introduction

Both color confinement and chiral symmetry breaking are the important non-perturbative aspects of *QCD* and are believed to have an strong connection with the gauge field topology [1, 2]. For the mechanism of color confinement, abelian degrees of freedom which obtained after the gauge fixing [3], seems to play a crucial role through the dual Meissner effect [4, 5, 6, 7]. For example in the maximal abelian gauge [8], the abelian dominance has been found [9, 10], in which the string tension can be almost reproduced by abelian field only. Although the reproducibility is not perfect [11], this feature is called as "abelian dominance" for confinement. Not only this abelian dominance but also "monopole dominance" was noticed [12, 13]. There is a evidence for the condensation of monopoles in the confinement phase [14, 15]. Furthermore the effective action was successfully constructed by the monopole field in which the string tension can be reproduced by monopole contributions only [16]. In the case above the topological object is "abelian monopole".

There is also another candidate that seems play crucial role for confinement. This is the center of the gauge group. For $SU(2)$ case, the center is $Z(2)$. Just like as "abelian dominance", "center dominance" is also observed. The string tension constructed from only $Z(2)$ variables carries almost part of $SU(2)$ string tension [17]. In this case, the topological object is "center vortex". [18]

If the topological objects mentioned above, abelian monopole and center vortex, play significant role for confinement, these two should be unified. There is an argument along this direction, but the final conclusion has not been performed yet [19, 20, 21].

For the chiral symmetry breaking, instanton is expected to play important roles. Instanton is related to the axial $U(1)$ anomaly [23, 24], and is also associated with a zero mode of the Dirac operator [1, 25]. There is a lattice data which suggests the local correlations between the topological charge and the chiral condensate [26].

Finite temperature lattice gauge simulations suggest that the color becomes deconfined and chiral symmetry is restored at the same critical temperature [27]. This might suggest that these two aspects of *QCD*, confinement and chiral symmetry breaking, can be explained in a unified way.

It is noted that the correlation between monopole and chiral symmetry breaking was observed in the maximal abelian gauge [28].

On the other hand for center degrees of freedom, it is proved by a numerical experiment that the center vortices are responsible for confinement and chiral symmetry breaking [29, 30]. When the center vortices are removed from $SU(2)$ configurations, confinement is lost and chiral symmetry is restored. Recently the importance of center has been accumulated [18].

Is is stressed that it is very hard to investigate the topology directly on center projected configurations because of the inherent discontinuity of $Z(2)$ link variables [30].

2. $Z(2)$ gauge model as an effective model of $SU(2)$

The success of center degrees of freedom, in which physical observables in $SU(2)$ such as the string tension can be well reproduced by $Z(2)$ variables only, implies the possibility that there

exists the effective action $S_{eff}(Z_2)$ by which the confinement and chiral symmetry breaking can be explained. It is highly expected the appearance of such effective action [29].

Suppose $\rho^C(\{Z_2\})$ is the distribution function obtained after the center projection of the original $SU(2)$ configurations. If we assume the shape of the effective action of $Z(2)$: $S^{trial}_{eff}(Z_2)$, using free parameters, we can ideally tune these parameters in $S^{trial}_{eff}(Z_2)$ such that the distribution generated by $S^{trial}_{eff}(Z_2)$ satisfies, $\rho^C(\{Z_2\}) \propto exp(-S^{trial}_{eff}(Z_2))$.

If this can be successfully applied, $S^{trial}_{eff}(Z_2)$ is nothing but $S_{eff}(Z_2)$.

It looks very important to tackle this problem, however, we do not go further in this letter.

Instead we will revisit the $Z(2)$ gauge model which was extensively studied as the effective theory of confinement [31, 32, 33, 34, 35].

The action of the 4 dimensional $Z(2)$ gauge theory can be written as (we adopt the simple plaquette action), $S_{Z_2} = -\beta \sum z_\mu(n) z_\nu(n+\mu) z_\mu(n+\nu) z_\nu(n)$.

where $z_\mu(n)$ is $Z(2)$ link variable (1 or -1) defined on the site n having the direction μ. The sum is over all plaquettes. This system has a phase transition at $\beta_c \simeq 0.44$, the confinement phase is at $\beta < \beta_c$ whereas the deconfinement phase is at $\beta > \beta_c$.

Let us suppose that the $Z(2)$ gauge model is the one obtained from $SU(2)$ gauge configurations after the center projection, *i.e.* $S_{eff}(Z_2) \simeq S_{Z_2}$ If this is the case, there should be remnants of $SU(2)$ gauge system in $Z(2)$ gauge configurations generated by S_{Z_2}.

We know, strictly speaking, this is not the case, however, it is very important to check whether there is a similarity between $S_{eff}(Z_2)$ and S_{Z_2} or not.

It is stressed that the center projected configurations have strong connection with the gauge field topology [29]. If there might be remnants of $SU(2)$ gauge system in the $Z(2)$ gauge model, we can see the topological remnants also in the $Z(2)$ gauge model.

The main purpose of this paper is to investigate the existence of the topological remnants in $Z(2)$ gauge model and it's correlation with the phases of $Z(2)$.

As noted in ref. [30], in order to investigate the topology in the discrete $Z(2)$ gauge configurations, we need to smooth the discontinuous $Z(2)$ variables.

3. Smoothing the $Z(2)$ by Metropolis type cooling

It is obvious that the discontinuity of center projected $Z(2)$ variables originates from the center projection, *i.e.* $SU(2) \to Z(2)$.

Smoothing is expected to do the reverse of the projection above,

$Z(2) \to SU(2)$,

such that it should not create any additional disorder keeping long range topological properties.

In other words, the center projection can be regarded as the removal of the off-diagonal part. In this sense, the smoothing is the creation or introduction of the off-diagonal elements from the diagonal $Z(2)$ variables.

It is well known that the cooling removes the short range disorder preserving the non-perturbative part of the configuration, and is successfully applied to extract the topological charge on the lattice. [36, 37, 38, 39].

For this reason we adopt the cooling technique to smooth the $Z(2)$ variables.

In general, cooling is performed by the successive local minimization of action. In $SU(2)$ case local minimization is realized by replacing the link $U_\mu(n)$ by $U_\mu^{new}(n)$,

$$U_\mu^{new}(n) = \sum_{\nu\neq\mu} U_\nu(n)U_\mu(n+\nu)U_\nu^\dagger(n+\mu)/k, \tag{3.1}$$

k is a normalization factor such that $U_\mu^{new}(n) \in SU(2)$.

In this heatbath type cooling, action will maximally decrease locally. So it is far from the smoothness we want.

Furthermore it is obvious that this procedure can not create any off-diagonal part from diagonal matrix.

Instead we adopt Metropolis type cooling in which the new trial link is defined as,

$$U_\mu^{new}(n) = R\,U_\mu(n), \;\; R \equiv \frac{I + i\vec{r}\cdot\vec{\sigma}}{\sqrt{1+|\vec{r}|^2}}, \tag{3.2}$$

where I is a unit matrix and $\vec{r}$ is a random vector with small length ($|\vec{r}|^2 \leq \varepsilon$) such that the $SU(2)$ matrix R should distribute around unit matrix to ensure the smoothness of this procedure. We accept new link $U_\mu^{new}(n)$ iff action decreases.

Smoothing is defined as follows:

Let $\{z\}$ is thermalized $Z(2)$ configuration and $z_\mu(n)$ denotes a link variable on $\{z\}$.

(1) Smoothing starts setting $SU(2)$ link variable $U_\mu(n)$, $U_\mu(n) \leftarrow z_\mu(n)\, I$, for all n and μ, and then,

(2) apply cooling to the $SU(2)$ configuration $\{U\}$.

It is noted that we do not apply cooling directly to $Z(2)$ gauge theory.

4. Lattice calculation of the topological charge

We prepare well thermalized $Z(2)$ lattice configurations of size 16^4 at $\beta = 0.2, 0.3, 0.4, 0.43$ (confinement phase), $0.45, 0.5, 0.6$ (deconfinement phase). At each β, 400 configurations separated 500 updating sweeps are used. For each configuration, smoothing by cooling technique is applied up to 500 smoothing sweeps measuring simultaneously the topological charge Q defined as, $Q = \frac{1}{32\pi^2}\sum \varepsilon_{\mu\nu\rho\sigma} tr[P_{\mu\nu}(n)P_{\rho\sigma}(n)]$,

where $P_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\mu)U_\mu^\dagger(n+\nu)U_\nu^\dagger(n)$.

The smoothing parameter ε is set to $\varepsilon = 0.03$.

Fig.1 shows the typical smoothing history of Q. For the early stage of smoothing, Q is not stable reflecting the discontinuity coming from $Z(2)$. On the other hand as smoothing goes on Q becomes stable.

For the almost configurations having non-vanishing Q, we observed that Q becomes stable after several hundred smoothing sweeps. So we adopt the value Q at 500 smoothing sweeps as the value of the topological charge.

We have checked that the result is almost the same for smaller ε ($\varepsilon \leq 0.1$), whereas the convergence of Q gets worse for larger ε ($\varepsilon \geq 0.1$) indicating the onset of disorder in eq.6.

The technical details about the calculation will be published elsewhere.

Fig.2 shows $< Q^2 >$ at various β. Clear discontinuity of $< Q^2 >$ can be seen around β_c which is the critical β of $Z(2)$ gauge theory. $< Q^2 >$ is finite for $\beta < \beta_c$ and $< Q^2 >$ is consistent to be 0 for $\beta > \beta_c$. This means that chiral symmetry is broken in the confinement phase whereas it is restored in the deconfinement phase.

This feature is very similar to that in $SU(2)$ gauge theory at finite temperature. The result shows that the existence of the topological remnant in $Z(2)$ gauge model as expected.

Compared with $SU(2)$, $Z(2)$ seems easy to handle, so the $Z(2)$ gauge model should be revisited as for the testbed to investigate the connection between confinement and chiral symmetry breaking.

5. Summary and Discussion

We found the existence of the topological remnants in $Z(2)$ gauge model. It is also observed that the topological nature changes drastically at the critical point of $Z(2)$ gauge model. $< Q^2 >$ is finite in the confinement phase and consistent to zero in the deconfinement phase.

Metropolis type cooling is introduced to smooth the discontinuous $Z(2)$ gauge variables and successfully applied to extract the topology.

Present result suggests that the $Z(2)$ gauge model with simple plaquette action might be the one obtained by center projection from $SU(2)$ preserving the non-perturbative nature of confinement and chiral symmetry breaking.

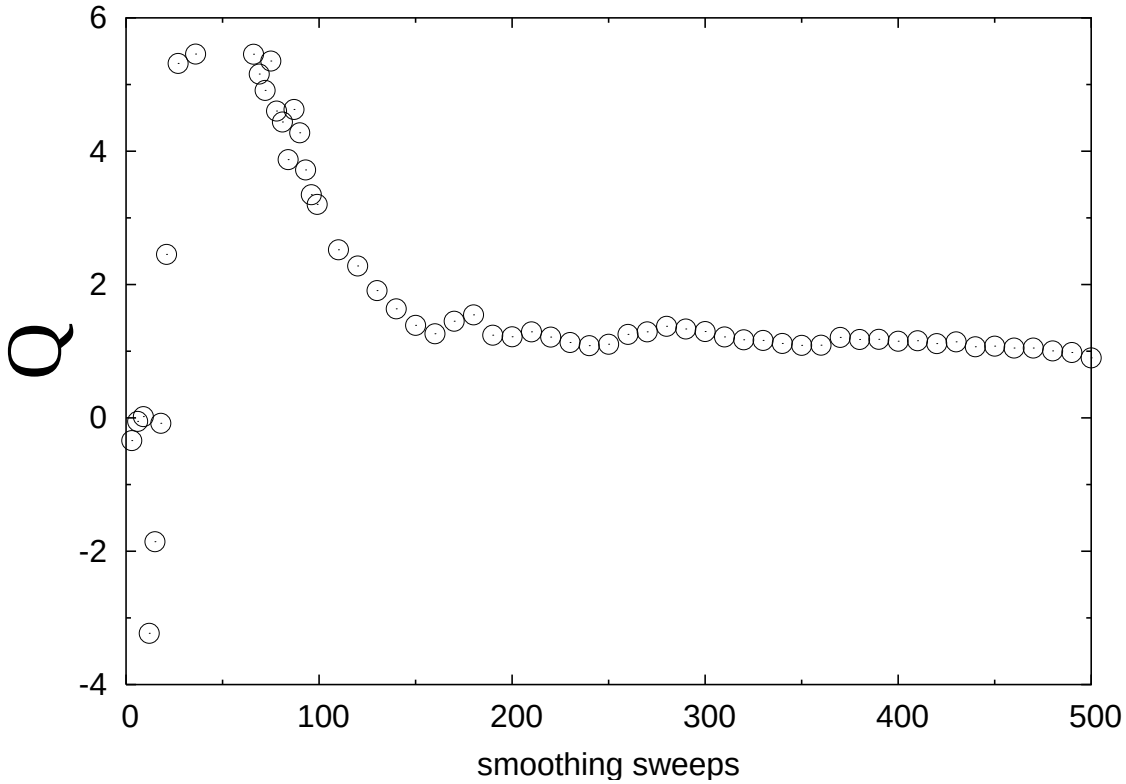

Figure 1: Topological charge Q as a function of smoothing sweeps. Q seems unstable for the early stage of smoothing reflecting the discontinuity of $Z(2)$ at the starting point.

It is very important to apply the appropriate smoothing, because the result obtained may depend on the way of smoothing. It is desirable to clarify the smoothing dependence or smoothing independence in near future.

Revisiting the $Z(2)$ and analyzing the nature of $Z(2)$ may reveal the non-perturbative nature of non-abelian gauge theories.

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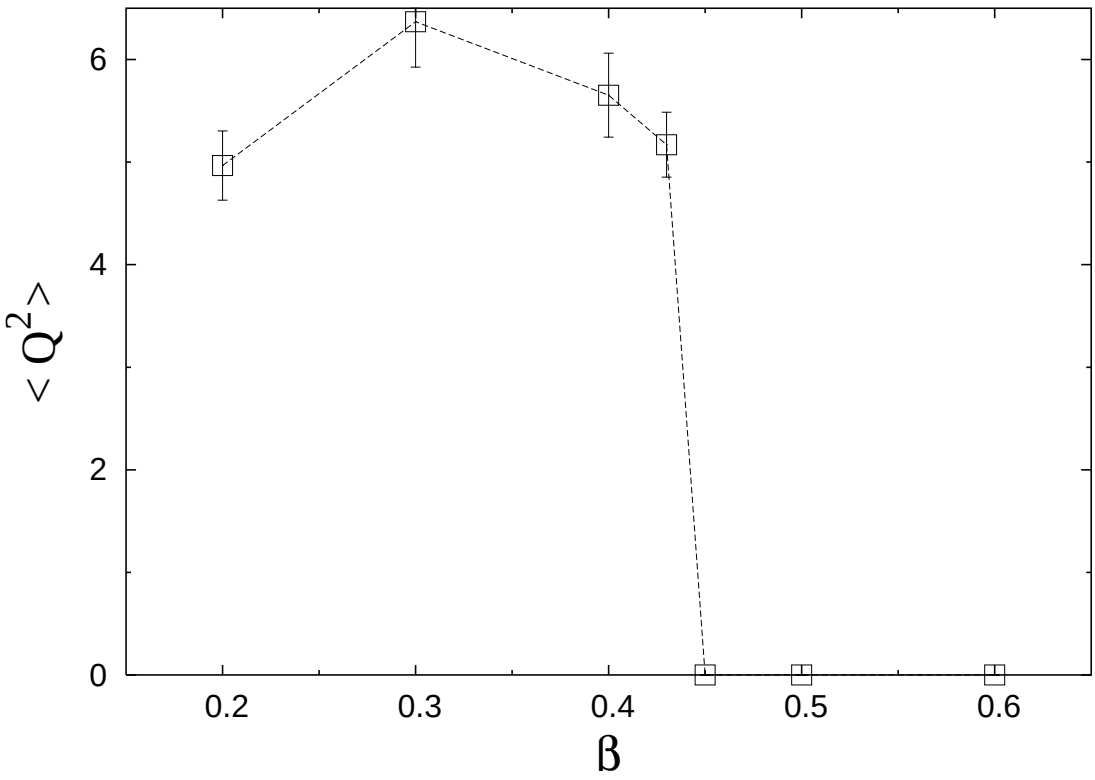

Figure 2: $< Q^2 >$ as a function of β. The phase transition point in the $Z(2)$ gauge model is $\beta_c \simeq 0.44$.

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