SU(3) **breaking effects in hyperon beta decay from lattice QCD**

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We present results of an exploratory study of flavor *SU*(3) breaking effects in hyperon beta decays using domain wall fermions. From phenomenological point of view, the significance of this subject is twofold: (1) to extract the element V_{us} of the Cabibbo-Kabayashi-Maskawa mixing matrix from the $\Delta S = 1$ decay process, and (2) to provide vital information to analysis of the strange quark fraction of the proton spin with the polarized deep inelastic scattering data. In this study, we explore the $\Xi^0 \to \Sigma^+$ beta decay, which is highly sensitive to the *SU*(3) breaking since this decay corresponds to the direct analogue of neutron beta decay under an exchange between the down quark and the strange quark. We expose the *SU*(3) breaking effect on $g_A/g_V = g_1(0)/f_1(0)$ up to the first order in breaking. The second-class form factors g_2 and f_3, of which non-zero values are the direct signals of the *SU*(3) breaking effect, are also measured. Finally, we estimate $f_1(0)$ up to the second-order correction and then evaluate $|V_{us}|$ combined with the KTeV experiment.

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1. Introduction

The octet baryons $(p, n, \Lambda, \Sigma, \Xi)$ admit various semileptonic decays (beta decays):

$$B_1 \rightarrow B_2 + e^- + \bar{\nu}_e \tag{1.1}$$

which are described by the vector and axial-vector transitions

$$\langle B_2 | V_\alpha(x) + A_\alpha(x) | B_1 \rangle = \bar{u}_{B_2}(p') \left(O_\alpha^V(q) + O_\alpha^A(q) \right) u_{B_1}(p) e^{-iq \cdot x}, \tag{1.2}$$

where $q \equiv p - p'$. Six form factors are needed to describe the hyperon beta decays: the vector, weak magnetism and induced scalar form factors for the vector current,

$$O_\alpha^V(q) = \gamma_\alpha f_1(q^2) + \sigma_{\alpha\beta} q_\beta \frac{f_2(q^2)}{M_{B_1}} - iq_\alpha \frac{f_3(q^2)}{M_{B_1}} \tag{1.3}$$

and the axial-vector, weak electricity and induced pseudo-scalar from factors for the axial current,

$$O_\alpha^A(q) = \gamma_\alpha \gamma_5 g_1(q^2) + \sigma_{\alpha\beta} q_\beta \gamma_5 \frac{g_2(q^2)}{M_{B_1}} - iq_\alpha \gamma_5 \frac{g_3(q^2)}{M_{B_1}}, \tag{1.4}$$

which are here given in Euclidean space by performing the Wick rotation from Minkowski space with γ-matrix conventions adopted in Refs. [1, 2]. The forward limits of the vector and the axial-vector form factors are well known as $g_V = f_1(0)$ and $g_A = g_1(0)$. Form factors f_3 and g_2 are known as the second-class form factors, which are identically zero in the certain symmetric limit (iso-spin symmetry, U-spin symmetry or V-spin symmetry as $SU(2)$ subgroups of the flavor $SU(3)$ symmetry). For an example, the second-class form factors in neutron beta decay are prohibited to be non-zero values because of G-parity conservation in the iso-spin symmetry limit [1].

From phenomenological point of view, the various ratios g_A/g_V in the hyperon beta decays provide vital information to analysis of strange quark spin fraction inside the proton [3]. However, the flavor $SU(3)$ breaking effects are simply neglected in such analysis [3], while sizeable effects are to be expected in general. Although there are various hyperon beta decays, we are especially interested in the $\Xi^0 \rightarrow \Sigma^+$ beta decay which is the direct analogue of neutron beta decay under an exchange between the down quark and the strange quark. If the flavor $SU(3)$ symmetry is manifest, the ratio g_A/g_V should be identical to that of neutron beta decay. In other words, this particular decay is highly sensitive to the $SU(3)$ breaking. Indeed, some model (center-of-mass correction approach [4]) and the $1/N_c$ expansion approach [5] predict that the $(g_A/g_V)_{\Xi\Sigma}$ is smaller than the $(g_A/g_V)_{np}$ by 8-10% and 20-30% respectively. However, the first and single experiment done by the KTeV collaboration at FNAL showed no indication of the $SU(3)$ breaking effect on $g_1(0)/f_1(0) = g_A/g_V$ [2]. The KTeV experiment also reported no evidence for a non-zero second-class form factor g_2 [2].

The $\Delta S = 1$ beta decay processes such as the $\Xi^0 \rightarrow \Sigma^+$ beta decay are also applicable to determine the element V_{us} of the Cabibbo-Kabayashi-Maskawa mixing matrix other than the K_{l3} decays [7]. Experimentally, the product $|V_{us} f_1(0)|^2 (1 + 3|g_1(0)/f_1(0)|^2)$ is obtained from the decay branching ratio and the value of $g_1(0)/f_1(0)$ is determined by the measured asymmetries. A theoretical input of $f_1(0)$, which is protected by the Ademollo-Gatto theorem against the first-order correction in breaking [6], is necessary to evaluate $|V_{us}|$ [7]. Although theoretical accurate

estimate of $f_1(0)$ are highly required for the precise determination of $|V_{us}|$, even a sign of the second-order correction, is somewhat controversial at present. The quark-model calculations show $f_1(0)/(f_1(0))_{SU(3)} < 1$, while heavy baryon chiral perturbation theory (HBChPT) and large N_c analysis, both predict $f_1(0)/(f_1(0))_{SU(3)} > 1$ [8].

In this study, we try to expose the flavor $SU(3)$ breaking effects in the hyperon beta decay from the theoretical first principle calculation.

2. Computational methodology

First of all, we define the finite-momentum three-point functions for the relevant components of either the local vector current ($\Gamma_\alpha = V_\alpha$) or the local axial current ($\Gamma_\alpha = A_\alpha$) with the interpolating operators $\mathscr{B}_1$ and $\mathscr{B}_2$ for the B_1 and B_2 states:

$$\langle \mathscr{B}_2(t',\mathbf{p}')\Gamma_\alpha(t,\mathbf{q})\overline{\mathscr{B}}_1(0,\mathbf{p})\rangle = G_\alpha^\Gamma(p,p') \times f(t,t',E_{B_1}(p),E_{B_2}(p')) + \cdots, \tag{2.1}$$

where the initial (B_1) and final (B_2) states carry fixed momenta $\mathbf{p}$ and $\mathbf{p}'$ respectively and then the current operator has a three-dimensional momentum transfer $\mathbf{q} = \mathbf{p} - \mathbf{p}'$. The ellipsis denotes excited state contributions which can be ignored in the case of $t' - t \gg 1$ and $t \gg 1$. We separate the correlation function into two parts: $G_\alpha^\Gamma(p,p')$ which is defined as

$$G_\alpha^\Gamma(p,p') = (-i\gamma \cdot p' + M_{B_2})O_\alpha^\Gamma(q)(-i\gamma \cdot p + M_{B_1}), \tag{2.2}$$

where $O_\alpha^\Gamma(q)$ corresponds to either Eq. (1.3) or Eq. (1.4), and the factor $f(t,t',E_{B_1}(p),E_{B_2}(p'))$ which collects all the kinematical factors, normalization of states, and time dependence of the correlation function.

In this study, the semileptonic hyperon decay process $B_1 \to B_2$ is measured at the rest frame of the B_2 state ($|\mathbf{p}'|^2 = 0$), which leads to $\mathbf{q} = \mathbf{p}$. Therefore, the squared four-dimensional momentum transfer is given by $q^2 = 2M_{B_2}(E_{B_1}(p) - M_{B_1}) - \Delta M^2$ where $\Delta M = M_{B_1} - M_{B_2}$ and $E_{B_1}(p) = \sqrt{M_{B_1} + \mathbf{p}^2}$. In the $SU(3)$ limit ($\Delta M = 0$), q^2 is always positive (space-like momentum). To extract the desired form factors, we take the trace of $G_\alpha^\Gamma(p,p'=0)$ with some appropriate projection operator $\mathscr{P}$ and then $\text{Tr}\{\mathscr{P}G_\alpha^\Gamma(p,p'=0)\}$ yields some linear combination of form factors in each Γ channel. We solve the simultaneous linear equations in terms of form factors at fixed $\mathbf{q}$.

3. Numerical results

We have performed quenched lattice calculations on a $L^3 \times T = 16^3 \times 32$ lattice with a renormalization group improved gauge action, DBW2 at $\beta = 6/g^2 = 0.87$ ($a \approx 0.15$ fm). Quark propagators were generated with three lighter quark masses $m_{ud} = 0.04$, 0.05 and 0.06 for up and down quarks and with two heavier quark masses $m_s = 0.08$ and 0.10 for the strange quark, using domain wall fermions (DWFs) with $L_s = 16$ and $M_5 = 1.8$. We refer to Ref. [9] for details of the quench DWF/DBW2 simulation. In this study, we take 5 different combinations between the up (down) quark and the strange quark as (m_{ud}, m_s)=(0.04, 0.08), (0.05, 0.08), (0.06, 0.08), (0.04, 0.10) and (0.05, 0.10), which yield different $SU(3)$ breaking patterns characterized by $\delta = (M_{B_1} - M_{B_2})/M_{B_1} = \Delta M/M_{B_1}$ in the range of 0.019 to 0.055.
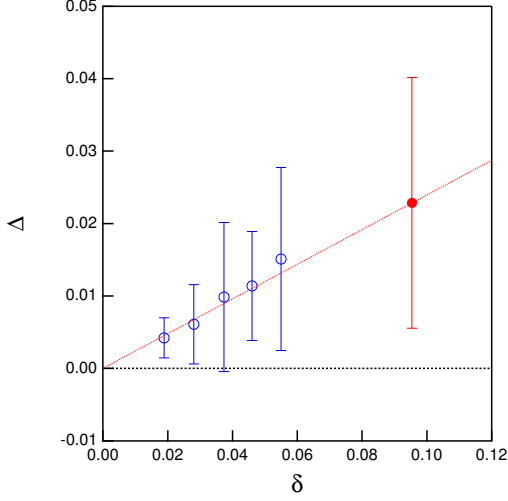
Figure 1: The deviation from unity in the double ratio $R(t)$ as a function of the $SU(3)$ breaking parameter $\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi}$.
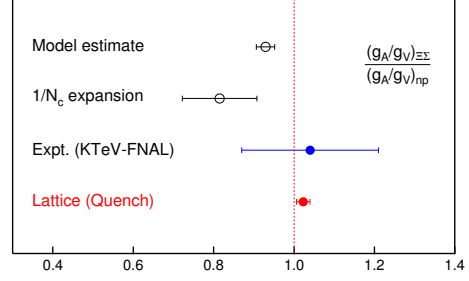
Figure 2: Comparison among model predictions, experimental data (KTeV-FNAL) and our lattice result for the ratio $\frac{(g_A/g_V)_{\Xi\Sigma}}{(g_A/g_V)_{np}}$.

Here we remark that the previous study of neutron beta decay with the same simulation parameters successfully yields a value of g_A/g_V as 1.212 ± 0.027, which just underestimates the experimental one by less than 5% [10]. This success encourages us to study the $SU(3)$ breaking effects in hyperon beta decay through comparison between neutron beta decay and the $\Xi^0 \to \Sigma^+$ beta decay.

3.1 First-order breaking in $g_1(0)/f_1(0)$

We first consider the following double ratio of zero-momentum three-point functions ($|\mathbf{q}|^2 = |\mathbf{p}|^2 = 0$) with fixed t:

$$R(t) = \frac{\text{Tr}[\mathscr{P}_5^k\langle \Sigma(t')\bar{u}(t)\gamma_5\gamma_k s(t)\overline{\Xi}(0)\rangle] \cdot \text{Tr}[\mathscr{P}^4\langle p(t')\bar{u}(t)\gamma_4 d(t)\bar{n}(0)\rangle]}{\text{Tr}[\mathscr{P}^4\langle \Sigma(t')\bar{u}(t)\gamma_4 s(t)\overline{\Xi}(0)\rangle] \cdot \text{Tr}[\mathscr{P}_5^k\langle p(t')\bar{u}(t)\gamma_5\gamma_k d(t)\bar{n}(0)\rangle]}, \quad (3.1)$$

where projection operators are defined as $\mathscr{P}^4 = \frac{1+\gamma_4}{2}\gamma_4$ and $\mathscr{P}_5^k = \frac{1+\gamma_4}{2}\gamma_5\gamma_k$ for the spatial index k. In this study, the polarization direction k is fixed to be along the z-axis. All the kinematical factors and normalization of states are exactly canceled out in this ratio. Here, we also take into account the relation between the lattice renormalizations of the local vector and axial-vector current operators as $Z_V = Z_A$ due to the good chiral property of DWFs [10]. As a result, $R(t)$ gives

$$R(t) \xrightarrow[t,(t'-t)\to\infty]{} \frac{g_1(q_{\max}^2) - \delta g_2(q_{\max}^2)}{f_1(q_{\max}^2) + \delta f_3(q_{\max}^2)} \bigg/ \left(\frac{g_1(0)}{f_1(0)}\right)_{SU(3)} = \frac{g_1(0)/f_1(0)}{(g_1(0)/f_1(0))_{SU(3)}} + \mathscr{O}(\delta^2), \quad (3.2)$$

where $q_{\max}^2 = -(\Delta M)^2$, which is proportional to δ^2. The q^2 dependence of form factors at $q^2 = q_{\max}^2$ can be involved in the second-order correction: $f_1(q_{\max}^2) = f_1(0) + \mathscr{O}(\delta^2)$ and $g_1(q_{\max}^2) = g_1(0) + \mathscr{O}(\delta^2)$. The non-zero value of the second-class form factors f_3 and g_2 starts from the first-order correction. The double ratio $R(t)$ exhibits the first order of the $SU(3)$ breaking effect on
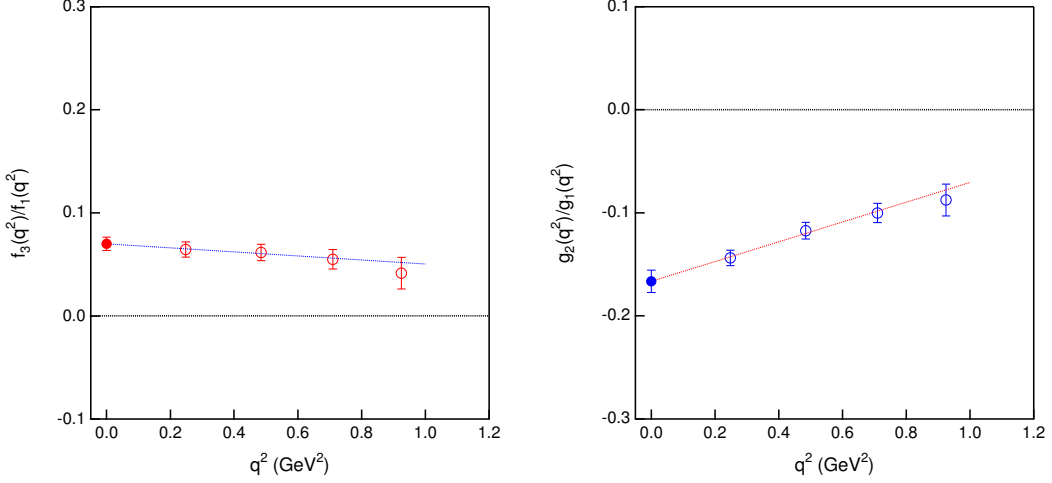
Figure 3: The ratios $f_3(q^2)/f_1(q^2)$ (left panel) and $g_2(q^2)/g_1(q^2)$ (right panel) as functions of q^2 as in the case of $(m_{ud}, m_s) = (0.05, 0.08)$.

$g_1(0)/f_1(0)$ since the axial-vector form factors are not protected by the Ademollo-Gatto theorem. The double ratio $R(t)$ becomes unity in the $SU(3)$ limit. Therefore, the effect of the first-order $SU(3)$ breaking on $g_A/g_V = f_1(0)/g_1(0)$ is exposed to this ratio by a deviation from unity. In other words, the double ratio can be described by $R(t) \rightarrow 1 + \Delta$ with the deviation Δ, which is expected to be proportional to the first-order correction of the $SU(3)$ breaking δ for small δ.

In Fig. (1), we plot the deviation from unity in the double ratio as a function of the $SU(3)$ breaking parameter δ. Surprisingly, it is observed that the deviation Δ is less than a few percents with $\sim 6\%$ $SU(3)$ breaking given among octet baryon masses. However, the deviation Δ is linearly correlated with the parameter δ as we expected. Thus, we may linearly extrapolate simulated Δ to the physical point at $\delta = (M_\Xi - M_\Sigma)/M_\Xi = 0.0954$. We finally obtain a value for the ratio of $(g_A/g_V)_{\Xi\Sigma}$ to $(g_A/g_V)_{np}$ as 1.023(18). Such tiny $SU(3)$ breaking effect on g_A/g_V in the $\Xi^0 \rightarrow \Sigma^+$ beta decay agrees well with the KTeV experiment where the value of 1.04(17) is measured. It is worth mentioning that our statistical error is at least one order smaller than that of the current experiment. In Fig. (2), we summarize our result and the experimental value combined with predictions from the center-of-mass correction approach [4] and the $1/N_c$ expansion approach [5]. A sign of our observed $SU(3)$ breaking correction is opposite to those predictions, although the first-order $SU(3)$ breaking on g_A/g_V is accidentally too small to justify neglect of the second-order correction in our analysis.

3.2 Second-class form factors

The kinematics of $|\mathbf{p}'|^2 = |\mathbf{p}|^2 = 0$ only allows the particular components of the vector (time component) or axial-vector currents (spatial components) to access the desired matrix element [10]. However, in the case if either $\mathbf{p}'$ or $\mathbf{p}$ are non-zero, three different types of three-point correlation functions are calculated in either channels:

$$\text{Tr}\{\mathscr{P}^4 G_4^V(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\}, \ \text{Tr}\{\mathscr{P}^4 G_{1,2,3}^V(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\}, \ \text{Tr}\{\mathscr{P}_5^3 G_{1,2}^V(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\} \quad (3.3)$$
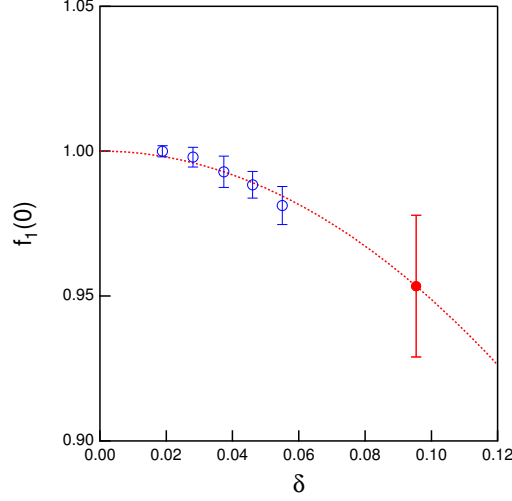
Figure 4: Extrapolation of $f_1(0)$ from simulated values (open circles) to the physical point (full circle) at $\delta = 0.0954$ using the simple fitting form as $f_1(0) = 1 + c_2\delta^2$ suggested by the Ademollo-Gatto theorem.

for the vector channel, and

$$\text{Tr}\{\mathscr{P}_5^3 G_3^A(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\}, \ \text{Tr}\{\mathscr{P}_5^3 G_{1,2}^A(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\}, \ \text{Tr}\{\mathscr{P}_5^3 G_4^A(\mathbf{p} \neq \mathbf{0}, \mathbf{p}' = \mathbf{0})\} \quad (3.4)$$

for the axial-vector channel. As a result, we can solve the simultaneous linear equations to get each form factor. In this study, we choose $|\mathbf{p}'|^2 = 0$ and $|\mathbf{p}|^2 = 1, 2, 3$ and 4 in units of $(2\pi/aL)^2$.

The second-class form factors f_3 and g_2 vanish in the $SU(3)$ flavor-symmetry limit and non-zero values are induced by the first-order correction. Although observation of non-zero second-class form factors corresponds to the direct signal of the $SU(3)$ breaking effects in hyperon beta decay, it is hard to measure those form factors in experiment. The KTeV experiment reported no evidence for a non-zero second-class form factor g_2, measuring $g_2(0)/f_1(0) = -1.7 \pm 2.0$ which corresponds to $g_2(0)/g_1(0) \simeq -1.4 \pm 1.7$.

Figs. 3 show ratios of $f_3(q^2)/f_1(q^2)$ (left panel) and $g_2(q^2)/g_1(q^2)$ (right panel) for the $\Xi^0 \to \Sigma^+$ beta decay as in the case of $(m_{ud}, m_s) = (0.05, 0.08)$ which yields $\delta = 0.028(1)$. We observe non-negligible values of both second-class form factors at simulated q^2 in the range of 0.25 to 0.93 GeV^2. The simplest linear form is adopted for the q^2 extrapolation of form factors to $q^2 = 0$ because of mild q^2 dependence. Furthermore, we linearly extrapolate measured $f_3(0)/f_1(0)$ and $g_2(0)/g_1(0)$ at five simulated points of δ to the physical point at $\delta = 0.0954$ and then obtain

$$\left(\frac{f_3(0)}{f_1(0)}\right)_{\Xi \to \Sigma} = +0.250(22), \quad \left(\frac{g_2(0)}{g_1(0)}\right)_{\Xi \to \Sigma} = -0.588(39), \quad (3.5)$$

which show firm evidence for non-zero second-class form factors.

3.3 Second-order correction on $f_1(0)$

The value of $f_1(0)$ for the semileptonic hyperon decays is equal to the $SU(3)$ Clebsh-Gordan coefficient up to the second order of the $SU(3)$ breaking, thanks to the Ademollo-Gatto theorem [6].

Here, in the case of the $\Xi^0 \to \Sigma^+$ process, it can be expressed by $f_1(0) = 1 + \mathcal{O}(\delta^2)$. Although theoretical accurate estimate of $f_1(0)$ are highly required for the precise determination of $|V_{us}|$, even a sign of the second-order correction, is somewhat controversial at present [8].

In Fig. 4, we plot our obtained $f_1(0)$ as a function of the $SU(3)$ breaking δ. The quadratic dependence of the $SU(3)$ breaking appears clearly. We extrapolate the measured values of $f_1(0)$ to the physical point at $\delta = 0.0954$ with a simple fitting form, $f_1(0) = 1 + c_2\delta^2$, according to the Ademollo-Gatto theorem and then obtain

$$f_1(0) = 0.953(24), \tag{3.6}$$

which indicates the negative correction of the second-order breaking on $f_1(0)$. By combining with a single estimate of $|V_{us}f_1(0)| = 0.216(33)$ from the KTeV experiment [2], we finally obtain

$$|V_{us}| = 0.219(27)_{\text{exp}}(5)_{\text{theory}}, \tag{3.7}$$

which is consistent with the value obtained from K_{l3} decays and the CKM unitary predicted value [11].

4. Summary

We have presented preliminary results of the flavor $SU(3)$ breaking effect in the $\Xi^0 \to \Sigma^+$ beta decay using quenched DWF simulations. We found a tiny first-order correction on g_A/g_V in agreement with the KTeV experiment. Although our result doesn't conflict with Cabibbo model fits [1], it doesn't mean that there is no $SU(3)$ breaking effect in hyperon beta decay. Indeed, we measured the non-zero value of the second-class form factors f_3 and g_2, which correspond to the direct evidence of the $SU(3)$ breaking effect. We finally observed that the second-order correction on $f_1(0)$ is negative. This leads to the closer value of $|V_{us}|$ to the value obtained from K_{l3} decays, while our observed tendency for the $SU(3)$ breaking correction is opposite against predictions of both HBChPT and large N_c analysis.

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