PROCEEDINGS OF SCIENCE

A study of the $B_s - \overline{B_s}$ mass and width difference in 2+1 flavor lattice QCD

R. T. Evans* **and A. X. El-Khadra**
Physics Department, University of Illinois, Urbana, Illinois, USA
E-mail: rtevans@uiuc.edu

M. Di Pierro

School of Computer Sci., Telecom. and Info. Systems, DePaul University, Chicago, Illinois, USA

Fermilab Lattice and MILC Collaborations

We present a preliminary calculation of the hadronic matrix elements relevant to $B_s - \overline{B_s}$ mixing. The calculation is done on MILC lattices with 2+1 sea quarks. We use the Asqtad action for the light valence quarks and the Fermilab action for the b quark.

XXIVth International Symposium on Lattice Field Theory
July 23-28, 2006
Tucson, Arizona, USA

*Speaker.

PoS(LAT2006)081

1. Introduction

Mixing in the $B_s - \overline{B}_s$ system is sensitive to the CKM matrix element V_{ts} and can help to constrain CP violating effects. It is the mass and width difference (Δm_s and $\Delta\Gamma_s$) between the mass eigenstates of this system that are measurable.

The $B_s - \overline{B}_s$ meson system's mass difference, Δm_s, is parametrized via an effective hamiltonian as

$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B(\mu_B) S_0(m_t, m_W) |V_{ts} V_{tb}^*|^2 \langle \overline{B}_s | Q(\mu_B) | B_s \rangle, \tag{1.1}$$

where η_B is a Wilson coefficient and $S_0(m_t, m_W)$ is known as the Inami-Lim function, and the scale $\mu_B \approx m_B$.

The hadronic matrix element is conventionally parametrized as

$$\langle \overline{B}_s | Q | B_s \rangle = \langle \overline{B}_s | \overline{b} \gamma_\mu (1-\gamma_5) s \overline{b} \gamma^\mu (1-\gamma_5) s | B_s \rangle = \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{B_s}. \tag{1.2}$$

f_{B_s} is the decay constant of the B_s meson, and B_{B_s} is the bag parameter.

The difference in decay rates of the eigenstates in the neutral B_s meson system is another measurable quantity which is sensitive to CP violation. The width difference

$$\Delta\Gamma_{B_s} = \frac{G_F^2 m_b^2}{6\pi^2 m_{B_s}} |V_{cb}^* V_{cs}|^2 [F(\frac{m_c^2}{m_b^2}) \langle \overline{B}_s | Q | B_s \rangle + F_S(\frac{m_c^2}{m_b^2}) \langle \overline{B}_s | Q_S | B_s \rangle][1 + O(\frac{\Lambda_{QCD}}{m_b})] \tag{1.3}$$

is determined by two hadronic matrix elements at the leading order in the heavy quark expansion. Q is the familiar operator from the mass difference, and Q_S is parametrized in terms of B_S or B'_S in a similar way

$$\langle \overline{B}_s | Q_S | B_s \rangle = \langle \overline{B}_s | \overline{s}(1-\gamma_5) b \overline{s}(1-\gamma_5) b | B_s \rangle = -\frac{5}{3} \frac{m_{B_s}^2}{(m_b + m_s)^2} m_{B_s}^2 f_{B_s}^2 B_S = -\frac{5}{3} m_{B_s}^2 f_{B_s}^2 B'_S. \tag{1.4}$$

Precise measurements of Δm_s have recently been made [7], yielding a determination of V_{ts} and a significant reduction of the allowed region in the $\rho - \eta$ plane due to this constraint [9]. The errors on V_{ts} are now completely dominated by theoretical uncertainties from lattice QCD calculations of $f_{B_s}^2 B_{B_s}$. $\Delta\Gamma_s$ has also already been measured and we can expect improved measurements by the end of the current Tevatron run [8]. The goal of this work is to use the unquenched MILC lattices for a precise determination of the above matrix elements in the B_s and B_d systems.

2. Lattice Parameters

We performed our calculations on the MILC coarse lattices ($a = 0.12\,fm$) with 2+1 sea quarks on 592 configurations. The sea quarks are simulated using the Asqtad improved action, where errors are introduced at $O(a^4, \alpha_s a^2)$. The valence light quark propagators were created using the Asqtad action, and the heavy b quark is handled using the Fermilab action, with errors starting at $O(a^2, \alpha_s a)$.

We used pre-existing staggered propagators of two valence quark masses, $m_q = 0.0415$ and 0.005. The first mass value is very close to the physical s quark mass and the second mass value is the closest available to the physical d quark mass, giving us a rough comparison between the

B_s and B_d systems. We used $\kappa_b = 0.086$ for the heavy quark. We performed the calculation for $m_q = 0.0415$ at two different time sources, whereas only one time source was used for $m_q = 0.005$. We used both a $1S$ wavefunction and a delta function to smear the heavy quark at the sink.

For tree level $O(\frac{\Lambda_{QCD}}{m_b})$ improvement of the operator we found that a rotation of the b quark via [1] is all that is necessary. In order to include $O(\alpha_s)$ effects additional six dimensional operators must be included. These additional matrix elements can be constructed from the open meson propagator described in Section 3.

3. The Open Meson Propagator

All possible 24 matrix elements that can be formed from the general 4-quark operator

$$\langle \overline{H}_q(x)|O_q^{\Delta h=2}|H_q(y)\rangle = \langle \overline{H}_q(x)|\overline{q}\Gamma_1 h\overline{q}\Gamma_2 h(0)|H_q(y)\rangle \tag{3.1}$$

can be calculated using only one inversion per quark flavor by placing the operator at the origin. After performing the Wick contractions and Fourier transforming Eq. (3.1) we obtain

$$\begin{aligned}\sum_{\vec{x},\vec{y}}\langle \overline{H}_q(x)|O_q^{\Delta h=2}|H_q(y)\rangle &= \mathbf{Tr}[\Gamma_1 E_{hq}(t_x)]\mathbf{Tr}[\Gamma_2 E_{hq}(t_y)] + \mathbf{Tr}[\Gamma_1 E_{hq}(t_y)]\mathbf{Tr}[\Gamma_2 E_{hq}(t_x)]\\ &+\mathbf{Tr}[\Gamma_1 E_{hq}(t_x)\Gamma_2 E_{hq}(t_y)] + \mathbf{Tr}[\Gamma_1 E_{hq}(t_y)\Gamma_2 E_{hq}(t_x)]\end{aligned} \tag{3.2}$$

where the traces are over color and spin indices. The open meson propagator

$$E^{ab}_{hq,ij}(t_x) = \gamma_5^{ac} G^{*dc}_{h,ki}(t_x,0) G^{db}_{q,kj}(t_x,0) \tag{3.3}$$

is all that is needed to construct the matrix element. This object is very small in size and can easily be saved and later used to construct all two-point and three-point functions necessary for our calculation [4].

4. Matrix Element Extraction

The mixing matrix element, Eq. (1.2), is extracted from the three-point function

$$C_Q(t_1,t_2) = \sum_{\vec{x},\vec{y}} \langle \overline{b}(\vec{x},t_1)\gamma_5 q(\vec{x},t_1)[Q(0)]\overline{b}(\vec{y},t_2)\gamma_5 q(\vec{y},t_2)\rangle. \tag{4.1}$$

The correlation function has naive valence quarks which contain doublers that cause higher energy 0^+ states to contribute. As can be seen in Fig. 1, these states oscillate in Euclidean time and make a significant contribution to the correlation function [2].

Fig 2. depicts the ratio of the three-point function and two-point functions

$$R(-t_1,t_2) = \frac{3}{8}\frac{C_Q(-t_1,t_2)}{C_{A4}(-t_1)C_{A4}(t_2)}, \tag{4.2}$$

where

$$C_{A4}(t) = \sum_{\vec{x}} \langle \overline{b}(\vec{x},t)\gamma_5 q(\vec{x},t)\overline{q}(0)\gamma_0\gamma_5 b(0)\rangle. \tag{4.3}$$

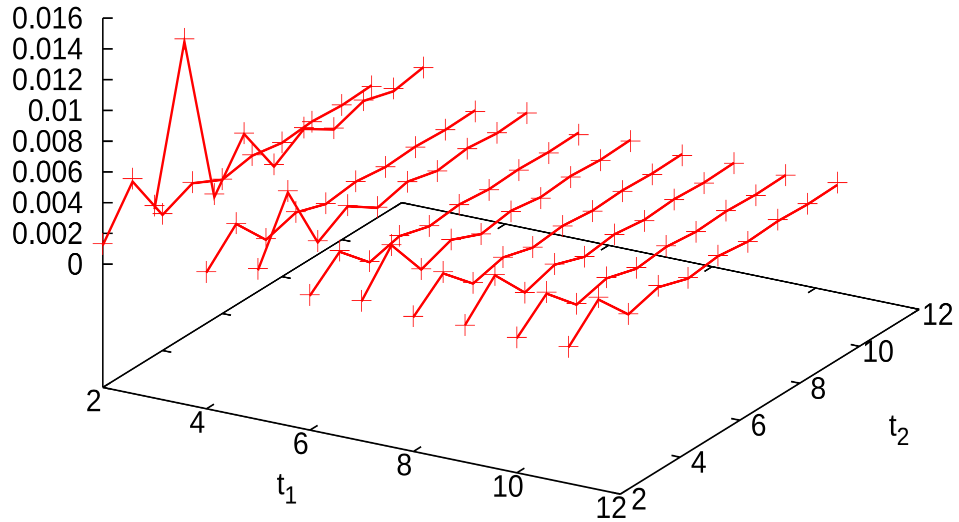

Figure 1: $C_Q(t_1,t_2)e^{m_0t_1+m_0t_2}$ with delta function sink, m_0 =ground state mass. Crosses are data and lines are the best fit to the data.

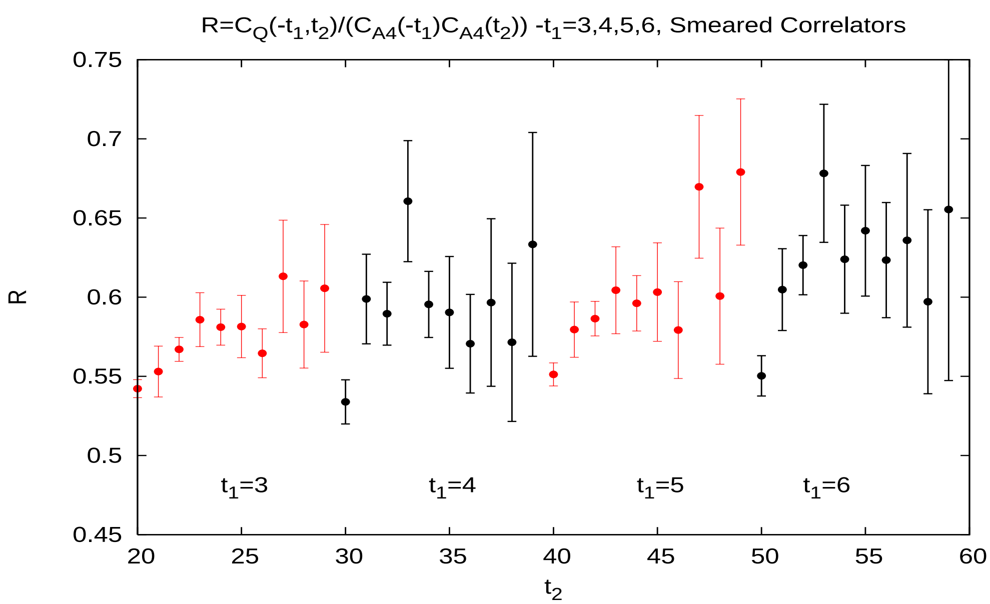

Figure 2: B is placed at the source and $\overline{B}$ at the sink, $t_2 = 2-11$.

In the limit $-t_1, t_2 \to \infty$ R becomes the bag parameter

$$B_{B_q} = \frac{3}{8}\frac{\langle \overline{B_q}|\overline{b}\gamma_\mu(1-\gamma_5)q\overline{b}\gamma^\mu(1-\gamma_5)q|B_q\rangle}{m_{B_q}^2|\langle b\gamma_0\gamma_5 q|B_q\rangle|^2}, \tag{4.4}$$

which we would hope to see as a plateau in Fig. 2. The oscillating states make the clear identification of a plateau in the ratio and fitting to it difficult. We are examining other ratios to determine the possibility of fitting to these, but are currently fitting to C_Q directly.

In order to extract the matrix elements of interest we performed constrained fits [3] simultaneously to 3 correlation functions: the two-point functions

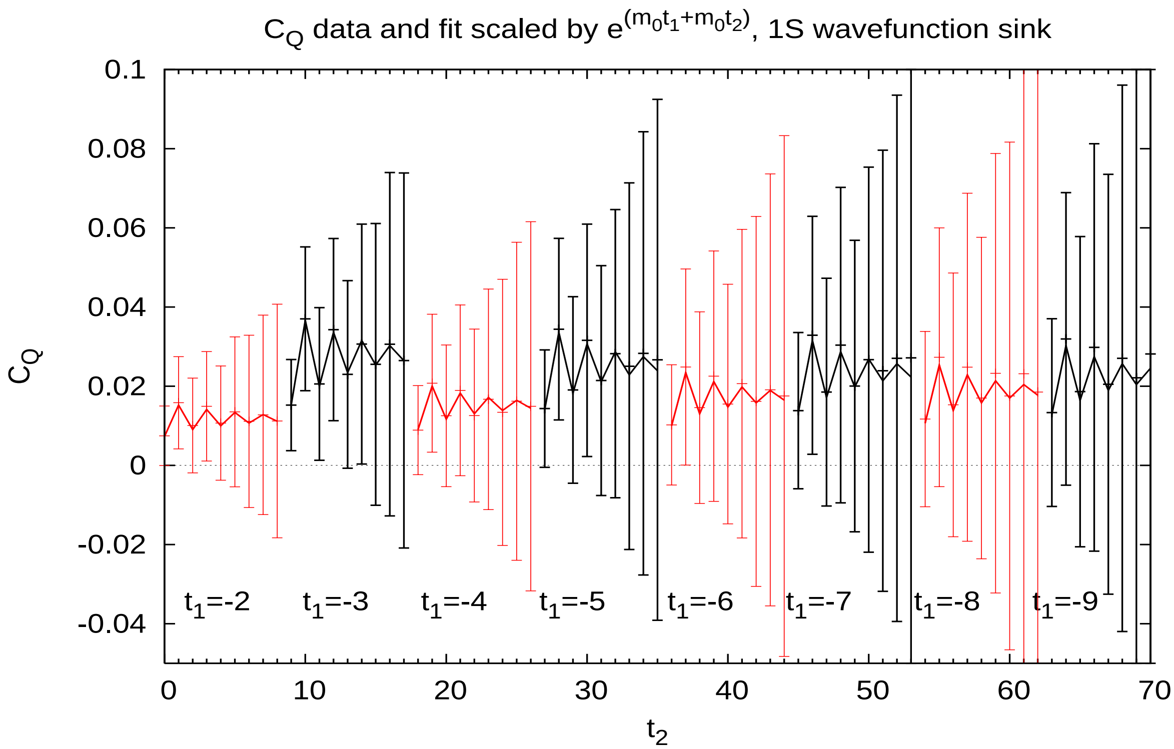

Figure 3: C_Q, 1S wavefunction sink, $t_2 = 2-11$

$$C_Z(t) \rightarrow_{t\rightarrow\infty} \frac{1}{2m_{B_q}} |\langle \bar{q}\gamma_5 b|B_q\rangle|^2 e^{-m_{B_q}t}, \quad C_{A_4}(t) \rightarrow_{t\rightarrow\infty} \frac{1}{2m_{B_q}} \langle \bar{q}\gamma_5 b|B_q\rangle \langle B_q|\bar{b}\gamma_0\gamma_5 q\rangle e^{-m_{B_q}t} \quad (4.5)$$

and three-point function

$$C_Q(-t_1,t_2) \rightarrow_{t_1,t_2\rightarrow\infty} \frac{1}{(2m_{B_q})^2} |\langle \bar{q}\gamma_5 b|B_q\rangle|^2 \langle \bar{B}|Q|B_q\rangle e^{-m_{B_q}t_1} e^{-m_{B_q}t_2}. \quad (4.6)$$

C_Z allows the overlap parameters in C_Q to be removed and the matrix element isolated. C_{A_4} is used to determine f_{B_q} and can be used to isolate B_{B_q}. The parameter most directly of phenomenological interest, $f_{B_q}\sqrt{B_{B_q}}$, can be extracted by combining just C_Z and C_Q.

In addition to the ground state other excited states contribute, in particular the opposite parity oscillating states arising from the naive valence quark. For our best fits to the data with a delta function sink we included the first 6 states (3 regular and 3 oscillating), with t_1 and t_2 taken over $t_{min} = 1$, $t_{max} = 12$, giving a $\chi^2 \approx 1.0$. The best fits using 1S smeared data were obtained by including the first 4 states from $t_{min} = 1$, $t_{max} = 11$, also resulting in a $\chi^2 \approx 1.0$. As illustrated in Figs. 1, 3, and 4 our fits give a reliable description of the data over almost the entire $t_1 - t_2$ plane. The parameter values extracted from these fits are listed in Table 1.

The fit results using different numbers of states, $2-8$, were typically consistent within 50% of the error bars of the best fit. The larger errors observed in the $m_q = 0.005$ fits are to be expected, as only one time source was used and statistical fluctuations of the data increase closer to the chiral

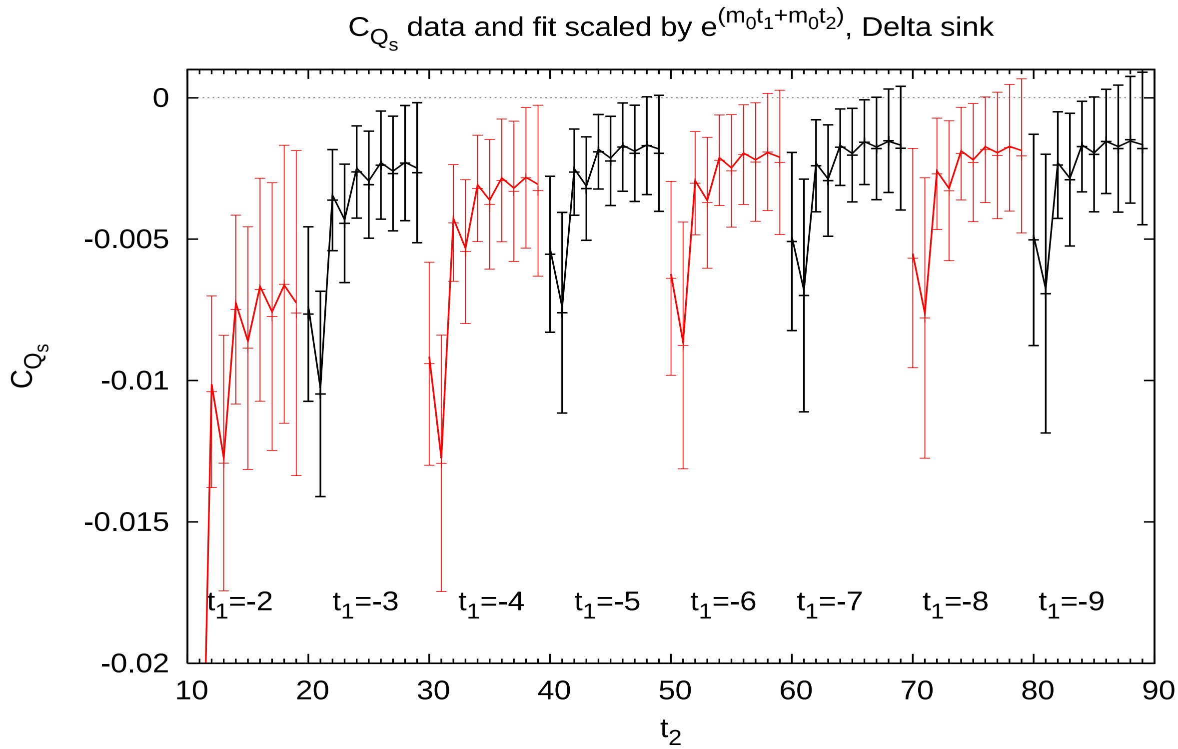

Figure 4: C_{Q_S}, delta function sink, $t_2 = 2 - 12$

limit. With more statistics and experience we hope to improve the robustness of the fits further. We calculated the matrix element of Q_S in an identical way, with similar results (see Fig. 4).

The errors reported in Table 1 are χ^2 errors from the fitting. The results do not include the renormalization coefficients or chiral extrapolations. The $m_q = 0.0415$ fit results are greatly improved by using smearing, with the errors being halved in some cases.

Smearing	m_q	B_{B_q}	$f_{B_q}\sqrt{B_{B_q}}$	B'_S	$f_{B_q}\sqrt{B'_S}$
delta	0.0415	0.62 +/- 0.06	0.160 +/- 0.006	2.44 +/- 0.15	0.329 +/- 0.007
	0.005	0.62 +/- 0.10	0.150 +/- 0.009	2.36 +/- 0.29	0.306 +/- 0.013
1S	0.0415	0.59 +/- 0.03	0.160 +/- 0.003	2.40 +/- 0.10	0.325 +/- 0.005
	0.005	0.69 +/- 0.13	0.144 +/- 0.010	2.64+/-0.39	0.274 +/- 0.014

Table 1: Smeared and unsmeared results in lattice units. The $m_q = 0.005$ results are derived from half the time sources of the $m_q = 0.0415$ results.

5. Summary and Outlook

The statistical uncertainties of this calculation are straightforward to reduce. Specifically, we plan to repeat the calculation on the same ensemble, but with more time sources. Improving the fitting procedure may also aid in reducing errors.

The calculation thus far is done with $O(a)$ improvement and only tree-level matching. We are planning to include the perturbative matching at one loop order at which point additional operators must be included. The three-point functions for the additional operators can easily be constructed from our stored open meson propagators, making the the inclusion of these operators straightforward once their coefficients have been calculated. The NLO operators in the $(\frac{\Lambda_{\rm QCD}}{m_b})$ expansion contribute significantly to $\Delta\Gamma_s$, and will also have to be calculated [6].

We are also planning to repeat this calculation on the available MILC ensembles for various sea quark/light valence quark masses and lattice spacings in order to observe the light quark mass and lattice spacing dependence of our results. A comparison of our $m_q = 0.0415$ and 0.005 results shows a mild dependence, although it should be stressed that the errors in the $m_q = 0.005$ fits are very large. With the full data set we plan to use staggered chiral perturbation theory to extract the parameters at the physical masses.

Acknowledgments

The numerical simulations for this work were carried out on the Fermilab lattice QCD clusters, which are a computing resource of the USQCD collaboration and are funded by the DOE. We are grateful to the Fermilab Computing Division for operating and maintaining the clusters. This work was supported in part by the DOE under grant no. DE-FG02-91ER40677.

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