PROCEEDINGS OF SCIENCE

Charm physics with highly improved staggered quarks

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We use a relativistic highly improved staggered quark action to discretize charm quarks on the lattice. We calculate the masses and the dispersion relation for heavy-heavy and heavy-light meson states, and show that for lattice spacings below .1 fm, the discretization errors are at the few percent level. We also discuss the prospects for accurate calculations at the few percent level of f_{D_s}, f_D, and the leptonic width of the ψ and ϕ.

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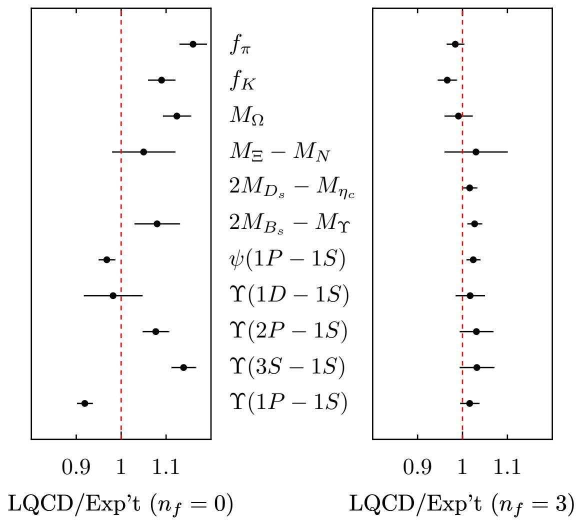

Figure 1: Comparison between quenched and unquenched simulation results.

1. Introduction

Improved staggered quarks have proved very effective in obtaining precise results of phenomenological interest in the unquenched light valence sector [1] (see figure 1 for a comparison of quenched and unquenched results.) On the other hand, non-relativistic effective field formulations are very successful in the bottom sector [2, 3]. Although a non-relativistic formulation can also be applied, in principle, to the charm sector, the errors are much larger.

Highly improved staggered quarks have very small discretization errors. Combining this with fine enough lattices may provide a good method of handling charm quarks.

CLEO-c is making precise measurements of several quantities (for example f_{D_s} f_D) in the charm system with small errors ($\approx 4\%$). For comparison, Fermilab results have errors of $\approx 8\%$ [4]. This provides a good opportunity to test our methods.

2. Improved Staggered Quarks

The massless one-link (Kogut-Susskind) staggered Dirac operator is defined as:

$$D(x,y) = \frac{1}{2au_0} \sum_{\mu=1}^{d} \eta_\mu(x) \left[U_\mu(x) \delta_{x+\hat{\mu},y} - H.c. \right] , \quad \eta_\nu(x) = (-1)^{\sum_{\mu<\nu} x_\mu} \tag{2.1}$$

with u_0 an optional tadpole-improvement factor.

This operator suffers from doubling: there are four “tastes” (non-physical flavours) of fermions in the spectrum, which couple through taste-changing interactions. These are lattice artifacts of order a^2, involving at leading order the exchange of a gluon of momentum $q \approx \pi/a$. Such interactions are perturbative for typical values of the lattice spacing, and can be corrected systematically a la Symanzik. By judiciously smearing the gauge field we can remove the coupling between quarks and high momentum gluons.

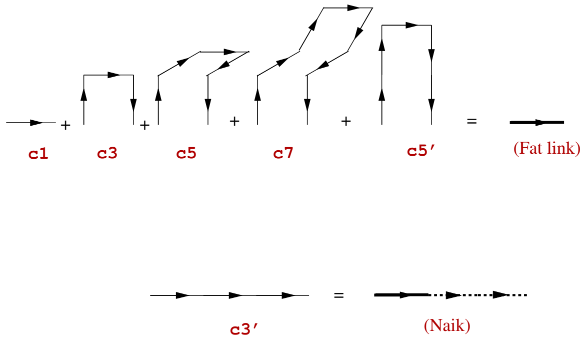

Figure 2: Paths used for smearing in the ASQTAD staggered action.

The most widely used improved staggered action is called ASQTAD, and removes all tree-level a^2 discretization errors [5, 6, 7]. The paths used to smear the gauge-fields are shown in figure 2.

The HISQ (highly improved staggered quarks) staggered Dirac operator involves two levels of smearing with an intermediate projection onto $SU(3)$. It is designed so that, as well as eliminating all a^2 discretization errors, it further reduces the one-loop taste-changing errors (for a more detailed discussion see [8].)

This action has been shown to substantially reduce the errors associated with the taste-changing interactions [8, 9, 10].

3. Charm Sector

When we put massive quarks on the lattice, the discretization errors grow with the quark mass as powers of am. Therefore to obtain small errors we would need $am \ll 1$. For heavy quarks this would require very small lattice spacings. On the other hand, to keep our lattice big enough to accommodate the light degrees of freedom, we need $La \gg m_\pi^{-1}$. The fact that we have two very different scales in the problem makes difficult a direct solution. What we can do instead is to take advantage of the fact that m is large, by using an effective field theory (NRQCD, HQET). This program has been very successful for b quarks [2, 3, 4].

The charm quark is in between the light and heavy mass regime. It is quite light for an easy application of NRQCD, but quite large for the usual relativistic quark actions, $am_c \lesssim 1$. However, if we use a very accurate action (HISQ) and fine enough lattices (fine MILC ensembles), it is possible to get results accurate at the few percent level.

4. Results

We use $2+1$-flavours unquenched configurations generated by the MILC collaboration [11, 12, 13]. We present here results obtained from an ensemble with $m_l = m_s/5$, where m_s is the light and m_s the strange quark mass, and with a lattice spacings of $1/11 fm$. The extent of the corresponding lattices is $28^3 x\, 96$.

The bare mass used for the charm valence quarks is fixed by adjusting the "Goldstone" pseudoscalar mass to the experimental value.

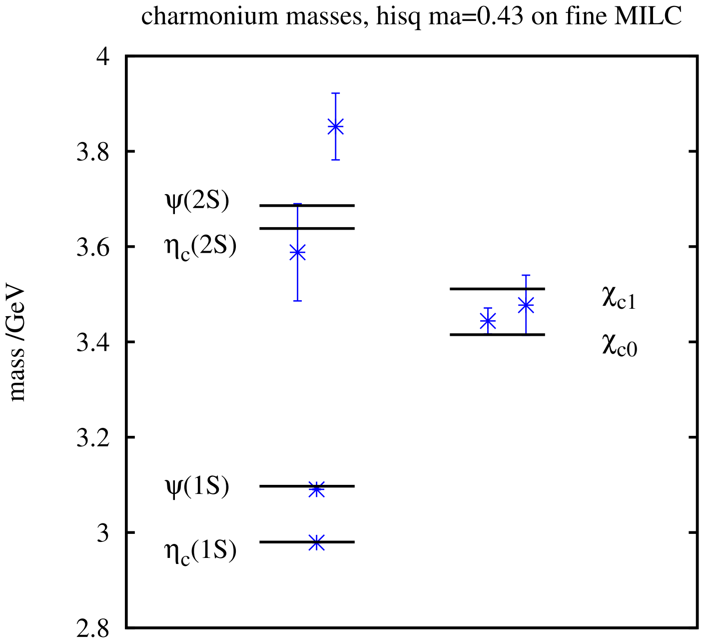

Figure 3: Charmonium spectrum on fine MILC configurations.

4.1 $c\bar{c}$ pseudoscalar and vector

We show in figure 3 some of our results for the charmonium spectrum, as a check of the formalism. We haven't optimized in any way our operators for the calculation of the excited states, which therefore have large errors.

In figure 4 we show the results for the pseudoscalar and vector mesons. In this sectors we have very small statistical errors. In the staggered formalism we have 16 different mesons of each type, with, in general, different masses, due to the taste-changing interactions. We can see that such mass splitting is mostly noticeable for the pseudoscalar states, but is almost negligible for the vector mesons, where it's below our statistical error. This is consistent with previous results [14], although the actual size of the splitting is very much reduced with respect to the one using one-link staggered fermions. The total splitting for η_c is only about 10 Mev for HISQ, and is also much smaller than the one for ASQTAD. We obtain an hyperfine splitting of $\approx 110 MeV$, to be compared with the experimental value of $117 MeV$.

4.2 Speed of Light

It is important to check that the discretization artifacts at the masses we use in our simulation do not spoil the relativistic invariance of the action (as is bound to happen at large enough mass.) One way to do this is by calculating the dispersion relation of a meson, or equivalently, that the squared speed of light is still 1 at small non-zero momenta.

In figure 5 we show the results for c^2 at a small non-zero momentum for pseudoscalar heavy-heavy and light-light mesons, where the heavy mass is set to the charm mass and the light one to the strange mass. We show the results for both HISQ and ASQTAD actions. c^2 is one at the strange

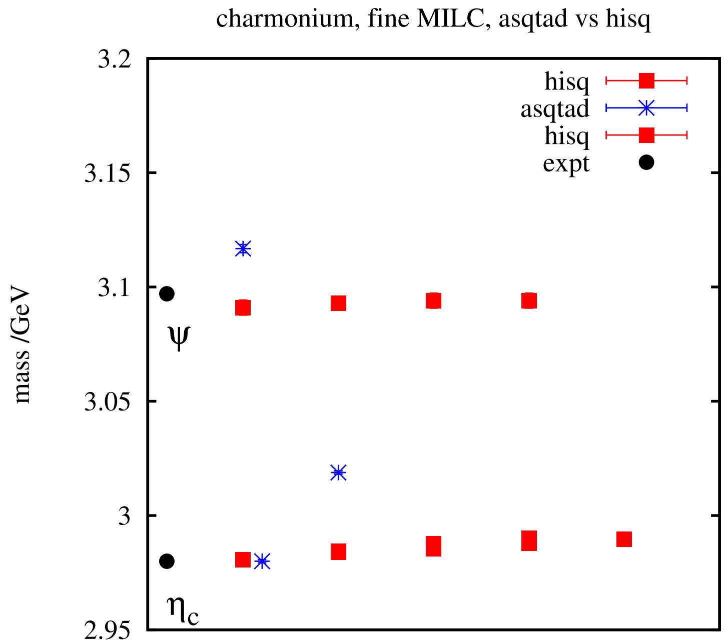

Figure 4: Charm pseudoscalar and vector states for HISQ and ASQTAD on fine MILC configurations.

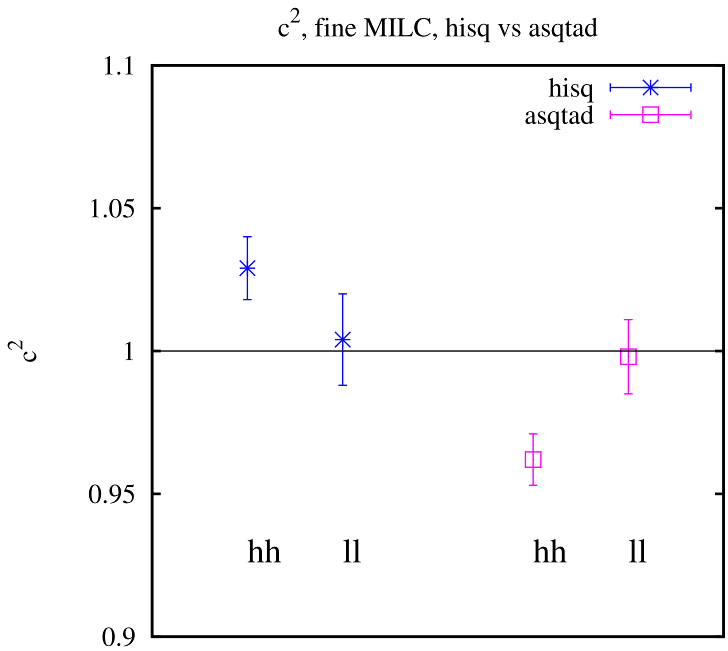

Figure 5: Speed of light squared for HISQ and ASQTAD on fine MILC configurations.

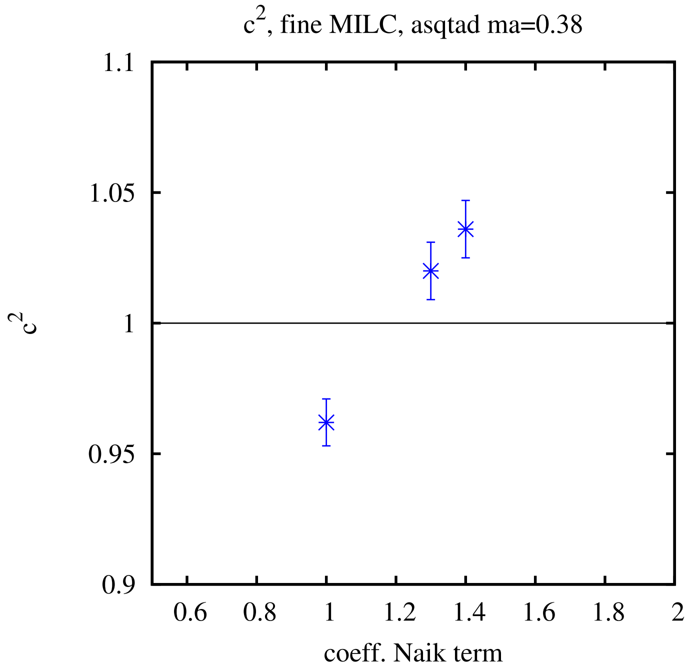

Figure 6: Speed of light squared as a function of the coefficient of the Naik term for ASQTAD on fine MILC configurations.

mass, but different from 1 at the charm mass, and that deviation is larger for the ASQTAD action than for the HISQ action.

This error can be corrected for by modifying the overall coefficient of the Naik term in the action. It can be shown that this simple modification to the action removes all discretization errors of order $\alpha_s(am)^2$, the largest ones remaining in our improved actions for large quark mass [8]. We plot in figure 6 the result of modifying such coefficient in c^2 for the ASQTAD action.

5. Conclusions and Outlook

The use of a highly improved quark action and fine enough lattices provides a very good way of studying the charmonium systems from first principles. We have shown that the discretization errors are well under control, and we can obtain precise results in the charm sector.

We are now using this formalism for the precision calculation of several interesting quantities, which can be checked against the new CLEO-c results. This includes f_{D_s} and f_D. We could also use this method to obtain leptonic decay widths $D \to \mu\nu_\mu$.

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