PROCEEDINGS OF SCIENCE

Kaon B-parameters for generic $\Delta S = 2$ four-quark operators in quenched domain wall QCD

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We present a study of B-parameters for generic $\Delta S = 2$ four-quark operators in domain wall QCD. Our calculation covers all the B-parameters required to study the neutral kaon mixing in the standard model (SM) and beyond it. We evaluate one-loop renormalization factors of the operators employing the plaquette and Iwasaki gauge actions. Numerical simulations are carried out in quenched QCD with both gauge actions on $16^3 \times 32 \times 16$ and $24^3 \times 32 \times 16$ at the lattice spacing $1/a \approx 2$GeV. We investigate the relative magnitudes of the non-SM B-parameters to the SM one, which are compared with the previous results obtained with the overlap and the clover quark actions.

XXIVth International Symposium on Lattice Field Theory
July 23-28, 2006
Tucson, Arizona, USA

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PoS(LAT2006)089

1. Introduction

The decay processes of the neutral kaon system contain rich physics in the SM and beyond it. The indirect CP violation parameter ε provides us a chance to examine the SM and make constraints on the physics beyond it. In order to determine ε we need the kaon matrix element of the effective Hamiltonian for $\Delta S = 2$ transition, whose general structure is written as $\langle \bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle$, where $H_{\rm eff}^{\Delta S=2} = G_F/\sqrt{2}\sum_i V_{\rm CKM}^i C_i(\mu)\mathscr{O}_i$ with G_F the Fermi constant, $V_{\rm CKM}^i$ the CKM factors, C_i the Wilson coefficients and $\mathscr{O}_i$ the relevant local operators for the process. Although it is known that only the left-left operator $\mathscr{O}_{\rm LL}$ is relevant in the SM, we need the operators with more general chiral structures for the physics beyond the SM. For example the supersymmetric model requires[1]

$$\mathscr{O}_1 = \bar{s}^a\gamma_\mu(1-\gamma_5)d^a\bar{s}^b\gamma_\mu(1-\gamma_5)d^b, \tag{1.1}$$

$$\mathscr{O}_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b, \tag{1.2}$$

$$\mathscr{O}_3 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a, \tag{1.3}$$

$$\mathscr{O}_4 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b, \tag{1.4}$$

$$\mathscr{O}_5 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a \tag{1.5}$$

and $\tilde{\mathscr{O}}_{1,2,3}$ obtained from the $\mathscr{O}_{1,2,3}$ by exchanging the left- and right-handed quarks. $\mathscr{O}_1$ is the same operator in the SM, while $\mathscr{O}_{2,3,4,5}$ and $\tilde{\mathscr{O}}_{1,2,3}$ are the non-SM operators.

In this report we present the results of $\langle \bar{K}^0|\mathscr{O}_i|K^0\rangle$ ($i = 1,\ldots,5$) obtained in quenched domain wall QCD. Although these operators are introduced by the supersymmetric model, we should note that $\langle \bar{K}^0|\mathscr{O}_i|K^0\rangle$ with $i = 1,\ldots,5$ are a complete set of the kaon matrix elements of the four-quark operators allowed by the symmetries. This implies that they are sufficient for analysis of any model beyond the SM. We are particularly interested in the relative magnitude of non-SM to SM matrix elements:

$$R_i(\mu) \equiv \frac{\langle \bar{K}^0|\mathscr{O}_i(\mu)|K^0\rangle}{\langle \bar{K}^0|\mathscr{O}_1(\mu)|K^0\rangle}, \quad i = 2,\ldots,5. \tag{1.6}$$

We also present the kaon B-parameter for the SM:

$$B_1(\mu) = B_K(\mu) \equiv \frac{\langle \bar{K}^0|\mathscr{O}_1(\mu)|K^0\rangle}{\frac{8}{3}\langle \bar{K}^0|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle}, \tag{1.7}$$

and those for the non-SM:

$$B_i(\mu) \equiv \frac{\langle \bar{K}^0|\mathscr{O}_i(\mu)|K^0\rangle}{C_i\langle \bar{K}^0|\bar{s}\gamma_5 d(\mu)|0\rangle\langle 0|\bar{s}\gamma_5 d(\mu)|K^0\rangle}, \quad i = 2,\ldots,5, \tag{1.8}$$

where $C_i \equiv \{\frac{5}{3}, -\frac{1}{3}, -2, -\frac{2}{3}\}$ are the convention factors introduced by the vacuum saturation approximation. Our results are compared with the previous works employing the overlap[2] and the clover quark actions[3].

2. Simulation details

2.1 Simulation parameters

In Table 1 we summarize the simulation parameters, part of which are the same as in the previous CP-PACS calculation of ε'/ε[4]. The domain wall fermion with the wall height $M = 1.8$ is

Table 1: Simulation parameters. P and R denote the expectation values for the plaquette and the 1×2 rectangular.

	plaquette	Iwasaki	
β	6.0	2.6	
$N_s \times N_t \times N_5$	$16^3 \times 32 \times 16$	$16^3 \times 32 \times 16$	$24^3 \times 32 \times 16$
a^{-1}[GeV]	2.12[5]	2.00[6]	
P	0.59374[9]	0.670632(10)[7]	
R	–	0.45283(2)[7]	
No. config.	180	400	200 (400 for $m_f a = 0.02$)
m_{5q}[MeV]	3.14(36)	0.292(44)	–

employed with the plaquette and the Iwasaki gauge actions in quenched approximation. We choose $N_s \times N_t \times N_5 = 16^3 \times 32 \times 16$ lattice at $\beta = 6.0$ for the plaquette gauge action, while $N_s = 16^3$ and 24^3 at $\beta = 2.6$ for the Iwasaki gauge action in order to investigate the volume dependence. As for the lattice spacing we refer to Refs.[5, 6] where the lattice spacing determined from the Sommer scale $r_0 = 0.5$fm is parameterized in terms of β. We take degenerate masses $m_f a = 0.02, 0.03, 0.04, 0.05, 0.06$ for the up, down and strange quarks in both gauge actions. For each value of $m_f a$ gauge configurations are independently generated with the algorithm of the five-hit pseudo-heat-bath combined with four overrelaxation sweeps, which we call an iteration. In Table 1 we list the numbers of configurations with the interval of 200 iterations for measurements. The anomalous quark masses m_{5q} for both gauge actions are calculated in Ref.[8]. Their values in the physical unit given in Table 1 are obtained by using the lattice spacing discussed above.

2.2 Calculational method

The ratios R_i are extracted from the plateau of

$$\frac{\sum_{\vec{z},\vec{y},\vec{x}} \langle \bar{K}^0(\vec{z}, N_t - 1) \mathcal{O}_i(\vec{y}, t) K^0(\vec{x}, 0) \rangle}{\sum_{\vec{x},\vec{y},\vec{z}} \langle K^0(\vec{z}, N_t - 1) \mathcal{O}_1(\vec{y}, t) K^0(\vec{x}, 0) \rangle}, \tag{2.1}$$

choosing $0 \ll t \ll N_t - 1$. Similarly B_K and B_i $(i = 2, \cdots, 5)$ are obtained from the three-point functions divided by two-point functions,

$$\frac{\sum_{\vec{z},\vec{y},\vec{x}} \langle \bar{K}^0(\vec{z}, N_t - 1) \mathcal{O}_i(\vec{y}, t) K^0(\vec{x}, 0) \rangle}{\frac{8}{3} \sum_{\vec{z},\vec{y}} \langle \bar{K}^0(\vec{z}, N_t - 1) A_0(\vec{y}, t) \rangle \sum_{\vec{y}',\vec{x}} \langle A_0(\vec{y}', t) K^0(\vec{x}, 0) \rangle} \quad \text{for} \quad B_K, \tag{2.2}$$

$$\frac{\sum_{\vec{z},\vec{y},\vec{x}} \langle \bar{K}^0(\vec{z}, N_t - 1) \mathcal{O}_i(\vec{y}, t) K^0(\vec{x}, 0) \rangle}{C_i \sum_{\vec{z},\vec{y}} \langle K^0(\vec{z}, N_t - 1) P(\vec{y}, t) \rangle \sum_{\vec{y}',\vec{x}} \langle P(\vec{y}', t) K^0(\vec{x}, 0) \rangle} \quad \text{for} \quad B_i. \tag{2.3}$$

In order to calculate the hadron correlation functions we solve quark propagators with the Coulomb gauge fixing employing wall sources placed at the edges of lattice where the Dirichlet boundary condition is imposed in the time direction.

We estimate errors by the single elimination jackknife procedure for all measured quantities.

2.3 Renormalization factors

In order to convert the matrix elements obtained on the lattice to those defined in the continuum, we make a one-loop perturbative matching of the lattice operators and the continuum ones at a scale $\mu = 1/a$, where the latter is defined in the $\overline{\rm MS}$ scheme with the naive dimensional regularization(NDR). Since the domain wall quarks retains the good chiral symmetry on the lattice, the four-quark operators are renormalized in the same way as in the continuum: multiplicative renormalization for $\mathscr{O}_1$, while mixing between $\mathscr{O}_2$ and $\mathscr{O}_3$ and between $\mathscr{O}_4$ and $\mathscr{O}_5$.

The one-loop perturbative renormalization factors with mean field improvement are written as[9]

$$\mathscr{O}_i^{\overline{\rm MS}}(\mu) = \frac{1}{(1-w_0)^2 Z_w^2} Z_{ij}(\mu a)\mathscr{O}_j(1/a)^{\rm lat}, \tag{2.4}$$

where w_0 is a function of domain wall height, Z_w is the renormalization factor of domain wall height and Z_{ij} are those of the four-quark operators. Indices i,j label the operator number. Numerical values of Z_{ij} with mean field improvement for the Iwasaki and the plaquette gauge actions are given in Ref.[10]. Employing $\beta = 6.0$, $a^{-1} = 2.12$GeV and $P = 0.59374$ for the plaquette gauge action we replace $M \to \tilde{M} = M + 4(u-1) = 1.311$ with $u = P^{1/4}$ and obtain

$$Z_{ij}^{\rm plaquette}(\mu = 1/a) = \begin{pmatrix} 0.7287 & 0 & 0 & 0 & 0 \\ 0 & 0.6845 & -0.00156 & 0 & 0 \\ 0 & 0.0352 & 0.8682 & 0 & 0 \\ 0 & 0 & 0 & 0.6325 & -0.0414 \\ 0 & 0 & 0 & -0.0689 & 0.7564 \end{pmatrix}. \tag{2.5}$$

For the Iwasaki gauge action we obtain $\tilde{M} = 1.420$ and

$$Z_{ij}^{\rm Iwasaki}(\mu = 1/a) = \begin{pmatrix} 0.8062 & 0 & 0 & 0 & 0 \\ 0 & 0.8124 & -0.00679 & 0 & 0 \\ 0 & 0.0156 & 0.9241 & 0 & 0 \\ 0 & 0 & 0 & 0.7847 & -0.0427 \\ 0 & 0 & 0 & -0.0477 & 0.8425 \end{pmatrix}. \tag{2.6}$$

for $\beta = 2.6$, $a^{-1} = 2.00$GeV, $P = 0.670632(10)$ and $R = 0.45283(2)$. The details of the mean field improvement for the domain wall fermion are explained in Refs.[9, 10]. We observe that the Iwasaki gauge action shows smaller one-loop corrections than the plaquette gauge action. For the axial vector current and the pseudoscalar density in B_K and B_i ($i = 2,\ldots,5$), we also use the one-loop perturbative renormalization factors with mean field improvement.

3. Results for R_i and B_i

Since the kaon matrix element of $\mathscr{O}_1$ is proportional to M_K^2, the ratios R_i in Eq.(1.6) should diverge toward the chiral limit. In order to keep the ratios finite in the chiral limit and tame the quark mass dependences of R_i, we introduce another definition:

$$\hat{R}_i(\mu) \equiv \frac{1}{(M_K^{\rm exp})^2}\left[m_M^2 \frac{\langle \bar{K}^0 | \mathscr{O}_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \mathscr{O}_1(\mu) | K^0 \rangle}\right]_{\rm lat}, \qquad i = 2,\ldots,5, \tag{3.1}$$

Table 2: Results for $\hat{R}_i$ and B_i extrapolated at the physical point with the quadratic fit. The renormalization scale is $\mu = 1/a$

	plaquette	Iwasaki	
$N_s \times N_t \times N_5$	$16^3 \times 32 \times 16$	$16^3 \times 32 \times 16$	$24^3 \times 32 \times 16$
$\hat{R}_1$	1	1	1
$\hat{R}_2$	$-18.22(69)(38)$	$-19.65(45)(21)$	$-18.97(16)(16)$
$\hat{R}_3$	4.74(17)(10)	5.21(12)(6)	5.039(41)(44)
$\hat{R}_4$	27.19(82)(24)	30.14(55)(11)	29.68(20)(13)
$\hat{R}_5$	8.11(24)(6)	9.01(16)(3)	8.794(58)(34)
B_1	0.561(13)	0.54361(65)	0.5233(55)
B_2	0.5603(80)(9)	0.5564(52)(10)	0.5369(17)(3)
B_3	0.731(11)(1)	0.7368(71)(15)	0.7124(24)(4)
B_4	0.691(13)(8)	0.7075(72)(31)	0.6996(22)(26)
B_5	0.616(12)(7)	0.6346(65)(25)	0.6226(19)(24)

where m_M is the pseudoscalar meson mass on the lattice and $M_K^{\rm exp}$ is the experimental kaon mass.

In Figure 1 we plot the results for $\hat{R}_i$ as a function of m_M^2. We observe little volume dependence for the Iwasaki gauge action. The results of the Iwasaki and the plaquette gauge actions show 10% discrepancies, which could be $O(a^2)$ effects due to the difference of the gauge actions. We extrapolate the results for $\hat{R}_i$ at the physical kaon mass, which is located around $m_f a \sim 0.02$, employing the quadratic and the logarithmic fitting functions:

$$\hat{R}_i = b_0 + b_1 m_M + b_2 (m_M)^2, \tag{3.2}$$

$$\hat{R}_i = b_0 + b_1 m_M + b_2 m_M \log(m_M). \tag{3.3}$$

The results are depicted in Figure 1, where both fitting functions give consistent results at the physical point. Table 2 summarizes the values of $\hat{R}_i$ extrapolated at the physical point with the quadratic fitting function. The first errors are statistical. The second denote differences of the central values for the two fitting functions, which are considered to be the systematic errors. We check numerically that the uncertainties due to a choice of the fitting functions are smaller than the statistical errors. The results for $\hat{R}_i$ show a striking feature that the non-SM matrix elements are much larger than the SM ones for both gauge actions. Especially, the magnitudes of the matrix elements $\langle \bar{K}^0 | \mathscr{O}_2(\mu) | K^0 \rangle$ and $\langle \bar{K}^0 | \mathscr{O}_4(\mu) | K^0 \rangle$ are one order larger than that of $\langle \bar{K}^0 | \mathscr{O}_1(\mu) | K^0 \rangle$.

In Figure 2 we also show the results for the B-parameters defined in Eqs.(1.7) and (1.8). Both gauge actions give consistent results ranging from $m_f a = 0.02$ to 0.06. We again find little volume dependence for the Iwasaki gauge action. In order to extrapolate B_K at the physical kaon mass, we use the following fitting function suggested by ChPT:

$$B_K = B(1 - 3c m_M \log(m_M) + b m_M). \tag{3.4}$$

As for B_i ($i = 2, \ldots, 5$) we employ two fitting functions:

$$B_i = b_0 + b_1 m_M + b_2 (m_M)^2, \tag{3.5}$$

$$B_i = b_0 + b_1 m_M + b_2 m_M \log(m_M). \tag{3.6}$$

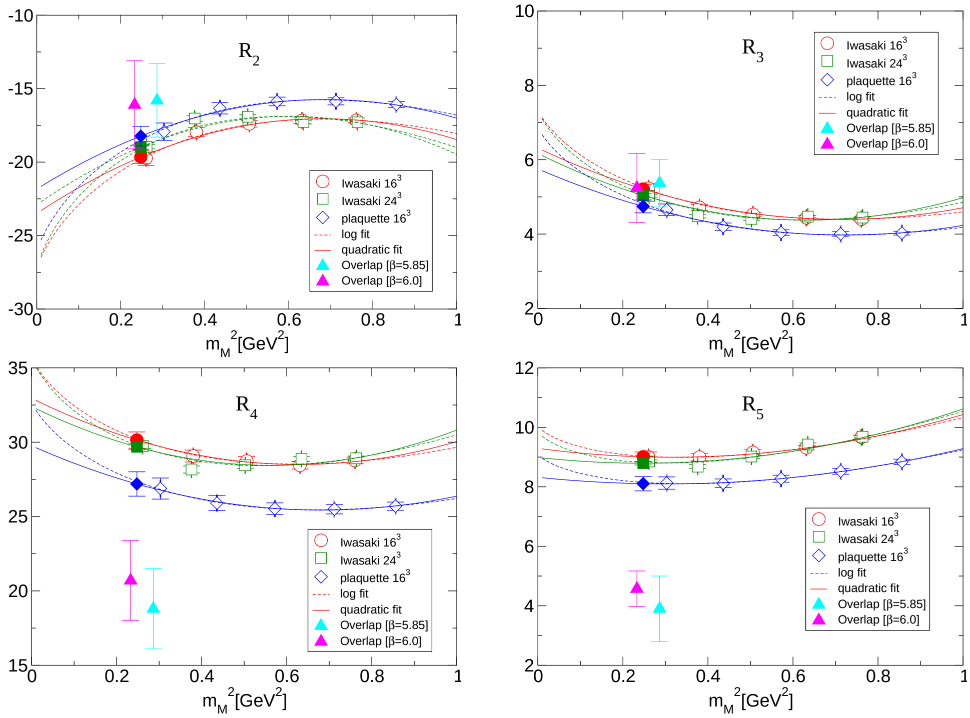

Figure 1: $\hat{R}_i$ as a function of m_M^2. Solid symbols denote the extrapolated values at $M_K^{\rm exp}$. The lattice spacing is fixed by the Sommer scale.

Figure 2 shows that the extrapolated values at the physical point with both fitting functions are consistent. This is also confirmed by the numerical values listed in Table 2, where the first errors are statistical and the second for differences of the central values for the two fitting functions.

4. Comparison with previous works

In Figure 1 we compare our results for R_i with those obtained using the overlap fermion and the plaquette gauge action[2]. The lattice spacing is fixed by the Sommer scale $r_0 = 0.5$fm. Both results show good consistencies for R_2 and R_3, while large deviations are observed for $R_4$and R_5.

As for the B-parameters we plot in Figure 2 the previous results with the overlap fermion[2] and the $O(a)$-improved Wilson quark action[3]. We again employ the Sommer scale to determine the lattice spacing. Large discrepancies are observed for all the B-parameters except B_5, though the previous results have rather large statistical errors.

Although the different choices of the quark and the gauge actions should allow the $O(a^2)$ uncertainties, the magnitudes of the inconsistencies between the previous results and ours are more than expected. We suspect that a possible source of discrepancies is difference of the renormalization methods: one-loop perturbation for our results and nonperturbative MOM scheme for the previous ones. We are now working on a nonperturbative renormalization for the domain wall fermion using the Schrödinger functional method[11].

This work is supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology (Nos. 13135204, 13135216, 15540251,

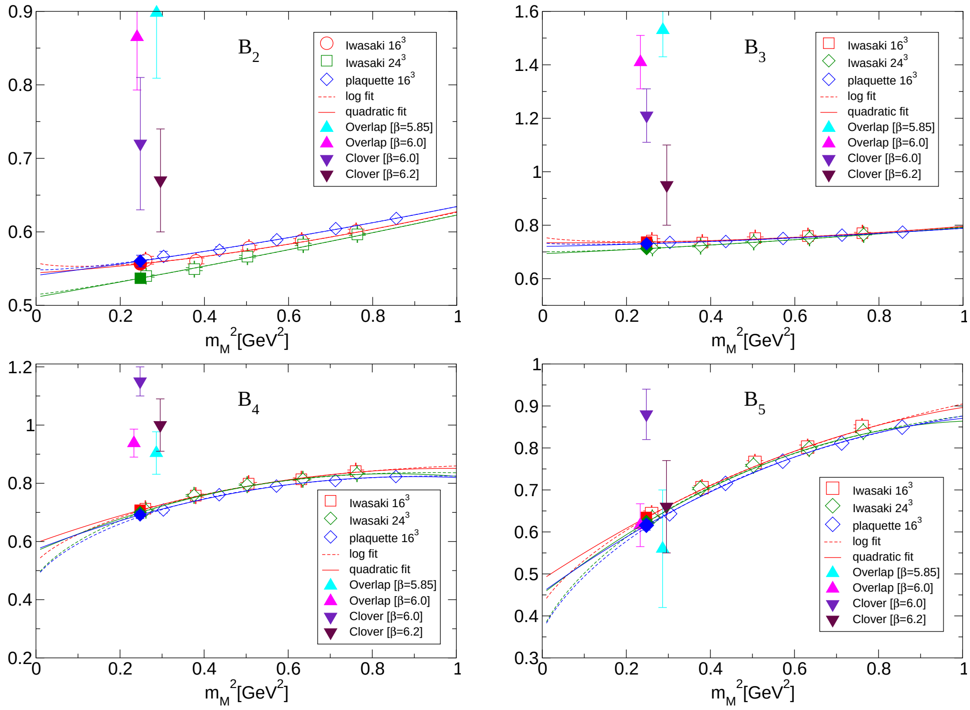

Figure 2: Bag parameters B_i as a function of m_M^2.Solid symbols denote the extrapolated values at $M_K^{\rm exp}$. The lattice spacing is fixed by the Sommer scale.

16540228, 17340066, 17540259, 17740171, 18104005, 18540250, 18740130, 18740139).

References

[1] F. Gabbiani, A. Masiero, Nucl. Phys. **B322** (1989) 235; J. S. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. **B415** (1994) 293; F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. **B477** (1996) 321.

[2] R. Babich *et al.*, hep-lat/0605016.

[3] C. R. Allton *et al.*, Phys. Lett. **B453** (1999) 30.

[4] CP-PACS Collaboration, J. Noaki *et al.*, Phys. Rev. **D68** (2003) 014501.

[5] S. Necco and R. Sommer, Nucl. Phys. **B622** (2002) 328.

[6] CP-PACS Collaboration, S. Takeda *et al.*, Phys. Rev. **D70** (2004) 074510.

[7] CP-PACS Collaboration, A. Ali Khan *et al.*, Phys. Rev. **D64** (2001) 114506.

[8] CP-PACS Collaboration, A. Ali Khan *et al.*, Phys. Rev. **D63** (2001) 114504.

[9] S. Aoki, T. Izubuchi, Y. Kuramashi and Y. Taniguchi, Phys. Rev. **D59** (1999) 094505; *ibid.* **D60** (1999) 114504; *ibid.* **D67** (2003) 094502.

[10] Y. Nakamura and Y. Kuramashi, Phys. Rev. **D73** (2006) 094502.

[11] Y. Taniguchi, hep-lat/0604002.