

Baryon correlators containing different diquarks from lattice simulations

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Point to point vacuum correlators containing diquarks in the color anti-triplet representation are computed both in the quenched approximation and dynamical overlap simulations with two flavors. The scalar, pseudoscalar and axial vector diquarks are combined with light quarks to form color singlets. The scalar ("good") diquark channel shows a stronger attraction than the axial vector ("bad") channel in the quenched data set. The pseudoscalar diquark channel shows a finite volume zero mode artifact: the correlator becomes negative at large distance when the quark mass is small. By separating configurations without zero modes from those with zero modes, we found that the zero modes have an important contribution in both the attraction in the scalar channel and the repulsion in the pseudoscalar channel. In the axial vector diquark channel, we did not find apparent zero mode effects.

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1. Introduction

It is possible that bound states of two quarks, diquarks, may be a component of the structure of hadrons. For a review of their phenomenology, see [1]. Although the constituent quark model has enjoyed a rather successful description of a huge body of hadronic states, it cannot explain all aspects of hadron phenomenology. One example is the paucity of exotics which are states beyond qqq baryons and $\bar{q}q$ mesons. As a special case: why do not a proton and a neutron in close contact merge into a single state? Recently, the possible existence of exotic particles [2, 3] such as the pentaquark [4] has motivated research, e.g. [5, 6, 7, 8, 9], to understand the role of diquarks in QCD.

In this work, we study diquarks by investigating the properties of point to point vacuum baryon correlators in quenched and two flavor dynamical overlap simulations. We calculate several current correlators in which two quarks are combined into scalar, pseudoscalar and axial vector diquarks in the color anti-triplet representation. Each correlator is normalized by its free field theory version so that we can see if there is attraction or repulsion in a specific channel. For each channel in the quenched data set, we separate the configurations with zero modes from those without zero modes. By comparing the correlators from the two groups, we can try to detect zero mode effects. The dynamical simulation is done with fixed topological charge $Q = 0$ or ± 1 . i.e., there is no zero mode or one zero mode with positive (negative) chirality in each data set. By comparing the correlators from different Q sectors, we can again examine the zero mode effects.

2. Methodology

Diquarks are not color singlets. To investigate their properties on the lattice, a diquark can be combined with a static (infinite heavy) quark to form a colorless state as was done in Refs. [7, 8]. In Ref. [11], gauge dependent diquark correlation functions were calculated in a fixed gauge, the Landau gauge, on the lattice. Whatever a diquark is, the environment it feels in a baryon with one heavy quark is different from that in a light baryon with three light quarks. Here we combine a light quark with a diquark with a specific quantum number in the color anti-triplet representation to get a color singlet and compute the point to point correlation function. By doing this, we cannot extract the mass splitting between spin 0 and spin 1 diquark states or the size of a diquark state as were done in Refs. [7, 8, 11] since the interaction between a diquark and a light quark also depends on the spin of the diquark. However, by comparing baryon correlators containing different diquarks, we can see which diquark is favored and which is not. We can also investigate the effects of the zero modes. Excess zero modes of Dirac operators are artifacts in quenched simulations.

The currents and correlation functions we considered are collected in Table 2. Here C is the charge-conjugation operator. We choose not to consider diquarks in the color sextet representation because they have much larger color electrostatic energy and thus are not favored phenomenologically [12]. In perturbative QCD, one-gluon exchange leads to attraction [13, 14] between two quarks in the color anti-triplet representation with $J^P = 0^+$. Instanton interactions [15, 16, 17] is also attractive in the scalar diquark channel. Because it is thought to be attractive, this channel is called the “good” diquark. We use the current J^5 in Table 2 to investigate this channel. In quenched simulations [18], the point to point scalar meson correlator was found to be negative, a quenching

J^P (diquark)	Color	Current	Correlator $R(x)$
0^+	$\bar{3}$	$J^5 = \varepsilon_{abc}[u^a C \gamma_5 d^b] u^c$	$\frac{1}{4} \text{Tr}[\langle \Omega T J^5(x) \bar{J}^5(0) \Omega \rangle x_\nu \gamma_\nu]$
0^-	$\bar{3}$	$J^I = \varepsilon_{abc}[u^a C d^b] u^c$	$\frac{1}{4} \text{Tr}[\langle \Omega T J^I(x) \bar{J}^I(0) \Omega \rangle x_\nu \gamma_\nu]$
1^+	$\bar{3}$	$J^3 = \varepsilon_{abc}[u^a C \gamma_3 d^b] u^c$	$\frac{1}{4} \text{Tr}[\langle \Omega T J^3(x) \bar{J}^3(0) \Omega \rangle x_\nu \gamma_\nu]$

Table 1: Currents and correlation functions

artifact. We are curious to see if we will see similar artifact in the pseudoscalar diquark channel by using J^I . (This effect is predicted by instanton liquid models.) The 1^+ diquark is called the ‘‘bad’’ diquark in the literature, since all models suggest that it is heavier than the scalar diquark.

The two point correlator for a current J is defined as $\langle \Omega | T J(x) \bar{J}(0) | \Omega \rangle$, where $|\Omega\rangle$ is the vacuum and T is the time order operator. For free massless quarks, the quark propagator in coordinate space takes the form (in Euclidean space)

$$\langle 0 | T q(x) \bar{q}(0) | 0 \rangle = \frac{1}{2\pi^2} \frac{x_\mu \gamma_\mu}{x^4}. \quad (2.1)$$

Here μ is summed over. Then for the current J^5 in Table 2, we have

$$\langle 0 | T J^5(x) \bar{J}^5(0) | 0 \rangle = -\frac{15}{4\pi^6} \frac{x_\mu \gamma_\mu}{x^{10}}. \quad (2.2)$$

Similarly, we can get the free correlators for other currents in Table 2. They are the same as the result in Eq.(2.2) except for a sign flip for J^I (for J^3 , $x_3 = 0$ is needed to get a same result). As was done in Ref. [19, 20], it is convenient to multiply the correlators with $x_\nu \gamma_\nu$ and take the trace in the Dirac indices to get a number for each of the currents. For example, from Eq.(2.2), we find $R_0(x) \equiv \frac{1}{4} \text{Tr}[\langle 0 | T J^5(x) \bar{J}^5(0) | 0 \rangle x_\nu \gamma_\nu] = -15/4\pi^6 x^8$. $R_0(x)$ is used to normalize the interacting correlator $R(x)$, i.e. we will examine the ratio $R(x)/R_0(x)$ for each current. In our lattice simulations, we use the free lattice correlators $R_0(x)$ to do the normalization to reduce lattice artifacts.

The point source for quark propagators is put on $(0, 0, 0, 0)$. The boundary condition in the time direction is anti-periodic, while in the space direction it is periodic. To avoid different boundary condition effects, we fix the time component n_t of x in $R(x)$ at zero, i.e. $x = (n_x, n_y, n_z, 0)$. We take all the diagonal points $(n, n, n, 0)$ and those lying approximately along the diagonal: $(n, n-1, n-1, 0)$ and $(n, n, n-1, 0)$.

Our quenched data set has 40 configurations with lattice size 16^4 and gauge coupling $\beta = 6.1$. The bare quark mass $am_q = 0.015, 0.025$ and 0.05 . The lattice spacing is 0.08 fm determined from the Sommer parameter. The dynamical data set has about 30 configurations for each bare quark mass $am_q = 0.015, 0.03$ or 0.05 and for each topological charge sector $|Q| = 0$ or 1 . The lattice size is 10^4 and $\beta = 7.2$. The lattice spacing is rather coarse: $a \sim 0.16$ fm.

3. Results and discussion

Fig. 1 shows the normalized correlation functions of the three currents from the quenched data set for quark mass $am_q = 0.015$ and 0.05 . As we can see, there is a strong attraction in the scalar diquark channel, which is similar to the attraction seen in the pseudoscalar meson channel in

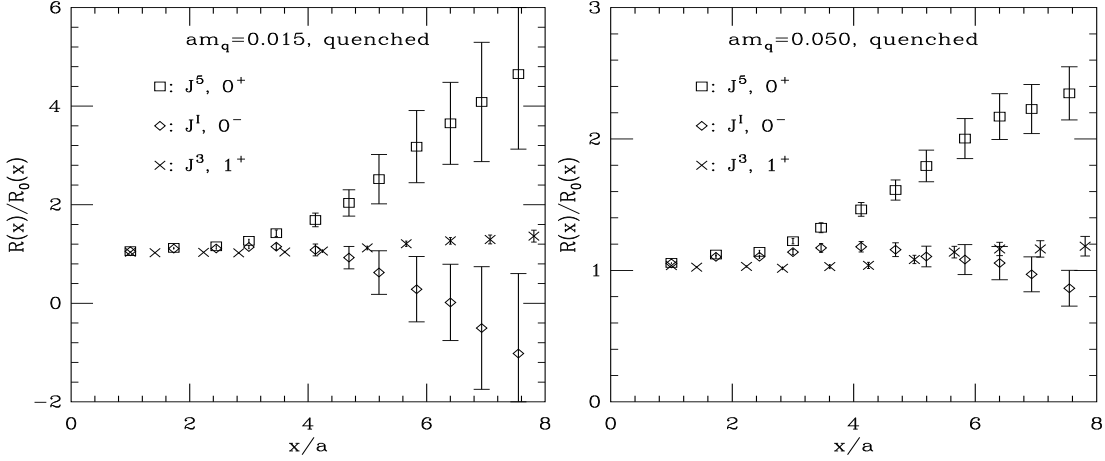


Figure 1: Comparison of the normalized correlation functions of the three currents from the quenched data set. The squares are for J^5 which contains a scalar (“good”) diquark structure. The diamonds are for J^1 which contains a pseudoscalar diquark structure. The crosses are for J^3 which contains an axial vector (“bad”) diquark structure.

Refs. [19, 21, 18]. Also we can see a quark mass dependence in this channel. As the quark mass decreases, the attraction increases. Since our lattice spacing a is 0.08 fm, the region we are looking at is $0 \text{ fm} < x < 0.64 \text{ fm}$. The correlator for J^1 has a tendency to go negative at large distance as the quark mass decreases. This behavior is very similar to what observed for the correlator of the scalar meson in Ref. [18]. It was argued in Ref. [18] that zero modes are the source of the negativity. We will investigate the zero mode contributions in all three channels later. The correlator for J^3 is flat and stays close to one. This means that there is little correlation among quarks in this channel.

To see zero mode effects, we separate the 40 configurations in the quenched data set into two groups: 35 of them with topological charge $Q \neq 0$ and the other 5 with $Q = 0$. i.e. the first group contains zero modes while the second one does not. From each group we compute the correlation functions. The results are compared in Fig. 2 for quark mass $am_q = 0.015$. The attraction in the current J^5 at large x is mainly from zero mode contributions. The repulsion in the current J^1 also has a big contribution from zero modes. At quark mass $am_q = 0.05$, the correlators from the two groups agree with each other within error bars (no graph is shown here). It is not surprising that zero mode effects become large at small quark masses since the zero mode contribution to the quark propagator is proportional to the inverse of the quark mass. For the current J^3 , we do not see apparent zero mode effects. In Fig. 3 we compare the three correlators obtained from configurations without zero modes. They are not so different as in Fig. 1 when there is no zero mode contribution. Nevertheless, it seems that there is more attraction in channel J^5 and J^1 than in channel J^3 around $x \sim 4a = 0.32 \text{ fm}$.

The results from the two flavor dynamical simulation cannot be compared to the quenched results directly because the dynamical simulation is done with fixed topological charge $|Q| = 0$, or 1. Thus we will only compare the dynamical results from different Q sectors to investigate the zero mode effects.

The comparisons of the correlators from the two topological sectors are given in Fig. 4. There is no difference within error bars between the correlators from the two sectors for both current J^5

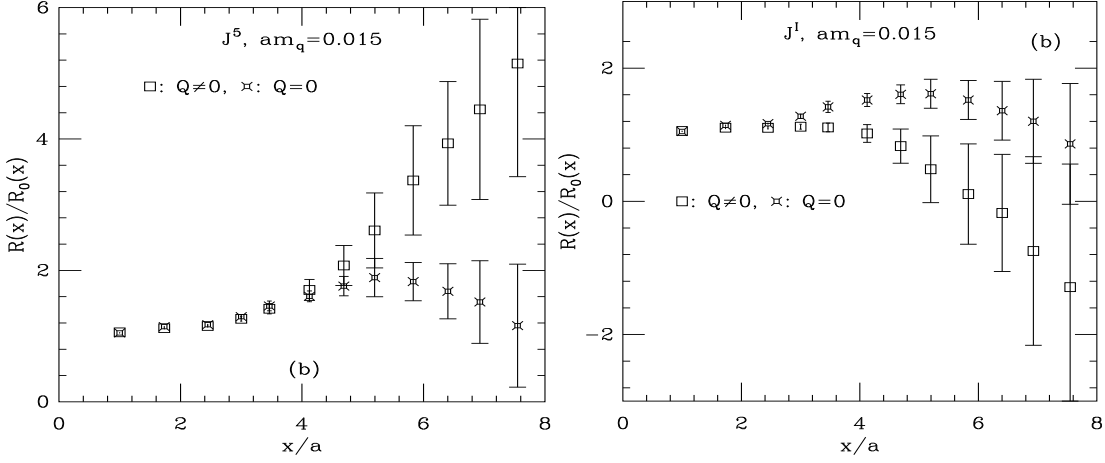


Figure 2: Comparison of correlators from the topological charge $Q \neq 0$ sector (squares) and the $Q = 0$ sector (fancy squares) in the quenched data set. The graph on the left is for J^5 , the one on the right is for J^1 .

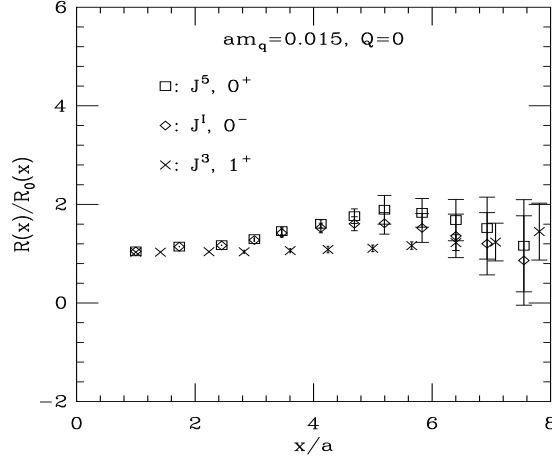


Figure 3: Comparison of the correlators of the three currents from the topological charge $Q = 0$ sector in the quenched data set. The three channels are not so different when there is no zero mode contribution. It seems that there is more attraction in channel J^5 and J^1 than in channel J^3 around $x \sim 4a = 0.32$ fm.

and J^1 . Similarly, no difference is seen for the current J^3 . In Fig. 2, the $Q \neq 0$ sector of the quenched simulation contains not only configurations with $|Q| = 1$ but also those with $|Q| > 1$. To compare with the dynamical simulation results here, we pick out the $|Q| = 1$ and $Q = 0$ configurations and calculate the correlators again. The results are given in Fig. 5. The difference between the two sectors is small, just like the dynamical results in Fig. 4. Comparing Fig. 5 with Fig. 2, we see that the attraction in the J^5 channel and the repulsion in the J^1 channel in the quenched simulation are mainly from configurations with big topological charge: $|Q| > 1$.

4. Conclusions

The quenched simulation shows that the attraction in the J^5 channel is the strongest. J^5 contains a scalar diquark structure. By comparing the correlators from different topological sectors, we

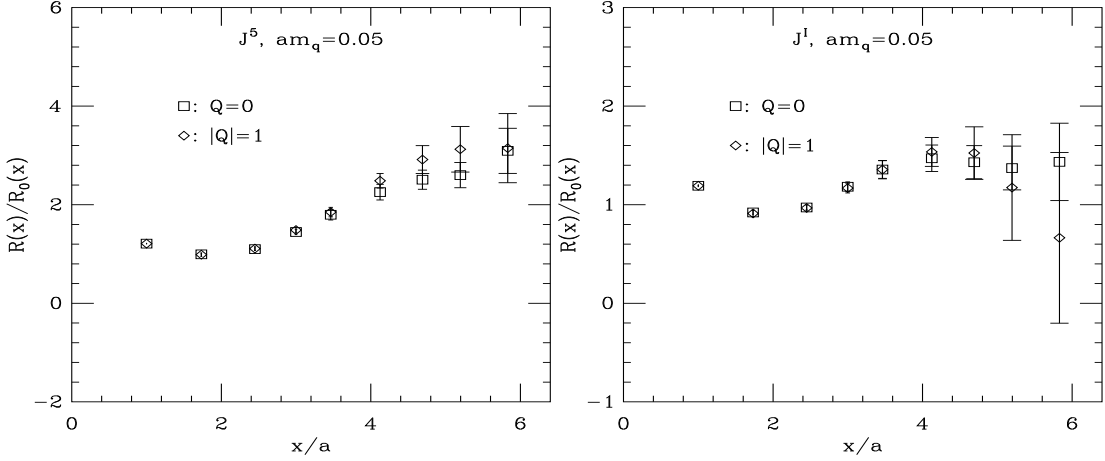


Figure 4: Comparison of correlators from the topological charge $Q = 0$ sector (squares) and the $|Q| = 1$ sector (diamonds) from two flavor dynamical simulations. The graph on the left is for J^5 , the one on the right is for J^I . There is no difference within error bars between the correlators from the two sectors.

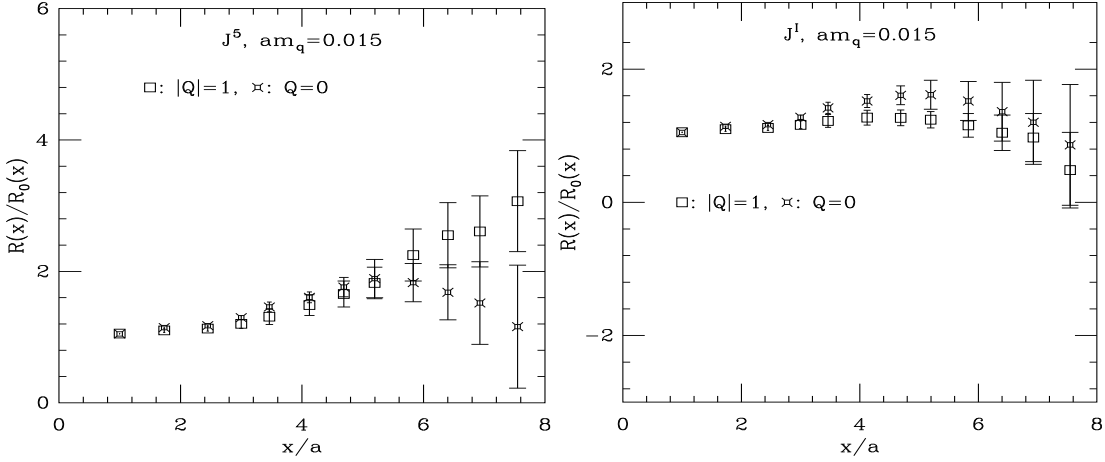


Figure 5: Comparison of correlators from the topological charge $Q = 0$ sector (fancy squares) and the $|Q| = 1$ sector (squares) from the quenched data set. The graph on the left is for J^5 , the one on the right is for J^I . The difference between the correlators from the two topological sectors is small.

found that both the attraction in the scalar diquark channel and the repulsion in the pseudoscalar diquark channel have big contributions from configurations with more than one zero mode. The correlators for the currents J^5 and J^I obtained from configurations without zero modes in the quenched data set are similar to each other. Both of them seem to have more attraction around $x \sim 4a = 0.32$ fm than the correlator for the current J^3 which contains an axial vector diquark structure. The correlators from the two flavor dynamical data set show no difference between the $Q = 0$ and $|Q| = 1$ sector. This confirms that zero mode effects are mainly from configurations with big topological charges. In this respect, quenched and full QCD in sectors of low Q are not too different.

In full QCD, small quark masses suppress high Q configurations. Since they are absent there, we suspect that different diquarks in light baryons are not that different. At a minimum, diquark contributions in quenched QCD (*not filtered by Q*) and full QCD are different, and results from

quenched simulations may be misleading.

Acknowledgments

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